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# Enriques surfaces covered by Jacobian Kummer surfaces

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## 1 Introduction

**Jacobian Kummer surface $X$**

C: a genus 2 curve
$\Downarrow$
$J(C) = \mathrm{Pic}^0(C)$: Jacobian of $C$
$\Downarrow$
$\overline{X} := \overline{Km}(J(C)) := J(C)/\{\pm 1\}$
*(Kummer's quartic surface)*
$\Downarrow$
$X := Km(J(C))$: the minimal desing. of $\overline{X}$.

$$\begin{array}{ccc} J(C) & & \\ \downarrow /\pm 1 & & \\ \overline{X} & \xleftarrow[\text{min.desing.}]{} & X = Km(J(C)) \end{array}$$

**Definition**
$X$ is Picard-general if $\rho(X) = 17$, which we assume in what follows.

$\mathrm{Aut}(X)$ has been studied by many authors.
One definitive result is the following
**Theorem(S. Kondo, 1998)**
$\mathrm{Aut}(X)$ is generated by

$$\begin{cases} 16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\ 60 & \text{Hutchinson's involutions } (\sigma_G), \\ 192 & \text{Keum's automorphisms } (\phi_{W,W'}). \end{cases}$$

Where

$$\alpha \in \{\text{2-torsion pts of } J(C)\},$$
$$\beta \in \{\text{theta characteristics of } C\}.$$

**Corollary of the Main Theorem**
$\mathrm{Aut}(X)$ is generated by

$$\begin{cases} 16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\ 60 & \text{Hutchinson's(HG) involutions } (\sigma_G), \\ 192 & \underline{\textit{Hutchinson-Weber(HW) involutions}}\ (\sigma_W). \end{cases}$$

Where did $\sigma_W$ come from ?

## 2 Main Result

**Main Theorem** There are $31 = 6 + 10 + 15$ fixed-point-free involutions on $X$, up to the isomorphism of the quotient Enriques surfaces.
They are exactly as follows.

## 3 free involutions on $X$

**Switches**

$$\Theta_\beta = \{p - \beta | p \in C\}.$$

For $p \in J(C)$, $(\Theta_\beta + p) \cap (\Theta_\beta - p) = \{q, -q\}$.

$$\sigma_\beta\colon \ \pm p \mapsto \pm q.$$
$$\sigma_\beta \in \mathrm{Bir}(\overline{X}) = \mathrm{Aut}(X).$$

$\beta$ runs over even theta characteristics of $C$; we obtain 10 free switches.

**HG involutions**
Restriction of the Cremona involution to $\overline{X}$:

$$\sigma_G : (x, y, z, t) \mapsto (\frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}).$$

$G$ : four points of $\overline{X}$, called Göpel subgroup.

$\sigma_G$ is well-defined, because
**Theorem**[Hutchinson] If we choose the four points of $G$ as the reference points of $\mathbb{P}^3$, the equation of $\overline{X}$ becomes

$$A(x^2t^2 + y^2z^2) + B(y^2t^2 + z^2x^2) + C(z^2t^2 + x^2y^2) + Dxyzt$$
$$E(yt + zx)(zt + xy) + G(zt + xy)(xt + yz) + H(xt + yz)(yt + zx)$$
$$= 0.$$

There are 15 Göpel subgroups.
**HW involutions**
Restriction of the Cremona involution $\sigma_W : (s_i) \mapsto (s_i^{-1})$ of $\mathbb{P}^4$ to $X_W$, where $W$: a Weber hexad (definition omitted), $X_W$: another quartic model of $X$.

$$\overline{X} \overset{|\mathcal{O}_{\mathbb{P}^3}(2) - W|}{\dashrightarrow} X_W \subset \mathbb{P}^4.$$

**Theorem**[Hutchinson] The equation of $X_W$ is

$$\sum_{i=1}^{5} s_i = \sum_{i=1}^{5} \lambda_i / s_i = 0, \quad \lambda_i \in \mathbb{C}^*.$$

We obtain 6 HW involutions.

## 4 Sketch of the Proof

We compute certain invariant, *the patching subgroups* of free involutions. For our $X$, it exactly classifies the isom. classes of quotient Enriques surfaces. The definition of it uses Nikulin's lattice theory.