

# Nonequilibrium dynamics of a periodically-driven impurity spin coupled to the bath of ultracold fermions by Kondo coupling

Koudai Iwahori\* and Norio Kawakami

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

## Abstract

The dynamics of the anisotropic Kondo model under the periodic driving is studied. The periodic driving we consider is the local magnetic field which couples with the impurity spin and the in-plane exchange interaction. At the special point in the parameter space (Toulouse limit), the anisotropic Kondo model can be transformed to the non-interacting resonant level model and thereby the exact results for arbitrary external fields can be obtained. It is shown that characteristic behavior of the dynamics changes with the field intensity, the field frequency, and the Kondo temperature. When the local magnetic field intensity is very large compared with the field frequency and the Kondo temperature, the time dependence of the impurity spin and the spin density of the fermionic bath shows an oscillation whose period is inversely proportional to the field intensity. We also find that another time scale, which is given by the inverse of the Kondo temperature, governs the response of the system to the external field.

*Keywords:* Kondo effect, periodically-driven systems, exact results

## 1 Introduction

The dynamics of a single impurity in a fermionic or bosonic bath is one of the fundamental problems in condensed matter physics, and plays an important role in polaronic systems, X-ray absorption problem, Kondo effect and so on. While these impurity problems have been originally studied in dilute magnetic alloys, recent experimental progress in ultracold atomic systems and nanostructures such as quantum dots provides new tools for exploring these systems. For example, the Kondo effect has been observed in quantum dots [8, 7, 4, 30] and polaronic systems have been experimentally realized in ultracold atomic systems [29, 22, 32, 18, 17]. The realization of the Kondo effect in ultracold atomic systems has been proposed theoretically [24, 2]. In addition to the equilibrium properties, the real time dynamics of the impurity systems, which is difficult to study in solid state systems, has also been studied by utilizing high controllability of the cold atomic systems [3, 5, 6, 13].

\*iwahori@scphys.kyoto-u.ac.jp

Motivated by the experimental relevance, the real time dynamics of quantum many body systems has been studied theoretically. Most of the studies have treated the real time dynamics due to the quantum quench, i.e. the dynamics after a sudden change of the system parameters. However, the properties of the real time dynamics of periodically driven quantum many body systems are not well understood, partly due to the difficulty of theoretical treatment. In quantum impurity systems such as the Anderson model or the Kondo model, considerable work which treats time-periodic driving has been done [11, 23, 27, 9, 25, 20, 10, 14, 15, 16, 34, 12]. These studies have considered the nanostructure systems in which some leads and quantum dots are coupled with each other, and thereby have focused on the transport quantities such as the current between two leads through a dot or the conductance.

In this paper, we investigate the periodically-driven Kondo model with particular focus on the ultracold atomic systems, where the time-dependent local densities can be observed by using recently developed single-site-resolved imaging technique [5, 6, 13]. Thus, we calculate the local quantities such as the impurity spin polarization and the spin density of the fermionic bath. At the special point of parameter space known as Toulouse limit [31], the Kondo model becomes exactly solvable. By using this property, we obtain the exact analytical expression for the time evolution and the useful relation between the spin density of the fermionic bath and the impurity spin polarization. Then, we find that characteristic behavior of the dynamics changes with the external field-intensity  $h$  and its frequency  $\Omega$ , and the Kondo temperature  $T_K$ . In  $h \gg \Omega, T_K$ , the impurity spin polarization and the spin density of fermionic bath oscillate with the frequency of  $h$ , while in  $\Omega \gg T_K, h$  the system cannot keep up with the external driving and observables approach temporally constant values.

## 2 Periodically-driven Kondo model

The system we study is the anisotropic Kondo model with the time-dependent exchange coupling and the time-dependent local magnetic field which couples with the impurity spin:

$$H(t) = \sum_{\sigma} \int dx : \psi_{\sigma}^{\dagger}(x) (-iu\partial_x) \psi_{\sigma}(x) : + \sum_{i=x,y,z} J_i(t) S^i s^i(0) - h(t) S^z \quad (1)$$

