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The reflexivity of a segre product of projective varieties

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THE REFLEXIVITY OF A SEGRE PRODUCT OF PROJECTIVE VARIETIES

(Joint work with Hajime Kaji, [Math. Ann. **342** (2008), 279–289])

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Main Theorem (Fukasawa-Kaji)

For a proj var $Y \subset \mathbb{P}^N$ of dim n , with Hessian rank r / char $p \geq 0$,

The Segre product $X = \mathbb{P}^m \times Y$: non-reflexive

$$\Leftrightarrow \begin{cases} (1) 0 < n - m < r, p = 2, m + n : \text{odd, or} \\ (2) n - m > r \text{ and } Y : \text{non-reflexive.} \end{cases}$$

Segre product := Product embedded by Segre embedding

Main Theorem'

$$\mathbb{P}^m \times Y : \text{reflexive} \Leftrightarrow \begin{cases} (i) m \geq n, \\ (ii) 0 \leq n - m < r, p \neq 2, \\ (iii) 0 \leq n - m < r, p = 2, m + n : \text{even} \\ (iv) n - m = r, \text{ or} \\ (v) n - m > r \text{ and } Y : \text{reflexive.} \end{cases}$$

Reflexivity

$$Y \text{ is reflexive} \stackrel{\text{def}}{\Leftrightarrow} C(Y) = C(Y^*)$$

$\mathbb{P}^{N*} := \{H \mid \text{hyperplane in } \mathbb{P}^N\}$ dual proj. space;

$Y \subset \mathbb{P}^N$;

$C(Y) := \{(y, H) \in Y_{\text{sm}} \times \mathbb{P}^{N*} \mid T_y Y \subset H\} \subset \mathbb{P}^N \times \mathbb{P}^{N*}$

$\downarrow \pi_Y$ conormal variety

$\mathbb{P}^{N*}$

$Y^* := \pi_Y(C(Y)) \subset \mathbb{P}^{N*}$; dual variety

$C(Y^*) \subset \mathbb{P}^{N**} \times \mathbb{P}^{N*} = \mathbb{P}^N \times \mathbb{P}^{N*}$

Rem. Y : reflexive $\Rightarrow Y^{**} = Y$ (Proj. Duality)

Hessian Rank r

$$r = \text{rank} \left[\frac{\partial^2(h|Y)}{\partial y_i \partial y_j} \right]_{i,j}$$

$(y, H) \in C(Y)$: general pt,
 h : rational function defining H ,
 (y_i) : local coord. of Y at y .

Rem. rank $d\pi_Y = N - 1 - (\dim Y - r) \leq \dim Y^*$

Monge-Segre-Wallace Criterion

Y is reflexive.

$\Leftrightarrow \pi_Y : C(Y) \rightarrow Y^*$ is generically smooth.

$\Leftrightarrow \text{rank } d\pi_Y = \dim Y^*$

Y	p	r	R/N	$\mathbb{P}^m \times Y$	R/N	cond.
$\mathbb{P}^n$	≥ 0	0	R	$\mathbb{P}^m \times \mathbb{P}^n (m \geq n)$	R	(i)
$\mathbb{P}^1 \times \mathbb{P}^1$	$\neq 2$	2	R	$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$	R	(ii)
$\mathbb{P}^2 \times \mathbb{P}^2$	2	4	R	$\mathbb{P}^2 \times \mathbb{P}^2 \times \mathbb{P}^2$	R	(iii)
$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$	2	2	N	$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$	R	(iv)
$\mathbb{P}^1 \times \mathbb{P}^2$	≥ 0	2	R	$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^2$	R	
$\mathbb{P}^1 \times \mathbb{P}^3$	≥ 0	2	R	$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^3$	R	(v)
$\mathbb{P}^1 \times \mathbb{P}^1$	2	2	R	$\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$	N	(1)
$\mathbb{P}^1 \times \mathbb{P}^2$	2	2	R	$\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$	N	
$F_d (d \equiv 1 \pmod p)$	> 0	0	N	$\mathbb{P}^m \times F_d (m < n)$	N	(2)

TABLE: The reflexivity of Segre products of projective varieties

R:= Reflexive / N:= Non-Reflexive
 F_d : Fermat hypersurf. of deg d in $\mathbb{P}^{n+1}$

PROOF OF MAIN THEOREM

- Determine rank $d\pi_X$ and $\dim X^*$.
- Use Monge-Segre-Wallace Criterion.

APPLICATION

Any Non-reflexive example of a Segre product gives a Negative Answer to Kleiman-Piense's question for Gauss maps.

Rem. The Gauss map of any Segre product is gen. smooth onto its image.