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Induced automorphisms on Hyperkähler manifolds

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Preceding theorems

There is a well defined non separated space of Hyperkähler manifolds together with a marking, i.e. an isometry between their second cohomology and a fixed lattice. The period map sends a marked Hyperkähler manifold to its Hodge structure. Let $X$ and $Y$ be two Hyperkähler manifolds. An isometry $f: H^2(X) \to H^2(Y)$ of parallel transport if and only if it is obtained by parallel transport along a family of Hyperkähler manifolds containing $X$ and $Y$. The following theorem of Huybrechts, Markman and Verbitsky tells us how to recover a manifold from its Hodge structure.

**Theorem**

(2) The period map is generically injective on each connected component of the moduli space of marked hyperkähler manifolds. Let $(X, f)$ and $(Y, g)$ be two marked manifolds with the same period. Then the following are equivalent:

- $f = g^{-1}$ is of parallel transport.
- $f \circ g = 1$ is the composition of a birational map and reflections on reduced irreducible divisors.
- $(X, f)$ and $(Y, g)$ lay in the same connected component of the moduli space of marked hyperkähler manifolds.

In the peculiar case of manifolds of $K3^{[n]}$-type, Markman also describes the group of parallel transport operators and Bayer and Macrì describe more in detail the birational geometry of moduli spaces of stable objects.

Throughout our discussion, we use some theory of lattices and their discriminant forms (see [5]), as our tools

**The tools**

We use some theory of lattices and their discriminant forms (see [5]), as our natural generalization of the moduli space of stable sheaves. Let $X$ be a K3 surface and let $\mathcal{A}$ be the torsion lattice. Let $\mathcal{H}$ be the transcendental lattice. In the following we focus on the $K3$ case, as the abelian is analogous.

**Induced automorphisms**

If we take a $K3$ (or abelian) surface with an automorphism $\gamma$, we can consider the induced action of $\gamma$ on coherent sheaves. When the automorphism preserves also stable sheaves with given Chern classes, it induces an automorphism on the moduli space of such sheaves. We call such automorphisms induced.

Together with Maks Wandel, we answered the following questions:

- When is a manifold of $K3^{[n]}$-type a moduli space of sheaves on a $K3$ surface?
- Which of its automorphisms are induced from the surface?

In the following we focus on the $K3$ case, as the abelian is analogous.

**Definition**

Let $\mathcal{H}(X) = H^2(X, \mathbb{Z})$ be the invariant lattice and let $\mathcal{A}(X) = H^2(X, \mathbb{Z})^G$ be the coinvariant lattice. We denote $\mathcal{H}(X)$ by $\mathcal{H}$.

To determine when a $K3$ is a moduli space of stable sheaves, one can think of moduli spaces of stable objects in the derived category of a surface as a natural generalization of moduli spaces of stable sheaves. Let $S$ be a $K3$ surface and let $\mathcal{M}(S)$ be the moduli space of stable sheaves on $S$. There is a natural notion of stability, there is a smooth moduli space $\mathcal{M}(S)$ of stable objects $E$ with $\chi(E) = 1$ for all prime ideals $\mathfrak{m}$ with square at least $2$. It has dimension $2\chi - 2$. Moreover there is the following isometry:

$$H^2(M(S), \mathbb{Z}) \cong \mathcal{H}(X).$$

(1)

Which holds if $\chi \geq 2$.

We also use Verbitsky’s global Torelli theorem and Markman’s determination of the fibres of the period map to recover a manifold from its second cohomology [3].

**References**


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