"Title"  Néron-Severi group of certain elliptic surfaces

"Author(s)"  Kuroda, Masamichi

"Citation"  代数幾何学シンポジウム記録 (2013), 2013: 162-162

"Issue Date"  2013

"URL"  http://hdl.handle.net/2433/214984

"Type"  Departmental Bulletin Paper

"Textversion"  publisher

Kyoto University
1 Introduction

P. Stiller computed the Mordell-Weil ranks and hence the Picard numbers of several families of elliptic surfaces by studying the action of certain automorphisms on the cohomology group ([Stiller 1987]). He considered five families of elliptic surfaces $\Sigma^i$ (1 $\leq$ $i$ $\leq$ 5, $n$ $\in$ $\mathbb{N}$) (Example 2, 3, 4, 5 and 6 in [Stiller 1987]). For each $\Sigma^i$, he proved that there exists a finite set $\text{Adm}_n$ of natural numbers such that the Mordell-Weil rank $r_n$ is given by

$$r_n = \sum_{d \mid n, d \geq \text{Adm}_n} \varphi(d),$$

where $\varphi$ is the Euler function. However, he did not give generators of the Néron-Severi groups of these surfaces. In [Kuroda], $\mathbb{Q}$-bases or $\mathbb{Z}$-bases of these groups are given explicitly. In this paper, we explain briefly properties with respect to such bases, and we give a basis in the most complicated case Example 1 of these five examples:

$$\Sigma^1: y^2 = x^3 + t^2x + t^n \quad (n \in \mathbb{N}, t \in \mathbb{P}^1 ),$$

$\text{Adm}_1 = \{ 1, 2, 3, 7, 8, 10, 12, 15, 18, 20, 42 \}$.

2 The Néron-Severi group of an elliptic surface

**Notations**

- $f : E \to \mathbb{P}^1$: an elliptic surface with a zero section
- $E \cap (t)$: the generic fiber of $f : E \to \mathbb{P}^1$
- $E(\mathbb{C}(t))$: the Mordell-Weil group of $E(\mathbb{C}(t))$
- $(P)$: the image in $E$ of the section corresponding to $P \in E(\mathbb{C}(t))$
- $\Sigma(E) = \{ t \in \mathbb{P}^1 \mid \Sigma_t = f^{-1}(t) \text{ is a singular fiber} \}$
- $F_{t, a} = (0 \leq a \leq m_1 - 1)$: the irreducible components of the fiber $E_t$
- $\Phi_t = \{ t \in \Sigma(E) \mid m_1 \geq 2 \}$

We have

$$\text{det}(M) = -\text{det}(N) \prod_{i=0} \text{det}(L_i).$$

**Remark 2.2** (i) $P_1, \ldots, P_r$ form a $\mathbb{Z}$-basis of $E(\mathbb{C}(t))_{\text{tor}}$.

3 $\mathbb{Q}$-bases of $\text{NS}(\Sigma^1)$

We give $r_1(n)$ rational points of the generic fiber $E^1_1/\mathbb{C}(t)$ of $\Sigma^1$ and calculate the determinant of the intersection matrix $M_n$ of the associated divisors (or $\text{det}(N)$, where $N$ is similar to (2)).

**Definition 3.1.** $\mathbb{C}(t)$-rational points $P_{d, j}$ of $E^1_d$ (for $d \in \text{Adm}_1$)

**Remark 3.2.** (i) $f : \Sigma^1 \to \mathbb{P}^1$ has a global section and is not smooth.

**References**