where  $\psi_{\sigma}^{\dagger}(x)$  and  $\psi_{\sigma}(x)$  are creation and annihilation operators of fermions in the bath and  $S^i (i = x, y, z)$  are the impurity spin operators.  $s^i(x) = \sum_{ss'} : \psi_s^{\dagger}(x) \sigma_{ss'}^i \psi_{s'}(x) :$  represent the spin density of the fermionic bath where  $\sigma_{ss'}^i$  are the Pauli matrices and the colons  $:\dots:$  denote the normal ordering. Since the scattering term is point like and the impurity spin only scatters fermions whose angular momentum is zero ( $s$ -wave), the effective model is reduced to a one-dimensional one [1]. We consider the local magnetic field  $h(t)$  and the couplings  $J_i(t)$  to be time-periodic functions:  $h(t + \tau) = h(t), J_i(t + \tau) = J_i(t)$  ( $t > 0$ ) and allow the couplings  $J_i(t)$  to have anisotropy; the coupling along the  $x, y$  direction  $J_x(t) = J_y(t) \equiv J_{\perp}(t)$  is different from that of the  $z$  direction  $J_z(t)$ . The initial state ( $t = 0$ ) is the ground state of the anisotropic Kondo model with  $h(0)$  and  $J_i(0)$ . The dependence of the results on the initial state (i.e.  $h(0)$  and  $J_i(0)$ ) is discussed in the section 4.

To simplify the problem, the coupling along the  $z$  direction is taken as

$$\frac{J_z}{\pi u} = \sqrt{2}(\sqrt{2} - 1) \text{ (Toulouse limit [31])}. \quad (2)$$

At this point, we can obtain the exact analytical results even though the Kondo model is a strongly correlated system. In addition, they have many intriguing properties common to

the antiferromagnetic Kondo model. By using the bosonization method, the Kondo model at Toulouse limit can be mapped to the non-interacting resonant level model [35]:

$$H(t) = \int dx : \tilde{\psi}^\dagger(x) (-iu\partial_x) \tilde{\psi}(x) : + \frac{J_\perp(t)}{\sqrt{2\pi\alpha}} (\tilde{\psi}^\dagger(0)\tilde{c}_d + h.c.) - h(t) (\tilde{c}_d^\dagger\tilde{c}_d - \frac{1}{2}) \quad (3)$$

The  $\tilde{\psi}(x)$  and  $\tilde{c}_d$  are fermionic operators. The  $z$  component of the impurity spin is represented by the occupation number of the resonant level  $S^z = \tilde{c}_d^\dagger\tilde{c}_d - 1/2$  and the spin density of the fermionic bath is  $s^z(x) = \sqrt{2} : \tilde{\psi}^\dagger(x)\tilde{\psi}(x) : (x \neq 0)$ . The Kondo temperature is determined from the impurity spin contribution to the specific heat as  $T_K = J_\perp^2 w / 4\alpha u$ , where  $w = 0.4128$  is the Wilson number [19].

### 3 Time evolution of operators

In this section, we calculate the time evolution of the system in the Heisenberg picture. The Heisenberg equation of the annihilation operators by the Hamiltonian in eq.(3) reads

$$\begin{aligned} i\frac{d}{dt}\tilde{c}_k(t) &= uk\tilde{c}_k(t) + \frac{J_\perp(t)}{\sqrt{2\pi\alpha L}}\tilde{c}_d(t) \\ i\frac{d}{dt}\tilde{c}_d(t) &= -h(t)\tilde{c}_d(t) + \frac{J_\perp(t)}{\sqrt{2\pi\alpha L}}\sum_k\tilde{c}_k(t) \end{aligned} \quad (4)$$

where the operator  $\tilde{c}_k$  is the Fourier component of the field operator  $\tilde{\psi}(x) = 1/\sqrt{L}\sum_k e^{ikx}\tilde{c}_k$ . Because eqs.(4) are simultaneous differential equations with time dependent coefficients, it is difficult to solve them for arbitrary functions  $h(t)$  and  $J_\perp(t)$ . Here, we solve these equations (4) by "quadrature by parts". First, divide the time-periodic functions  $h(t)$  and  $J_\perp(t)$  into discrete  $M$  time steps:

$$\begin{aligned} h(t) &= \sum_{N=0}^{\infty} \sum_{n=1}^M \theta(t - (N - (n-1)/M)\tau) \theta((N + n/M)\tau - t) h^{(n)} \\ J_\perp(t) &= \sum_{N=0}^{\infty} \sum_{n=1}^M \theta(t - (N - (n-1)/M)\tau) \theta((N + n/M)\tau - t) J_\perp^{(n)} \end{aligned} \quad (5)$$

Second, solve eqs.(4) for each time step. Because the local magnetic field  $h$  and the coupling  $J_\perp$  in each time step are time independent, it is easy to solve the Heisenberg equation. Then, connect the solutions for each time step. Due to the linear dispersion, it is easy to connect the solutions. By taking the continuum limit  $M \rightarrow \infty$ , we obtain the exact analytical expression of the time evolution for arbitrary time-periodic functions  $h(t)$  and  $J_\perp(t)$ .

