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Higher dimensional analogues of Bogomolov-Gieseker type inequality coming from anomaly cancellations based on positivity constrains with regards to abundance conjecture

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1 Introduction

Recently,A.Bayer,A.Bertram,E.Macri,and Y.Toda [2] deduced the Bogomolov-Gieseker type inequalities [3] on 3-folds in the context of stability conditions by using the techniques of tilting complexes,in these inequlities which have special aspects that they have not necessary the 3rd Chern classes c_3,also have it on several cases.On the hands, it is very well-known that the moduli spaces of stable vector bundles are determined by the 1st or 2nd Chern classes c_1, c_2 in spite of the dimension of the fixed base variety,by the results of M.Maruyama.Moreover,some kinds of higher dimensional analogues of Miyaoka-Yau type inequalities on the higher dimensional varieties of general type have been expected to be needed for considering the abundance conjecture for higher dimensional cases,as remarked by Y.Kawamata,Y.Miyaoka,N.Nakayama,after the proof of the existence of the minimal models for 3-folds established by S.Mori. In this paper,we will try to deduce some kinds of higher dimensional analogues of Bogomolov-Gieseker type inequality with regards to the above results and remarks in terms of a little "alternative" view point (of particle physics literature),as a test case,whose methods are like a "positivity constrains" given by D.Anselmi et al [1] based on Zamolodchikov's c-theorem [4].

2 Trace anomaly coefficients with positivity constrains and Zamolodchikov's c-theorem

We will briefly review some of anomalies and positivity constrains by taking the trace anomaly for a four-dimensional field theory with flavor currents and stress tensor [1]. For the covariant current (cf.Kazuo Fujikawa:Path Integral and Quantum Anomalies,Iwanami shoten (2001))

$J^{\alpha\mu}(x) = \lim_{y\to x}\{\mathrm{Tr}[T^a\gamma^\mu(\frac{1-\gamma_5}{2})f(D^2/M^2)\frac{1}{iD}\delta(x-y)]\}$, the trace anomaly (as a 2-form) of a supersymmetric gauge theory containing chiral superfields Φ_i^a in irreducible representations R_i of the gauge group G,with conserved current $J_\mu(x)$ for a non-anomalous flavor symmetry F of the theory,and with adiding a aource $B_\mu(x)$ for the current and also with $U(1)$ symmetry, contains

$$\Theta = \frac{1}{2g^3}\tilde{\beta}(g)(F^a_{\mu\nu})^2 + \frac{1}{32\pi^2}\tilde{b}(g)B^2_{\mu\nu} + \frac{\tilde{c}(g)}{16\pi^2}(W_{\mu\nu\rho\sigma})^2 - \frac{a(g)}{16\pi^2}(\tilde{R}_{\mu\nu\rho\sigma})^2 + \frac{\tilde{c}(g)}{6\pi^2}V^2_{\mu\nu},$$

where $W_{\mu\nu\rho\sigma}$ is the Wely tensor, $\tilde{R}_{\mu\nu\rho\sigma}$ is the dual of the curvature,and $B_{\mu\nu}, V_{\mu\nu}$ are the field strenghts of B_μ, V_ν,respectively.All anomaly coefficients are depend on the coupling $g(\mu)$ at renomalization scale μ.The first term is the internal trace anomaly,where $\tilde{\beta}(g)$ is the numerator of the NSVZ (Novikov-Shifman-Vainshtein-Zakharov) beta function

$$\tilde{\beta}(g(\mu)) = -\frac{g^3}{16\pi^2}[3T(G) - \textstyle\sum_i T(R_i)(1-\gamma_i(g(\mu)))],$$

where $T(G), T(R_i)$ are the Dynkin indicies of the adjoint representation of G and the representaton R_i of the chiral superfield Φ_i^α,respectively,and $\gamma_i/2$ is the anomalous dimension of Φ_i^α.The various extremal trace anomalies are contained in the three (central) coefficients $\tilde{b}(g), \tilde{(c)}(g)$ and $\tilde{a}(g)$.The free field (i.e. one-loop) with values of $\tilde{c}, a$ are already given as axial anomalies,for examples:

$$c = \frac{1}{120}(12N_1 + 3N_{1/2} + N_0),$$
$$a = \frac{1}{720}(124N_1 + 11N_{1/2}2N_0).$$

where $N_0, N_{1/2}, N_1$ are the numbers of scalars,Majorana spinors,and gauge vectors,respectively.

In supersymmetric gauge theory with $N_v = \dim G$ gauge multiplets and N_χ chiral multiplets,these anomalies as in UV (ultraviolet) limit fixed point in free field theory are:

$$c_{UV} = \frac{1}{24}(3N_V + N_\chi), a_{UV} = \frac{1}{48}(9N_V + N_\chi).$$

Zamolodchikov's c-theorem [4] For stress tensors $T_{\rho\sigma}$ and trace anomaly Θ on two-dimensional field theory, there exists a function $C(g(\mu))$ as a linear combination of the (suitable scaled) correlation functions $< T_{zz}T_{zz} >, < T_{zz}\Theta >$, $< \Theta\Theta >$ with the properties

$$\mu\frac{\partial}{\partial\mu}C(g(\mu)) \geq 0, \frac{\partial}{\partial g}C(g(\mu))|_{g=g^*} = 0, C(g^*) = c^*,$$

where c^* is the Virasoro central chrage of the critical theory at the fixed point $g = g^*$. or, the fixed point value of the extremal trace anomaly coefficient $\Theta = \frac{1}{24\pi}c^*R$, where R is the scalar curvature.The above thee properties imply $c_{UV} - c_{IR} > 0$.Along with ($N = 1$ SUSY) QCD, Cardy's conjecture asserts that there exist a universal c-theorem based on Euler anomaly,which implies $a_{UV} - a_{IR} > 0$.By the above results,for simplicity,if assuming that $a_{IR} = 0, c_{IR} = 0, b_{UV} = \lim_{x\to 0} b(g(1/x)) = 0, b_{IR} = \lim_{x\to\infty} b(g(1/x)) = 0$ and a_{UV}, c_{UV} are given as aboves, we have a (very weak) positivity inequality for Chern classes being contributed by a, c functions in the trace anomaly as follows:

Main Results If we expand the chern character for the trace anomaly Θ as before in 4-forms, with forgetting all the terms in Θ except for $a_{UV}.c_{UV}$, $\mathrm{ch}_4 := \mathrm{ch}(\Theta) = r + \frac{i}{2\pi}\mathrm{tr}\Theta + \frac{i^2}{2(2\pi)^2}\mathrm{tr}\Theta^2 + \cdots \sim \mathrm{tr}(e^{i(a_{UV}+c_{UV})/2\pi})$, where r is the dimesion of the representation of G,then $\mathrm{ch}_4 > 0$.

References

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