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京都大学学術情報リポジトリ (KURENAI)
HOMOLOGICAL MIRROR SYMMETRY FOR CUSP SINGULARITIES

ATSUSHI TAKAHASHI

1. STATEMENT AND THE RESULT

We associate two triangulated categories to a triple $A := (\alpha_1, \alpha_2, \alpha_3)$ of positive integers called a signature: the bounded derived category $D^b\text{coh}(X_A)$ of coherent sheaves on a weighted projective line $X_A := \mathbb{P}^{\alpha_1, \alpha_2, \alpha_3}$ and the bounded derived category $D^b\text{Fuk}^{-}(f_A)$ of the directed Fukaya category for a "cusp singularity" $f_A := x^{\alpha_1} + y^{\alpha_2} + z^{\alpha_3} + q^{-1}xyz$, $(q \in \mathbb{C}^*)$. Here, we consider $f_A$ as a tame polynomial if $\chi_A := 1/\alpha_1 + 1/\alpha_2 + 1/\alpha_3 - 1 > 0$ and as a germ of a holomorphic function if $\chi_A \leq 0$.

Then, the Homological Mirror Symmetry (HMS) conjecture for cusp singularities can be formulated as follows:

**Conjecture 1.1 ([T1]).** There should exist an equivalence of triangulated categories
$$D^b\text{coh}(X_A) \simeq D^b\text{Fuk}^{-}(f_A).$$

Combining results in [GL] with known results in singularity theory, one can easily see that the HMS conjecture holds at the Grothendieck group level, i.e., there is an isomorphism
$$(K_0(D^b\text{coh}(X_A)), \chi + \text{tr} \chi) \simeq (H_2(Y_A, \mathbb{Z}), -I),$$
where $Y_A$ denotes the Milnor fiber of $f_A$.

The HMS conjecture is shown if $\alpha_3 = 1$ (Auroux-Katzarkov-Orlov [AKO], Seidel [Se1], van Straten, Ueda, ...). Also the cases $A = (4,4,2), (6,3,2)$, which correspond to two of three simple elliptic hypersurface singularities, are known ([AKO], [U], [T2], ...).

The following is our main theorem:

**Theorem 1.2.** Assume that $\alpha_3 = 2$ or $A = (3,3,3)$. Then the HMS conjecture holds.

The keys in our proof are; the reduction of surface singularities to curve singularities (the stable equivalence of Fukaya categories given in [Se2] section 17), the use of A'Campo's divide [A1][A2] in order to describe the Fukaya category, and mutations of

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exceptional collections (distinguished basis of vanishing Lagrangian cycles). We shall give devides for cusp singularities with $\alpha_3 = 2$ and also quivers with relations associated to them.

2. Devides and Quivers with Relations

2.1. $\chi_A > 0$. After applying suitable mutations, we shall obtain the extended Dynkin quiver of type $A = (\alpha_1, \alpha_2, \alpha_3)$ ($\circ$ denotes the vertex to remove in order to get the Dynkin quiver of the same type). It is known by [GL] that $D^b\text{coh}(X_A)$ is equivalent to the derived category of extended Dynkin quiver of type $A$.

- $x_1^{\alpha_1} + x_2^2 + x_3^2 + x_1x_2x_3$ ($\alpha_1$: even ($D_{2l}$)):

- $x_1^{\alpha_1} + x_2^2 + x_3^2 + x_1x_2x_3$ ($\alpha_1$: odd ($D_{2l+1}$)):

- $x_1^3 + x_2^3 + x_3^3 + x_1x_2x_3$ ($\tilde{E}_6$):
2.2. $\chi_A \leq 0$. Note that the number of vertices (= Milnor number of the singularity) is given by $\alpha_1 + \alpha_2 + \alpha_3 - 1$.

$x_1^4 + x_2^3 + x_3^2 - x_1x_2x_3 (E_7)$:

$x_1^5 + x_2^3 + x_3^2 - x_1x_2x_3 (E_6)$:

$x_1^{\alpha_1} + x_2^{\alpha_2} + x_3^{\alpha_3} + x_1x_2x_3 (\alpha_1 \geq 6)$:

$x_1^{\alpha_1} + x_2^{\alpha_2} + x_3^{\alpha_3} + x_1x_2x_3 (\alpha_1, \alpha_2 \geq 4)$:
REFERENCES


DEPARTMENT OF MATHEMATICS, GRADUATE SCHOOL OF SCIENCE, OSAKA UNIVERSITY, TOYONAKA OSAKA, 560-0043, JAPAN

E-mail address: takahashi@math.sci.osaka-u.ac.jp