The exact expression thus obtained for the time evolution of annihilation operators  $\tilde{\psi}(x, t)$  and  $\tilde{c}_d(t)$  in the steady state (after an infinite number of periodic time steps) is

$$\begin{aligned} \tilde{c}_d(t) &= \sum_k T_{dk}(t)\tilde{c}_k, \quad \tilde{\psi}(x, t) = \tilde{\psi}(x - ut) - i \sum_k \frac{J_\perp(t - x/u)}{\sqrt{2\pi\alpha u^2}} \theta(x) T_{dk}(t - x/u) \tilde{c}_k \\ T_{dk}(t) &= \frac{-ie^{-iukt}}{1 - e^{iuk\tau} M_{dd}} \int_0^\tau ds \frac{J_\perp(t-s)}{\sqrt{2\pi\alpha L}} \exp \left[ i \int_{\tau-s}^\tau (h(s'+t) - \Delta(s'+t)) ds' + iuks \right] \\ M_{dd} &= \exp \left[ \int_0^\tau (ih(t) - \Delta(t)) dt \right], \quad \Delta(t) = \frac{J_\perp^2(t)}{4\pi\alpha u}. \end{aligned} \quad (6)$$

These results are applicable for arbitrary time-periodic functions  $h(t), J_{\perp}(t)$  and contain the whole information about the real time dynamics of the system. The point is that the time evolution can be described solely by the function  $T_{dk}(t)$  (transition matrix from  $\tilde{c}_k$  to  $\tilde{c}_d$ ) in the steady state. One can obtain similar exact analytical expressions for the time-dependent current between two leads through some nanostructures by using the Keldysh Green function [33, 27, 28]. N. S. Wingreen *et al.* [33] obtained a similar exact analytical expression for the time-dependent current between two leads through a non-interacting resonant level which is applicable for arbitrary external driving. Results by A. Schiller *et al.* [27, 28] are for the two-channel Kondo model and they treated the case of sinusoidal and rectangular external field. A similar expression to our results by A. Schiller *et al.* [27] can be obtained by substituting a sinusoidal field into eq.(6).

## 4 Observables in the steady state

As mentioned in section 2, the impurity spin polarization and the spin density of the fermionic bath are given by

$$S^z(t) = \tilde{c}_d^{\dagger}(t)\tilde{c}_d(t) - \frac{1}{2}, s^z(x, t) = \sqrt{2}\tilde{\psi}^{\dagger}(x, t)\tilde{\psi}(x, t). \quad (7)$$

Extract the phase factor  $e^{-iuk t}$  from  $T_{dk}(t)$  and define  $\tilde{T}_{dk}(t) = e^{iuk t}T_{dk}(t)$ , where  $\tilde{T}_{dk}(t)$  is a time-periodic function:  $\tilde{T}_{dk}(t) = \tilde{T}_{dk}(t + \tau)$ . Then, the expectation value of the impurity spin polarization in the steady state is calculated as follows:

$$\langle S^z(t) \rangle = \sum_{k, k'} \left( \tilde{T}_{dk}(t) \right)^* \tilde{T}_{dk'}(t) e^{iu(k-k')t} \langle \tilde{c}_k \tilde{c}_{k'} \rangle - \frac{1}{2} \quad (8)$$

