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Induced automorphisms on Hyperkähler manifolds

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On Hyperkähler manifolds

Hyperkähler manifolds are simply connected complex analytic varieties endowed with a symplectic holomorphic 2-form, \( \omega \), and a complex auto-dual holomorphic 2-form, \( \omega^* \). In this setting, the geometry is determined by the Hodge theory of the manifold. The Hodge structure of the manifold is given by the direct sum of the Hodge numbers, \( H^k(X) \), which are organized into the Hodge diamond:

\[
\begin{array}{c}
\vdots \\
H^0(X) \\
| \\
H^1(X) \\
| \\
\vdots \\
\end{array}
\]

It follows that the moduli space of hyperkähler manifolds is a quotient of the moduli space of coherent sheaves by the group of automorphisms of the moduli space. The moduli space of hyperkähler manifolds is a smooth projective variety, and it is a compactification of the moduli space of coherent sheaves. The moduli space of hyperkähler manifolds is a homogeneous space for the group of automorphisms of the moduli space.

Preprint

There is a well-defined non-separating surface of hyperkähler manifolds together with a marking, i.e., a holonomy group of a fixed curvature tensor. The period map sends a marked hyperkähler manifold to its Hodge structure. Let \( X \) and \( Y \) be two hyperkähler manifolds. An isometry \( f : H^2(X) \to H^2(Y) \) is a parallel transport if and only if it is obtained by parallel transport along a family of hyperkähler manifolds containing \( X \) and \( Y \). The following theorem of Huybrechts, Markman and Verbitsky tells us how to recover a manifold from its Hodge structure.

Theorem

(2) The period map is generically injective on each connected component of the moduli space of marked hyperkähler manifolds. Let \( (X, f) \) and \( (Y, g) \) be two marked manifolds with the same period. Then the following are equivalent:

- \( f = g^{-1} \) is parallelogram transport.
- \( f = g^{-1} \) is the composition of a biholomorphically isometric isomorphism.
- \( X, f \) and \( (Y, g) \) lay in the same connected component of the moduli space of marked hyperkähler manifolds.

In the peculiar case of manifolds of Kähler type, Markman also describes the group of parallel transport operators and Bayer and Macrì describe more in detail the birational geometry of moduli spaces of stable objects.

References