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Inductive construction of log flips in terms of division algorithms,part I

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1. Introduction

Let $f : X \longrightarrow Y$ be o projective morphism from a threefold X with only terminal singularities to a normal threefold Y and $Q \in Y$ such that $C = f^{-1}(Q)$ is a curve and $-K_X$ is f-ample.

1. For arbitrary small opne set $U \ni Q$, call $f^{-1}(U) \supset C \to U \ni Q$ an *extremal neighborhood*(or,*extremal nbd*,for short).It is said to be *flipping* (resp.*divisorial*) if the exceptional setb is a curve (resp. a divisor).An extremal nbd is said to be *irreducible* if C id irreducible.

2. Fact: a general member D of $| -K_X |$ has only Du Val singularities.The 6 types of irreducible extremal nbd are $k1A, k2A, cD/3, IIA, IC, kAD$ according to the singularities of D([1],(2.2)). The first two (or ,the last four) are said to be semistable (resp.exceptional).

3.(V.V.Shokurov,[3]) Reduction from the existence of the limitting or special log flips with indices greater than two or with the "types" to the existence of the index two exceptional special log flips in order to prove the existence of 3-fold log flips:

i) ([3],(6.1)) A small projective birational contraction f of a connected curve is *limitting* for a log canonical divisor $K + S + B$ if the following conditions hold:

a) $K + S$ is stricly log terminal;

b) S is an irreducible surface that intersects the connected curve and is nonpositive relative to f;

c) every irreducible components of the fractional part $\{B\}$ is negative relative to f;

d) the log divisor $K + S + B$ is negative to f;

e) in a neiborghhood of the contrcted curve,$K + S + B'$ is not log canonical for any $B' > B$ with the same support as B.

Moreover, f is called *special* if in additon f is extremal,and

f) B is integral,that is, $\{B\} = 0$;

g) $K + S + B$ is stricly log terminal.

ii) ([3],(1.10);weak form) A small proper morhphism $f : X \to Z$ of an algebraic 3-fold X that is finite ober the generic point of Z, has a stricly log terminal model for $K + B$,even if $X'$is not Q-factorial and $K + S + B$ not log canonical.

iii) Reduction(([3],(6.5)).The above ii) are implied by the existence of special flips,and even by the existence of special flips of the types (1),(2) as below.

iv) Types of special flips ([3],(6.6)):

(1) $B = 0$ and S is an irreducible sirface negative relative to f.

(2) $S + B = S_1 + S_2$ is the sum of two irreducible surfaces S_1 and S_2 negative relative to f.

v) ([3],(6.7)) Flips of type (2) exit.

Note: Reduction iii) $\Longleftrightarrow$ the existence of two irreducible divisors in the non-log case([1],(3.12)).

2. Division algorithm(partly)([1],(2.8),(2.9))

Example([1],(2.1)): Let $f : X \supset C(\simeq \mathbb{P}^1) \to Y \ni Q$ be an extremal nbd of type k2A with two terminal singular points P_1, P_2 of indices $m_1, m_2 > 1$ and axial mutiplicities $\alpha_1, \alpha_2 \geq 1$,respectively.

Theorem 1 ([1],(2.2)):Let U_i be the $\mathbb{Z}_{m_i}$-quotient of a "hypersurface" of $\mathbb{C}^4$, $U_i := (\xi_i, \eta_i, \zeta_i, u; \xi_i\eta_i = g_i(\xi_i^{m_i}, u))/\mathbb{Z}_{m_i}(1, -1, a_i, 0)$, where a_i is an integer $\in [1, m_i]$ prime to m_i, and $g_i(T, u) \in \mathbb{C}[[u]][T]$ is a monic polynnomial in T of degree $(=: \rho_i)$, such that $g_i(\xi_i^{m_i}, u)$ is square-free. Let $P_i := 0$ and $C_i := \xi_i - axis/\mathbb{Z}_{m_i}$,

$\Longrightarrow$ U_i is defined to be a formal scheme along $C_i \simeq \mathbb{C}^1$ with only terminal singularities.and the associated some patching conditions hold.

Note: A critrion for a divisorial contration in division algorithm $\Longleftrightarrow$ *permissible pencils of exceptional rational curves*,alternatively.(M.Miyanishi and S.Tsunoda,[2],or Y.Kawamata).

3. Reduction by division algorithm to the exceptional cases from semistable cases

Proposition 1. As the notations and conditions as Theorem1. Let (X,S+B) the log pair satisfying (exceptional) special flip conditions,S ths sub-boundary consits with irreducible and reduced prime divisors,B the subboundary consists with prime divisors with multiplicities < 1,and $B = B_1 + B_2$ (type $t = 2$).

Moreover,the intersection of B_1, B_2 contains an extremal curve C.And C has two terminal singular points P_1, P_2 of indicies $m_1, m_2 > 1$ and $n_1, n_2 > 1$, and axial multiplicities $\alpha_1, \alpha_2 \geq 1$, $\beta_1, \beta_2 \geq 1$ along $(B_1)_{|C}$, $(B_2)_{|C}$, respectively, as cited section2,

Then $D \in | - K_X|$ be a Du Val member,with similar datum as Theorem 1.

Moreover,consider the "$\mathbb{Z}_{m_i} \times \mathbb{Z}_{n_i}$"-quotient of "hypersurface" of $\mathbb{C}^4$ as follows:

$U_i := \dfrac{(\xi_i, \eta_i, \zeta_i, u; \xi_i\eta_i = g_i(\zeta_i^{m_i}, u)h_i(\zeta_i^{n_i}, u))}{\mathbb{Z}_{m_i}(1, -1, a_i, 0) \times \mathbb{Z}_{n_i}(1, -1, b_i, 0)}$, then the similar patching conditions as Thereom 1 are hold.(*to be continued.*)

References

[1] S.Mori, *On semistable extremal neighborhoods*, Adv.Stud.Pure Math.v.35(2002),157-184.

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[3] V.V.Shokurov, *3-fold log flips*,Math.USSR Izv.,**26** (1993),95-202.