The bracket  $\langle \dots \rangle$  is the expectation value obtained by the ground state of the Kondo model or equivalently the resonant level model with  $h(0)$  and  $J_{\perp}(0)$ . For arbitrary initial values of  $h(0)$  and  $J_{\perp}(0)$ , the expectation value can be split into two parts. One is the free part which contains the delta function  $\delta_{k, k'}$ , and the other is the scattering part  $S_{k, k'}(h(0), J_{\perp}(0))$ . Namely, the expectation value can be described as  $\langle \tilde{c}_k \tilde{c}_{k'} \rangle = \theta(-k)\delta_{k, k'} + S_{k, k'}(h(0), J_{\perp}(0))$ . In the steady state ( $t \rightarrow \infty$ ), the initial correlation term  $S_{k, k'}(h(0), J_{\perp}(0))$  vanishes due to the oscillation term  $e^{iu(k-k')t}$  in eq.(8). Thus, the information of the initial state vanishes after an infinite number of periodic steps. The impurity spin polarization is time periodic  $\langle S^z(t) \rangle = \langle S^z(t + \tau) \rangle$  because  $|\tilde{T}_{dk}(t)|^2 = |\tilde{T}_{dk}(t + \tau)|^2$ .

From eq.(6), one finds the intriguing relation;

$$\langle s^z(x, t) \rangle = \begin{cases} -\frac{\sqrt{2}}{u} \langle \frac{d}{dt} S^z(t - x/u) \rangle & x > 0 \\ 0 & x < 0 \end{cases} \quad (9)$$

This relation between the spin density of the fermionic bath and the impurity spin polarization means that the fermion whose spin is opposite to the impurity spin is scattered and propagates with the Fermi velocity  $u$  due to the antiferromagnetic interaction. Since the impurity spin polarization  $\langle S^z(t) \rangle$  has a period of  $\tau$ , the spin density of the fermionic bath is time periodic  $\langle s^z(x, t) \rangle = \langle s^z(x, t + \tau) \rangle$  and spatially periodic  $\langle s^z(x + u\tau, t) \rangle = \langle s^z(x, t) \rangle$  ( $x > 0$ ).

## 5 Case of the sinusoidal local magnetic field and constant coupling

Here, we discuss a specific case: the local magnetic field is sinusoidal  $h(t) = h \sin(2\pi t/\tau)$  and the coupling  $J_{\perp}(t)$  is time independent. Decompose the time  $t$  into periodic intervals:

$$t = N\tau + s, \quad N \in \mathbb{Z}, \quad 0 < s < \tau \quad (10)$$

Then, the time dependence of the impurity spin polarization in the steady state reads

$$\begin{aligned} \langle S^z(t) \rangle &\xrightarrow{N \rightarrow \infty} \sum_{n \in \mathbb{Z}} S_n^z e^{in\Omega t} \\ S_{n:\text{odd}}^z &= \frac{1}{\pi} \frac{e^{in\pi/2}}{n\Omega/\Delta - 2i} \sum_{m \in \mathbb{Z}} J_{n+m}\left(\frac{h}{\Omega}\right) J_m\left(\frac{h}{\Omega}\right) \left( \text{Log}((n+m)\Omega/\Delta - i) - \text{Log}(m\Omega/\Delta + i) \right) \\ S_{n:\text{even}}^z &= 0, \quad \Omega = \frac{2\pi}{\tau}, \quad \Delta = \frac{T_K}{\pi w} \sim 0.77T_K, \end{aligned} \quad (11)$$

where  $J_n$  are the integer Bessel functions. The time dependence of the spin density of the fermionic bath can be obtained from the general relation (9). The time evolution of the impurity spin polarization  $\langle S^z(t) \rangle$  and the spin density of the fermionic bath  $s^z(x, t)$  in a period ( $0 < s < \tau$ ) are shown in Fig.1. Because the spin density of the fermionic bath is spatially periodic  $\langle s^z(x, t) \rangle = \langle s^z(x + u\tau, t) \rangle$  ( $x > 0$ ), the point at  $x = Mu\tau$  ( $M \in \mathbb{N}$ ) is described. When the external driving is slow and strong (see the figure of  $\tau = 10/\Delta, h = 10\Delta$ ), the impurity spin polarization saturates to  $\pm 1/2$  in the time scale of  $1/T_K$  and oscillates with the period  $2\pi/h$ . This oscillation comes from the interference between the impurity spin and the fermions in the bath (in the language of the resonant level model, interference between the fermion in the resonant level and the fermions in the bath). Corresponding to the saturation of the impurity spin polarization in the time scale of  $1/T_K$ , wave packets whose width is of the order  $1/T_K$  are observed in the spin density of the fermionic bath, as seen from the relation (9). Because the dispersion of the fermionic bath is linear, these wave packets propagate without decaying. When the external driving becomes faster (see the figure of  $\tau = 0.3125/\Delta$ ), the impurity spin cannot keep up with the local magnetic field and the amplitude of the impurity spin polarization becomes smaller. The amplitude of the impurity spin polarization at  $\tau = 0.3125\Delta$  is about 10 times smaller than that of the  $\tau = 10\Delta$  case, but the amplitude of the spin density of the fermionic bath at  $\tau = 0.3125\Delta$  is as large as that of the  $\tau = 10\Delta$  case. This is because the spin density of the fermionic bath can be described by the time derivative of the impurity spin polarization, which is amplified by the frequency  $\Omega$ . In  $\Omega \gg T_K$ , while the amplitude of the impurity spin polarization becomes smaller, the amplitude of the spin density of the fermionic bath becomes larger with increasing  $\Omega$  under the condition that  $h/\Omega$  is fixed.

