

# Enriques surfaces covered by Jacobian Kummer surfaces

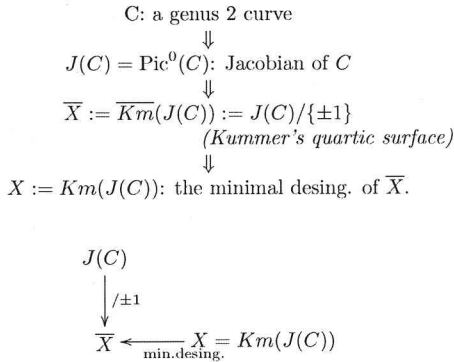
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## 1 Introduction

### Jacobian Kummer surface $X$



### Definition

$X$  is Picard-general if  $\rho(X) = 17$ , which we assume in what follows.

$\text{Aut}(X)$  has been studied by many authors.

One definitive result is the following

### Theorem(S. Kondo, 1998)

$\text{Aut}(X)$  is generated by

- 16 × 4 Klein's involutions  $(t_\alpha, \sigma_\beta, p_\alpha, p_\beta)$ ,
- 60 Hutchinson's involutions  $(\sigma_G)$ ,
- 192 Keum's automorphisms  $(\phi_{W, W'})$ .

Where

- $\alpha \in \{2\text{-torsion pts of } J(C)\}$ ,
- $\beta \in \{\text{theta characteristics of } C\}$ .

### Corollary of the Main Theorem

$\text{Aut}(X)$  is generated by

- 16 × 4 Klein's involutions  $(t_\alpha, \sigma_\beta, p_\alpha, p_\beta)$ ,
- 60 Hutchinson's(HG) involutions  $(\sigma_G)$ ,
- 192 Hutchinson-Weber(HW) involutions  $(\sigma_W)$ .

Where did  $\sigma_W$  come from ?

## 2 Main Result

**Main Theorem** There are 31 = 6 + 10 + 15 fixed-point-free involutions on  $X$ , up to the isomorphism of the quotient Enriques surfaces.

They are exactly as follows.

## 3 free involutions on $X$

### Switches

$$\Theta_\beta = \{p - \beta | p \in C\}.$$

$$\text{For } p \in J(C), (\Theta_\beta + p) \cap (\Theta_\beta - p) = \{q, -q\}.$$

$$\sigma_\beta: \pm p \mapsto \pm q.$$

$$\sigma_\beta \in \text{Bir}(\bar{X}) = \text{Aut}(X).$$

$\beta$  runs over even theta characteristics of  $C$ ; we obtain 10 free switches.

### HG involutions

Restriction of the Cremona involution to  $\bar{X}$ :

$$\sigma_G: (x, y, z, t) \mapsto \left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}\right).$$

$G$ : four points of  $\bar{X}$ , called Göpel subgroup.

$\sigma_G$  is well-defined, because

**Theorem**[Hutchinson] If we choose the four points of  $G$  as the reference points of  $\mathbb{P}^3$ , the equation of  $\bar{X}$  becomes

$$\begin{aligned}
 &A(x^2t^2 + y^2z^2) + B(y^2t^2 + z^2x^2) + C(z^2t^2 + x^2y^2) + Dxyz \\
 &E(yt + zx)(zt + xy) + G(zt + xy)(xt + yz) + H(xt + yz)(yt + zx) \\
 &= 0.
 \end{aligned}$$

There are 15 Göpel subgroups.

### HW involutions

Restriction of the Cremona involution  $\sigma_W: (s_i) \mapsto (s_i^{-1})$  of  $\mathbb{P}^4$  to  $X_W$ , where  $W$ : a Weber hexad (definition omitted),  $X_W$ : another quartic model of  $X$ .

$$\bar{X} \xrightarrow{|\mathcal{O}_{\mathbb{P}^3(2)} - W|} X_W \subset \mathbb{P}^4.$$

**Theorem**[Hutchinson] The equation of  $X_W$  is

$$\sum_{i=1}^5 s_i = \sum_{i=1}^5 \lambda_i / s_i = 0, \quad \lambda_i \in \mathbb{C}^*.$$

We obtain 6 HW involutions.

## 4 Sketch of the Proof

We compute certain invariant, *the patching subgroups* of free involutions. For our  $X$ , it exactly classifies the isom. classes of quotient Enriques surfaces. The definition of it uses Nikulin's lattice theory.