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<tr>
<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>代数幾何学シンポジウム記録 (2008), 2008: 133-133</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2008</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/215041">http://hdl.handle.net/2433/215041</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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ON THE STRUCTURE OF CERTAIN $K3$ SURFACES
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$(\mathbb{I}_4^*) := \{ X \rightarrow P^1_{t} \mid \text{an elliptic } K3 \text{ surface } X \text{ with a section and two singular fibers both of type } \mathbb{I}_4^* \text{ at } t = 0, \infty \in P^1_{t} \text{ over } \mathbb{C}\}$

$E_1, E_2 : \text{elliptic curves}$

$S = \text{Km}(E_1 \times E_2)$ has an elliptic fibration having a section and 3 singular fibers of type $\mathbb{I}_4^*, \mathbb{I}_1^*, \mathbb{I}_{12}$ at $\infty, 2, -2 \in P^1 (0 \leq \mu_1, \mu_2 \leq 1)$.

$\text{X: the double covering surface of } S \text{ induced by } f : P^1_t \rightarrow P^1_{t+1}$

Conversely is it possible to recover such a Kummer surface $S$ from given $X \in (\mathbb{I}_4^*)^2$ ?

$(\mathbb{I}_4^*)_2 := \{ X \in (\mathbb{I}_4^*) \mid a_1 = a_3 \}$

$(\mathbb{I}_4^*)_* := \{ X \in (\mathbb{I}_4^*) \mid a_1 \neq a_3 \}$

The answer of the question (●)

$X \in (\mathbb{I}_4^*)_2 \Rightarrow \text{YES.}$

$X \in (\mathbb{I}_4^*)_* \text{ and } \rho(X) = 18 \Rightarrow \text{NO.}$

Definition. (Shioda-Inose structure)

A $K3$ surface $X$ admits a Shioda–Inose structure if there exists a symplectic involution $\iota$ on $X$ with the rational quotient map $\pi : X \rightarrow Y$ of $\iota$ such that $Y$ is a Kummer surface and $\pi_* : H^2(X, \mathbb{Z}) \rightarrow H^2(Y, \mathbb{Z})$ induces a Hodge isometry $T_X(2) \cong T_Y$.

If $X$ admits a Shioda–Inose structure, the complex torus such that $Y = \text{Km}(\mathbb{Z})$ gives a diagram

$X \rightarrow \text{Z}$

of rational maps of degree 2. This diagram induces a Hodge isometry $T_X \cong T_Y$.

Theorem 1.

If $X \in (\mathbb{I}_4^*)_2$, then $X$ admits a Shioda–Inose structure.

Theorem 2.

If $X \in (\mathbb{I}_4^*)_*$ and Picard number of $X$ is 18, then $X$ does not admit a Shioda–Inose structure. However $X$ is isomorphic to a Kummer surface of product type.