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1 Introduction

Recently,A.Bayer,A.Bertram,E.Macri,and Y.Toda [2] deduced the Bogomolov-Gieseker type inequalities [3] on 3-folds in the context of stability conditions by using the techniques of tilting complexes,in these inequalities which have special aspects that they have not necessary the 3rd Chern classes $c_3,\text{also have it on several cases.}$On the hands, it is very well-known that the moduli spaces of stable vector bundles are determined by the 1st or 2nd Chern classes $c_1,c_2$ in spite of the dimension of the fixed base variety,by the results of M.Maruyama.Moreover,some kinds of higher dimensional analogues of Miyaoka-Yau type inequalities on the higher dimensional varieties of general type have been expected to be needed for considering the abundance conjecture for higher dimensional cases,as remarked by Y.Kawamata,Y.Miyaoka,N.Nakayama,after the proof of the existence of the minimal models for 3-folds established by S.Mori. In this paper,we will try to deduce some kinds of higher dimensional analogues of Bogomolov-Gieseker type inequality with regards to the above results and remarks in terms of a little "alternative" view point (of particle physics literature),as a test case,whose methods are like a "positivity constrains" given by D.Anselmi et al [1] based on Zamolodchikov’s c-theorem [4].

2 Trace anomaly coefficients with positivity constrains and Zamolodchikov’s c-theorem

We will briefly review some of anomalies and positivity constrains by taking the trace anomaly for a four-dimensional field theory with flavor currents and stress tensor [1]. For the covariant current (cf.Kazuhiro Fujikawa:Path Integral and Quantum Anomalies,Iwamnari shoten (2001))

\[ J^{\mu}(x) = \lim_{\Delta x \to 0} [\text{Tr}(T^{\mu} T^{\nu} (\frac{1}{2}\Omega_2) (D^2/M^2)^{1/2} \delta(x-y))] \],

the trace anomaly (as a 2-form) of a supersymmetric gauge theory containing chiral superfields $\Phi^{\mu}$ in irreducible representations $R_i$ of the gauge group $G$,with conserved current $J_{\mu}(x)$ for a non-anomalous flavor symmetry $F$ of the theory,and with adding a aource $B_{\mu}(x)$ for the current and also with $U(1)$ symmetry,contains

\[ \Theta = \frac{1}{2\pi^2} \tilde{\beta}(g)(F^{\mu}_{\nu})^2 + \frac{1}{32\pi^2} \tilde{b}(g)B^{\mu}_{\nu} B^{\nu}_{\rho} + \frac{\tilde{c}(g)}{16\pi^2} (W^{\mu\rho\nu\sigma})^2 - \frac{a(g)}{16\pi^2} (R^{\mu\nu})^2 + \frac{\tilde{c}(g)}{6\pi^2} V^{\mu}_{\nu}, \]

where $W^{\mu\rho\nu\sigma}$ is the Weyl tensor, $R^{\mu\nu}$ is the dual of the curvature, and $B^{\mu}_{\nu},V^{\mu}_{\nu}$ are the field strengths of $B_{\nu},V_{\nu}$, respectively. All anomaly coefficients are depend on the coupling $g(\mu)$ at renomalization scale $\mu$.The first term is the internal trace anomaly,where $\tilde{\beta}(g)$ is the numerator of the NSVZ (Novikov-Shifman-Vainshtein-Zakharov) beta function

\[ \tilde{\beta}(g(\mu)) = -\frac{g^3}{16\pi^2} [3T(G) - \sum_i T(R_i)(1 - g(\mu))], \]

where $T(G),T(R_i)$ are the Dynkin indicies of the adjoint representation of $G$ and the representaton $R_i$ of the chiral superfield $\Phi^{\mu}$,respectively, and $g/2$ is the anomalous dimension of $\Phi^{\mu}$.The various external trace anomalies are contained in the three (central) coefficients $b(g),c(g)$ and $a(g).$The free field (i.e. one-loop) with values of $\tilde{c},\tilde{a}$ are already given as axial anomalies,for examples:

\[ c = \frac{1}{120} (12 N_1 + 3 N_{1/2} + N_0), \]
\[ a = \frac{1}{720} (124 N_1 + 11 N_{1/2} 2N_0), \]

where $N_0,N_{1/2},N_1$ are the numbers of scalars,Majorana spinors,and gauge vectors, respectively.

In supersymmetric gauge theory with $N_f = \text{dim}G$ gauge multiplets and $N_V$ chiral multiplets,these anomalies as in UV (ultraviolet) limit fixed point in free field theory are:

\[ c_{UV} = \frac{1}{24} (3 N_V + N_f), a_{UV} = \frac{1}{48} (9 N_V + N_f). \]

Zamolodchikov’s c-theorem [4] For stress tensors $T_{\mu\nu}$ and trace anomaly $\Theta$ on two-dimensional field theory, there exists a function $C(g(\mu))$ as a linear combination of the (suitable scaled) correlation functions $< T_{\mu\nu} T^{\mu\nu},< T_{\mu\nu} \Theta >$, $\Theta$ with the properties

\[ \partial \frac{\partial}{\partial g} C(g(\mu)) \geq 0, \quad \partial \frac{\partial}{\partial g} C(g(\mu))_{UV} = 0, \quad C(g^*) = c^*, \]

where $c^*$ is the Virasoro central chrage of the critical theory at the fixed point $g = g^*$. or, the fixed point value of the extremal trace anomaly coefficient $\Theta = \frac{1}{24\pi} c^* R$, where $R$ is the scalar curvature.The above thee properties imply $c_{UV} - c_{IR} > 0$.Along with $(N = 1 \text{ SUSY})$ QCD, Cardy’s conjecture asserts that there exist a universal c-theorem based on Euler anomaly,which implies $a_{UV} - a_{IR} > 0$.By the above results,for simplicity,if assuming that $a_{IR} = 0$ and $b_{IR} = 0$, then $a_{UV} = \lim_{x \to 0} g(1/x) = 0$, $b_{UV} = \lim_{x \to 0} g(1/x) = 0$, and $a_{UV},c_{UV}$ are given as above,we have a (very weak) positivity inequality for Chern classes being contributed by $a,c$ functions in the trace anomaly as follows:

Main Results If we expand the chern character for the trace anomaly $\Theta$ as before in 4-forms, with forgetting all the terms in $\Theta$ except for $a_{UV},c_{UV}$, $\chi_{aUV} := \text{ch}(\Theta) = r + \frac{1}{2\pi^2} R \Theta + \frac{1}{2\pi^2} Tr(\Theta) + \cdots \sim (e^{a_{UV} + c_{UV}})/(2\pi)^2$, where $r$ is the dimension of the representation of $G$,then $\chi_{aUV} > 0$.

References


