## Title : Effects of current quark mass and quantum and thermal fluctuations to the inhomogeneous chiral phase transition in the QCD phase diagram

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Quantum chromodynamics (QCD) is a fundamental theory of strong interaction and the elucidation of their phase structure is a very challenging problem for the understanding of nature. One of the most important properties of QCD is the spontaneously chiral symmetry breaking. In particular, there has been actively studied focusing on the inhomogeneous chiral phase, which is characterized by spatially modulated chiral order parameters. By earlier studies, it has been confirmed that the inhomogeneous chiral phase appears in low temperature and middle density region. Nevertheless, we should keep in mind that these studies have been restricted some simplified situations; vanishing current quark masses (chiral limit), no fluctuations, and so on. Thus, in order to understand the inhomogeneous chiral phase, further theoretical improvements are needed. In this thesis, we develop our understandings to the inhomogeneous chiral phase.

First, we investigate the effect of the current quark masses on the inhomogeneous chiral phase. In the chiral limit, the thermodynamic potential has a wine-bottle shape and the equivalent bottom points of that bottle are called chiral circle. In this case, the ground state of the system in the inhomogeneous phase is realized by a spatially modulated order parameter which is winding with a constant frequency around the chiral circle. On the other hand, if the current guark masses are turned on, the system is qualitatively changed. Because of the existence of the mass term, the chiral circle is deformed and the absolute minimum appears in the thermodynamic potential. In this case, the inhomogeneous order parameter should be localized around that point. In fact, we show that the deformation of the function form of the inhomogeneous order parameter from that in the chiral limit becomes larger with increasing temperature. Also we discuss the order of the phase transition between the inhomogeneous and normal phases (termination boundary) which is of the second order in the chiral limit. Due to the effect of the current quark masses, the order changes from the second to the first one.

Next, we investigate the effects of the quantum and the thermal fluctuations on the termination boundary. It has been suggested by earlier studies in the condensed matter physics and the pion condensation that the fluctuation induced by the quantum or thermal effect changes the order of the inhomogeneous phase transition from the second to the first one. On the other hand, it is known that the termination boundary in the inhomogeneous chiral phase transition is typically of the second order within the mean-field physical the approximation. Because responses such as number susceptibility on the phase boundary are different with their order, it is important to elucidate these effects. Nevertheless, there are still a few studies discussing the effect of fluctuations on the termination boundary. In this thesis, we improve the procedure developed in the previous studies; we evaluate the qualitative and quantitative difference of the inhomogeneous chiral phase taking account of the fluctuations simultaneously. As a result, we find the order of the phase transition on the termination boundary is changed from the second to the first order in each temperature and the effects of the fluctuations reduce the inhomogeneous chiral phase.