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<th>Title</th>
<th>ℚ-bases of the Néron-Severi groups of certain elliptic surfaces</th>
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1 Introduction

P. Stiller computed the Mordell-Weil ranks and hence the Picard numbers of several families of elliptic surfaces by studying the action of certain automorphisms on the cohomology group ([Stiller 1987]). He considered five families of elliptic surfaces $L_i^m (1 \leq i \leq 5, n \in \mathbb{N})$ (Example 1, 2, 3, 4 and 5 in [Stiller 1987]). For each $L_i^m$, he proved that there exists a finite set $Adm_i$ of natural numbers such that the Mordell-Weil rank $r_i(n)$ is given by

$$r_i(n) = \sum_{d \mid n, d \in Adm_i} \varphi(d),$$

where $\varphi$ is the Euler function. However, he did not give generators of the Néron-Severi groups of these surfaces. In [Kuroda], Q-bases or Z-bases of these groups are given explicitly. In this paper, we explain briefly with respect to such bases, and we give a basis in the most complicated case Example 1 of these five examples:

$$E_i^k : y^2 = x^3 + t^n x + t^n \quad (n \in \mathbb{N}, t \in \mathbb{P}_C^1),$$

$Adm_i = \{1, 2, 3, 7, 8, 10, 12, 15, 18, 20, 42\}$.

2 The Néron-Severi group of an elliptic surface

Notations

- $f : E \to \mathbb{P}_C^1$: an elliptic surface with a zero section
- $E/(C(t))$: the generic fiber of $f : E \to \mathbb{P}_C^1$
- $E(C(t))$: the Mordell-Weil group of $E/(C(t))$
- $\Sigma(E)$: the image in $\mathbb{P}_C^1$ of $f$-1(t) is a singular fiber
- $F_i, F_i, \phi$: irreducible components of the fiber $E$ where $F_i$ is the unique component of $E$ intersecting with $\infty$
- $C_0 := C_0$, $t_0 \in \mathbb{P}_C^1 \setminus \Sigma(E)$: a general fiber

Properties with respect to Q-bases of $NS(E)$

Definition 3.1. C(t)-rational points $P_d$ of $E_d^i$ (d $\in Adm_i$) are given by

$$P_{d, i} = (-1, \sqrt{-1}, j), \quad j \in (\mathbb{Z}/d)^*$$

$$P_{d, i} = (-\sqrt{-1}, -t, j), \quad j \in (\mathbb{Z}/d)^*$$

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We give $r_i(n)$ rational points of the generic fiber $E_d^i/(C(t))$ of $E_d^i$ and calculate the determinant of the intersection matrix $M_n$ of the associated divisors (or $det(N)$, where $N_0$ is similar to (1)).

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We use the same symbol $P_{d, i}$ for the $P_{d, i}$.

Remark 3.2. (i) $f : E_d^i \to \mathbb{P}_C^1$ has a global section and is not smooth.

We give $E_d^i/(C(t))$ is generated by points of the form

$$x = t(d) \sum_{k=0}^{\frac{n}{2}} a_k t^k, \quad y = t(d) \sum_{k=0}^{\frac{n}{2}} b_k t^k \quad ([\text{Shioda} 1990]).$$

We can give the coefficients $a_k, b_k$ such that $det(N) = 0$ and $det(M_n) = 1$ if $n = 1, 2, 3, 4, 6, 7, 8, 12$.

In [Kuroda], these coefficients are given explicitly.

Theorem 3.3.

(i) $NS(E_d^i)$ has a Z-basis $C_0, \ldots, D_{m_i}, F_i, a$ ($d \in Adm_i$, $d, n, j \in (\mathbb{Z}/d)^*$, $t \in \Sigma(E_d^i), 1 \leq a \leq m_i - 1$).

(ii) $E_d^i$ is rational (i.e., n = 1, 2, 3, 4, 6, 7, 8, 12 or 12) $\Rightarrow$ these divisors form a Z-basis of $NS(E_d^i)$.

References


