Effects of interlayer interfacial stiffness on ultrasonic wave propagation in composite laminates at oblique incidence

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Abstract

The transmission characteristics of ultrasonic wave impinging obliquely on composite laminates are analyzed. Incorporating the influence of thin resin-rich regions between adjacent plies by spring-type interfaces, the amplitude transmission coefficient of a unidirectional composite laminate immersed in water is calculated by the stiffness-matrix method. Using Floquet’s theorem, the dispersion relation for the infinitely laminated structure is also calculated. Comparison between two results reveals that the frequency band-gaps in the dispersion relation agree well with the low-transmission frequency ranges of the finite laminated case. Comparing with the experimental transmission coefficients for an 11-ply carbon-epoxy composite laminate, the theoretical results are verified.

Keywords: Composite laminate; Interlayer interfacial stiffness; Stiffness matrix method; Floquet wave; Band gap; Oblique incidence

1. Introduction

The increasing application of composite laminates in aerospace, automotive, and marine industries demands effective techniques of ultrasonic non-destructive testing for their damage assessment as well as property characterization. Most composite laminates made of polymer matrices such as carbon-epoxy and glass-epoxy laminates have thin resin-rich regions with a few microns thickness between neighbouring plies, and such interlayer imperfections have significant effects on the mechanical performance of the laminated structure [1]. It is hence essential to understand the effects of such regions on the wave propagation behavior so as to evaluate the soundness...
of interlayer interfaces ultrasonically. In the present study, modelling the interlayer resin-rich regions as spring-type interfaces, the transmission characteristics of ultrasonic waves impinging obliquely on unidirectional composite laminates are investigated theoretically with the use of the stiffness-matrix method [2], [3] and Floquet’s theorem [4]. Some experimental results for a carbon-epoxy composite laminate are also shown to verify the theory.

2. Analysis of wave transmission characteristics through a unidirectional composite laminate

A unidirectional composite laminate shown in Fig. 1(a) is considered, which consists of \( N \) plies (density \( \rho \), thickness \( h \), complex elastic moduli \( C_{ij} \)) and \( N-1 \) interlayer spring-type interfaces (normal and shear stiffnesses \( K_N, K_{T1}, K_{T2} \)) and is immersed in water (density \( \rho_f \), wave velocity \( V_f \)). If the harmonic plane longitudinal wave with unit amplitude impinges on the laminate from \( x_3 > Z_0 \) with the direction determined by two angles, the angle of incidence \( \theta \) and the angle for plane of propagation \( \varphi \), then the reflected wave with amplitude \( R \) for \( x_3 > Z_N \) and transmitted wave with amplitude \( T \) for \( x_3 < Z_N \) are generated, respectively. Here \( R \) and \( T \) are the complex reflection and transmission coefficients, and can be calculated by the stiffness matrix method [2], [3].

A periodic system as shown in Fig. 1(b) is next considered, which is constructed by extending the composite laminate in Fig. 1(a) to an infinitely laminated structure. Using Floquet’s theorem [4], the characteristic equation for the \( x_3 \) component of the Floquet wavenumber \( \zeta \) can be given as

\[
A_3 \cos(3\zeta h) + A_2 \cos(2\zeta h) + A_1 \cos(\zeta h) + A_0 = 0 ,
\]

where

\[
A_3 = \det(K_{21}) ,
\]

\[
A_2 = \frac{\det(K_{22} - K_{11} + K_{21}) + \det(K_{22} - K_{11} - K_{21})}{2} \left[2 - \det(K_{22} - K_{11})\right] ,
\]

\[
A_1 = \frac{\det(K_{22} - K_{11} + K_{21}) + \det(K_{22} - K_{11} + K_{12}) + \det(K_{21} - K_{12})}{2} \left[2 - 2 \det(K_{21})\right] ,
\]

\[
A_0 = \frac{\det(K_{22} - K_{11} + K_{12} - K_{21}) + \det(K_{22} - K_{11} - K_{12} + K_{21})}{4} - A_2 .
\]

In the above expression, \( K_{IJ} (I, J = 1, 2) \) are \( 3 \times 3 \) submatrices of the \( 6 \times 6 \) stiffness matrix \( K' \) [2], [3] for a unit-cell consisting of a ply and a spring-type interface defined as

\[
K' = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}.
\]

Fig. 1. (a) A unidirectional composite laminate with \( N \) plies and \( N-1 \) spring-type interlayer interfaces immersed in water and (b) an infinitely laminated composite.
3. Results and discussion