Finally, we comment on some static (time-averaged) properties of observables. When the static component of the local magnetic field  $h(t)$  exists and the frequency  $\Omega$  is much larger than the Kondo temperature, we can observe an oscillation of the time-averaged impurity spin polarization with increasing the static component of the local magnetic field. This oscillation comes from the resonance between the oscillation of the local magnetic field  $h(t)$  and that of the impurity spin (for example, see Fig.1(a)). We also find that the time-averaged impurity spin polarization does not become zero even if the static component of the local magnetic field is zero when we also consider the time dependence of the coupling strength  $J_{\perp}(t)$ . This result is consistent with that obtained by the M. Heyl *et al.* [12] in the linear response regime when

the coupling term is periodically switched on and off. Our result is not restricted in the linear response regime, but also applicable to more general forms of driving. The detail of these observations will be reported elsewhere.

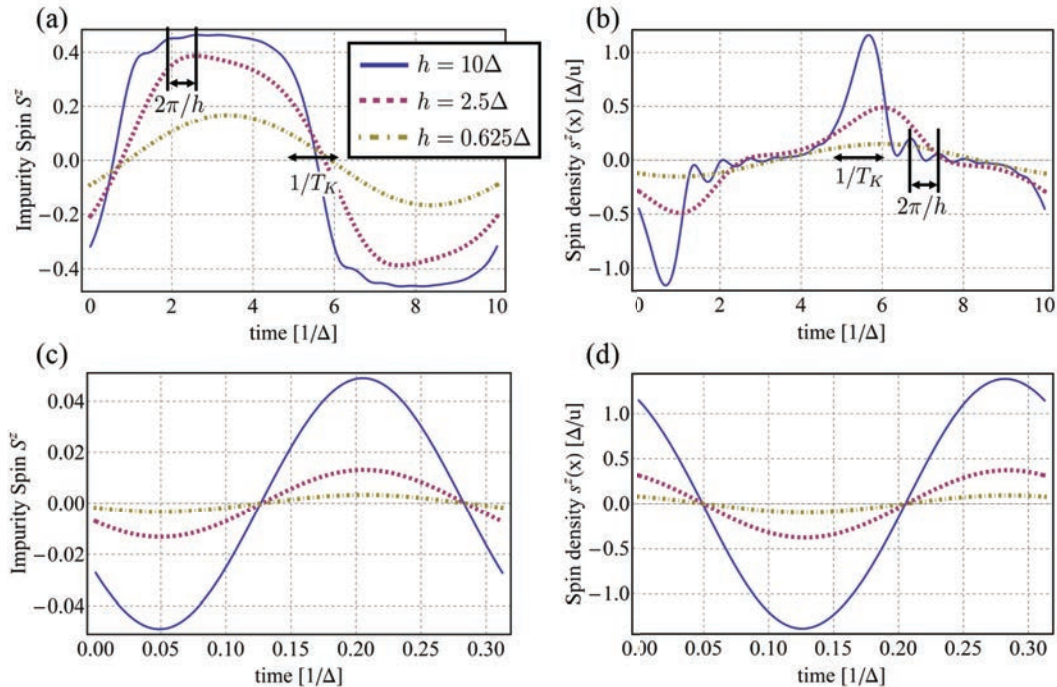


Figure 1: Time evolution of the impurity spin polarization (a), (c) and the spin density of the fermionic bath (b), (d) in a period for several choices of local magnetic field intensities  $h$  ( $h = 10\Delta$ , blue solid line;  $h = 2.5\Delta$ , red dashed line;  $h = 0.625\Delta$ , yellow dashed dotted line). (a) and (b) are the figures for  $\tau = 10/\Delta$ , while (c) and (d) are for  $\tau = 0.3125/\Delta$ .

