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Sakai-Sugimoto model in QCD,five-dimensional Yang-Mills theory,and the Chern character appearing in the associated chiral anomaly

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1 Introduction

- D-branes probed systems [5] in the context of AdS/CFT correspondence:
  - the study of quiver moduli [2].
  - the Bogomolov-Gieseker type inequality on Calabi-Yau 3-folds [1].
- D4/D8/$\overline{\text{D8}}$ probed system model (the so-called **Sakai-Sugimoto model** [4]):
  - mostly successful to describing Hadron models in QCD (quantum chromodynamics) without quark models.
  - to give the configurations of "skyrmions" (just like solitons) in five-dimensional Yang-Mills (YM) theory. (M.Atiyah et al,1988).

In this note,based on the result [4],we discuss a chiral anomaly [4] associated to the WZW term of Sakai-Sugimoto model with regards to the Chern character appearing in the conjectural form of stability condition [1].

2 Sakai-Sugimoto D4/D8/$\overline{\text{D8}}$ model and the associated five-dimensional Yang-Mills theory

Brief explanations [4] defines the chiral symmetry $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ gravitational model in 10-dimensional space, having the configurations of N_c D4 branes in (01234)-direction with compactification for (0123)-direction as S^1 by $\tau \sim \tau + 2\pi/U_{KK}$,and of N_f D8-$\overline{\text{D8}}$ branes ($\overline{\text{D8}}$ means having the inverse charge of D8) in (012356789)-direction,where N_c, N_f the numbers of colors and flavors,resp.,U_{KK} the Kalza-Klein mass,and τ the angle coordinate in (4)-direction of S^1-compactification.

(1) Then,D4-brane solution is $ds^2 = \left(\frac{2U}{3}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2) + \left(\frac{3}{2U}\right)^{3/2}(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2)$, where $f(U) := 1 - \frac{1}{U^3}$,$x^\mu(\mu = 0,1,2,3)$ the 4-dimensional Minkowski coordinates,τ the above angle coordinate, U the radius coordinate in the 5-dimensional space orthogonal to D4-brane with $U \geq U_{KK}$,and $d\Omega_4^2$ the metric of 4-dimensional sphere S^4 surrounding D4-brane [4, (3.1)]. (Assume $U_{KK} = 1$ for simplicity in the sequel.)

(2) Then,for introducing flavor-branes [3] of D8-$\overline{\text{D8}}$ as probes (assuming $N_f \ll N_c$),i.e.embedding D8 on D4-background as (1) with $U = U(\tau)$, the action of D8 is $S_{D8} \propto \int d^4x d\tau e^{-\phi}\sqrt{\det(-g_{D8})}$,where ϕ is dilaton,and g_{D8} is the metric induced on the world-sheet of D8.

(3) Chiral fermions ϕ_L, ϕ_R are induced from open string between D4 and D8($\overline{\text{D8}}$).

(4) Setting $(y,z) = \sqrt{U^2-1}(\cos(U_{KK}\tau), \sin(U_{KK}\tau))$,the configuration of continuing D8($\overline{\text{D8}}$) is given by $y = 0$. Then,world-volume of probe D8 is given by x^μ, z and S^4-direction surrounding D4,i.e.(8+1)-dimensional.

(5) **Assume that S^4-dependency is ignored.**Then, the 5-dimensional YM theory is given as $S^{YM}_{D8} = \kappa \int d^4x dz \mathrm{tr}\{\frac{1}{2}K^{-1/3}F^2_{\mu\nu} + KU^2_{KK}F^2_{\mu\nu}\}$, where $\kappa = \frac{\lambda N_c}{108\pi^3}$,$K(z) = 1 + z^2$,$\lambda = g^2_{YM}N_c$,$g_{YM}$ the YM coupling at U_{KK}, and $F_{\mu\nu}$ the field strengths.The associated Chern-Simons (CS) term is $S^{CS}_{D8} = \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}}\omega_5(A)$, where $\omega_5(A) = \mathrm{tr}(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^3)$ the 5-form, $A = A(x^\mu, z)$ the gauge field on D8,and $F = (F_{\mu\nu})$.

3 Chiral anomaly on probe D8 and the associated Chern characters

Main Result By [4, (5.76),(5.77)], $S^{CS}_{D8} = \mu\int_{D8} C_3 \mathrm{tr}F^3 = \mu\int_{D8} F_4\omega_5(A)$, where $F_4 = dC_3$ the RR(Ramond-Ramond) 4-form field strength,$\mu = \frac{1}{48\pi^3}$ and $d\omega_5 = \mathrm{tr}F^3$ holds. For the infinitesimal gauge transformation $\delta_\Lambda A = d\Lambda + [\Lambda, A]$, $\delta_\Lambda\omega_5(A) = d\omega^1_4(\Lambda, A)$, where $\omega^1_4(\Lambda, A) = \mathrm{tr}(\Lambda d(\mathrm{Ad}A + \frac{1}{2}A^3))$. Then,the gauge transformation of CS-term $\delta_\Lambda S^{D8}_{CS} = \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} d\omega^1_4(\Lambda, A)$ is given by taking $z \to \pm\infty$ for the gauge potentials $A_L(z,\Lambda)$ (resp.$A_R(z,\Lambda)$) of ϕ_L(resp.ϕ_R). So,$\delta_\Lambda S^{D8}_{CS}$ induces WZW terms. By considering $\mathrm{tr}F^3$ as the third term of the Chern character $\mathrm{ch}(F) = \mathrm{tr}e^{\sqrt{-1}F/(2\pi)}$, the followings are settled: Let X be the smooth projective complex 3-fold as smooth compactification of $M^4 \times \mathbb{R} \times \mathbb{R} \ni (x^\mu, z, u)$ and let $(\tilde{x}^\mu) = (x^\mu, z, u)$ with $\tilde{x}^6 = u$.Let $B_1 = A_{\mu\nu}(\mu, \nu = 0, 1, \cdots, 5)$ and $B_2 = A_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ the 2-forms in $H^2(X, \mathbb{C})$ such that B_2 is ample when X as a complex 3-fold. Define $\tilde{A} = \tilde{A}(x^\mu, z, u) = \tilde{A}(\tilde{x})$ the gauge field on X such that $\tilde{A}_u = 0$.Let the 2-forms $\tilde{F} = (\tilde{A}_{\mu\nu})(\mu, \nu = 0, 1, \cdots, 5)$ on X and $\mathrm{ch}_X(\tilde{F})$ the Chern character for $\tilde{F}$ on X, and define $Z_{B_1,B_2}(\tilde{F}) = \int e^{-(B_1+\sqrt{-1}B_2)}\mathrm{ch}_X(\tilde{F})$.Then;

Theorem As the chiral anomaly works even when $A_z = 0$ gauge [4, pp.869],$Z_{B_1,B_2}(\tilde{F})|_{M^4\times\mathbb{R}} = \delta_\Lambda S^{D8}_{CS}$ is satisfied. Therefore,if [1, I,Conj.3.2.7] holds for $Z_{B_1,B_2}(\tilde{F})$, then $\delta_\Lambda S^{D8}_{CS}$ or $Z_{B_1,B_2}(\tilde{F})$ reproduces the chiral anomaly.

References

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