Using the properties shown in Table 1, the squared amplitude transmission coefficient of 11-ply unidirectional composite laminates is calculated and illustrated in Figs. 2(a) – (l) as functions of the frequency and the angle of incidence for various planes of propagation and interfacial stiffnesses. In the present analysis, it is assumed that all plies as well as interlayer interfaces possess the same properties, and that the interfacial shear stiffness is orientation-independent, i.e., $K_{T1} = K_{T2} = K_T$. It is found that the transmission characteristics are influenced by not only the frequency and the angle of incidence but also the plane of propagation and the interfacial stiffnesses. There are wide frequency ranges where the transmission coefficient becomes small such as at around $\theta = 15^\circ$ in Fig. 2(b) and $\theta = 8^\circ$ in Fig. 2(d). Since such regions are due to the critical angle at the water-ply interface, they are influenced by the plane of propagation (anisotropic nature of the ply) but not the interfacial stiffnesses. On the other hand, certain low-transmission zones with finite frequency bands can be seen such as at about 8 MHz in Fig. 2(l), and their bandwidths are influenced by the interfacial stiffnesses.

Table 1. Material properties of the ply (transverse isotropy with $x_1$ as fiber direction is assumed) and water.

<table>
<thead>
<tr>
<th>$C_{11}$ (GPa)</th>
<th>$C_{13}$ (GPa)</th>
<th>$C_{44}$ (GPa)</th>
<th>$C_{55}$ (GPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$h$ (mm)</th>
<th>$\rho_i$ (kg/m$^3$)</th>
<th>$V_f$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>109 – 2.92 $i$</td>
<td>4.8 – 0.18 $i$</td>
<td>14.8 – 0.29 $i$</td>
<td>3.7 – 0.16 $i$</td>
<td>6.2 – 0.30 $i$</td>
<td>1.5 × 10$^3$</td>
<td>0.19</td>
<td>1.0 × 10$^3$</td>
</tr>
</tbody>
</table>

Fig. 2. (a) – (l) Theoretical and (m) – (p) experimental squared amplitude transmission coefficients of 11-ply unidirectional composite laminates.
Fig. 3. The dispersion relation of Floquet waves for two different interfacial qualities with fixed corresponding angle of incidence from water $\theta = 4^\circ$ and $\varphi = 90^\circ$.

The relation between the $x_3$ component of the Floquet wavenumber and the frequency calculated by Eq. (1) for the laminate parameters given in Table 1 (imaginary parts of ply stiffness are neglected) is shown in Fig. 3 for two different interfacial qualities with fixed propagation direction expressed in terms of the corresponding angle of incidence from water $\theta = 4^\circ$ and $\varphi = 90^\circ$. It is seen that in certain finite frequency ranges, the Floquet wavenumbers have non-zero imaginary parts. These ranges are the so-called frequency band-gaps, which imply that the corresponding mode cannot propagate in the infinitely laminated structure since the Bragg reflection occurs. Comparing these dispersion relations with the transmission coefficients of immersed composite laminate of finite thickness in Fig. 2 (indicated by blue arrows), the low-transmission frequency ranges such as at around 8 MHz are in good agreement with the band-gaps in Fig. 3. This indicates that the band-gaps do appear even in the laminate with a finite number of plies. It is also found that some band-gaps seen in Fig. 3 do not form clear low-transmission ranges in Fig. 2, such as all “QT1” and “QT2” band-gaps in Figs. 3(a), (b). This is because the wave field in the laminate of finite thickness at $\theta = 4^\circ$ in Figs. 2(h), (l) is mainly governed by the quasi-longitudinal mode, and not sensitive to whether the fast quasi-transverse mode can transmit through the laminate or not (slow quasi-transverse mode is not concerned as it cannot couple with water).

The variation of the squared amplitude transmission coefficient experimentally measured for an 11-ply carbon-epoxy unidirectional composite laminate with the frequency and the angle of incidence for four different planes of propagation is depicted in Figs. 2(m) – (p). The low-transmission zones seen over the wide frequency ranges due to the critical angles as well as those seen in the limited frequency ranges due to the Bragg reflection are well reproduced by the theory in Figs. 2(e) – (h). It should be noted that the latter feature cannot be described unless the influence of interlayer interfaces is taken into account, as shown in Figs. 2(a) – (d).

The results shown here indicate that the quality of the interlayer interfaces of composite laminates can be evaluated by making use of the band-gap generation behavior.

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References