## 6 Conclusion

We have investigated the periodically-driven Kondo model at Toulouse limit. We have obtained the exact analytical time evolution of operators for arbitrary time-periodic local magnetic field and the in-plane exchange interaction. The relation between the spin density of the fermionic bath and the time derivative of the impurity spin polarization has also been obtained.

By calculating the impurity spin polarization and the spin density of the fermionic bath when the local magnetic field has a sinusoidal time dependence and the exchange interaction is time independent, we have found that their behavior in the time evolution changes with the local magnetic field intensity  $h$  and its frequency  $\Omega$ , and the Kondo temperature  $T_K$ . When the field intensity  $h$  is much larger than the frequency  $\Omega$  and the Kondo temperature  $T_K$ , they oscillate with the frequency of  $h$ , while in  $\Omega \gg h, T_K$ , they approach temporally constant values.

The future work is to characterize the nonequilibrium steady state when the external driving is very fast so that the observables approach temporally constant values when  $\Omega \gg h, T_K$ . In such cases, we naturally expect novel properties of many body systems caused by the fast driving

[26, 21]. These issues are under consideration. This work is partly supported by KAKENHI (No.25400366 and No.15H05855).

## References

- [1] Ian Affleck and Andreas W.W. Ludwig. Critical theory of overscreened Kondo fixed points. *Nucl. Phys. B*, 360(2-3):641–696, 1991.
- [2] Johannes Bauer, Christophe Salomon, and Eugene Demler. Realizing a Kondo-Correlated State with Ultracold Atoms. *Phys. Rev. Lett.*, 111:215304, Nov 2013.
- [3] J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, and T. Giamarchi. Quantum dynamics of impurities in a one-dimensional Bose gas. *Phys. Rev. A*, 85:023623, Feb 2012.
- [4] Cronenwett, Sara M. and Oosterkamp, Tjerk H. and Kouwenhoven, Leo P. A tunable kondo effect in quantum dots. *Science*, 281(5376):540–544, 1998.
- [5] Takeshi Fukuhara, Adrian Kantian, Manuel Endres, Marc Cheneau, Peter Schauß, Sebastian Hild, David Bellem, Ulrich Schollwöck, Thierry Giamarchi, Christian Gross, Immanuel Bloch, and Stefan Kuhr. Quantum dynamics of a mobile spin impurity. *Nat. Phys.*, 9(4):235–241, 2013.
- [6] Takeshi Fukuhara, Peter Schauß, Manuel Endres, Sebastian Hild, Marc Cheneau, Immanuel Bloch, and Christian Gross. Microscopic observation of magnon bound states and their dynamics. *Nature*, 502(7469):76–79, 2013.
- [7] D. Goldhaber-Gordon, J. Göres, M. A. Kastner, Hadas Shtrikman, D. Mahalu, and U. Meirav. From the Kondo Regime to the Mixed-Valence Regime in a Single-Electron Transistor. *Phys. Rev. Lett.*, 81:5225–5228, Dec 1998.
- [8] David Goldhaber-Gordon, Hadas Shtrikman, D. Mahalu, David Abusch-Magder, U. Meirav, and M. A. Kastner. Kondo effect in a single-electron transistor. *Nature*, 391(6663):156–159, 1998.
- [9] Y. Goldin and Y. Avishai. Nonlinear Response of a Kondo System: Direct and Alternating Tunneling Currents. *Phys. Rev. Lett.*, 81:5394–5397, Dec 1998.
- [10] Y. Goldin and Y. Avishai. Nonlinear response of a Kondo system: Perturbation approach to the time-dependent Anderson impurity model. *Phys. Rev. B*, 61:16750–16772, Jun 2000.
- [11] Matthias H. Hettler and Herbert Schoeller. Anderson Model Out of Equilibrium: Time-Dependent Perturbations. *Phys. Rev. Lett.*, 74:4907–4910, Jun 1995.
- [12] M. Heyl and S. Kehrein. Nonequilibrium steady state in a periodically driven Kondo model. *Phys. Rev. B*, 81:144301, Apr 2010.
- [13] Sebastian Hild, Takeshi Fukuhara, Peter Schauß, Johannes Zeiher, Michael Knap, Eugene Demler, Immanuel Bloch, and Christian Gross. Far-from-Equilibrium Spin Transport in Heisenberg Quantum Magnets. *Phys. Rev. Lett.*, 113:147205, Oct 2014.
- [14] A. Kaminski, Yu. V. Nazarov, and L. I. Glazman. Suppression of the Kondo Effect in a Quantum Dot by External Irradiation. *Phys. Rev. Lett.*, 83:384–387, Jul 1999.
- [15] A. Kaminski, Yu. V. Nazarov, and L. I. Glazman. Universality of the Kondo effect in a quantum dot out of equilibrium. *Phys. Rev. B*, 62:8154–8170, Sep 2000.
- [16] Sigmund Kohler, Jrg Lehmann, and Peter Hnggi. Driven quantum transport on the nanoscale. *Physics Reports*, 406(6):379 – 443, 2005.
- [17] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, and R. Grimm. Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture. *Nature*, 485(7400):615–618, 2012.
- [18] M Koschorreck, D Pertot, E Vogt, B Frohlich, M Feld, and M Kohl. Attractive and repulsive Fermi polarons in two dimensions. *Nature*, 485(7400):619–622, 2012.
- [19] Dmitry Lobaskin and Stefan Kehrein. Crossover from nonequilibrium to equilibrium behavior in the time-dependent Kondo model. *Phys. Rev. B*, 71:193303, May 2005.

- [20] Rosa López, Ramón Aguado, Gloria Platero, and Carlos Tejedor. Kondo Effect in ac Transport through Quantum Dots. *Phys. Rev. Lett.*, 81:4688–4691, Nov 1998.
- [21] Hirokazu Miyake, Georgios A. Siviloglou, Colin J. Kennedy, William Cody Burton, and Wolfgang Ketterle. Realizing the Harper Hamiltonian with Laser-Assisted Tunneling in Optical Lattices. *Phys. Rev. Lett.*, 111:185302, Oct 2013.
- [22] S. Nascimbène, N. Navon, K. J. Jiang, L. Tarruell, M. Teichmann, J. McKeever, F. Chevy, and C. Salomon. Collective Oscillations of an Imbalanced Fermi Gas: Axial Compression Modes and Polaron Effective Mass. *Phys. Rev. Lett.*, 103:170402, Oct 2009.
- [23] Tai-Kai Ng. ac Response in the Nonequilibrium Anderson Impurity Model. *Phys. Rev. Lett.*, 76:487–490, Jan 1996.
- [24] Yusuke Nishida. SU(3) Orbital Kondo Effect with Ultracold Atoms. *Phys. Rev. Lett.*, 111:135301, Sep 2013.
- [25] Peter Nordlander, Ned S. Wingreen, Yigal Meir, and David C. Langreth. Kondo physics in the single-electron transistor with ac driving. *Phys. Rev. B*, 61:2146–2150, Jan 2000.
- [26] Takashi Oka and Hideo Aoki. Photovoltaic Hall effect in graphene. *Phys. Rev. B*, 79:081406, Feb 2009.
- [27] Avraham Schiller and Selman Hershfield. Solution of an ac Kondo Model. *Phys. Rev. Lett.*, 77:1821–1824, Aug 1996.
- [28] Avraham Schiller and Selman Hershfield. Out-of-equilibrium Kondo effect: Response to pulsed fields. *Phys. Rev. B*, 62:R16271–R16274, Dec 2000.
- [29] André Schirotzek, Cheng-Hsun Wu, Ariel Sommer, and Martin W. Zwierlein. Observation of Fermi Polarons in a Tunable Fermi Liquid of Ultracold Atoms. *Phys. Rev. Lett.*, 102:230402, Jun 2009.
- [30] Jörg Schmid, Jürgen Weis, Karl Eberl, and Klaus v. Klitzing. A quantum dot in the limit of strong coupling to reservoirs. *Physica B*, 256258:182 – 185, 1998.
- [31] G. Toulouse. *C. R. Acad. Sci. B*, 268:1200, 1969.
- [32] Sebastian Will, Thorsten Best, Simon Braun, Ulrich Schneider, and Immanuel Bloch. Coherent Interaction of a Single Fermion with a Small Bosonic Field. *Phys. Rev. Lett.*, 106:115305, Mar 2011.
- [33] Ned S. Wingreen, Antti-Pekka Jauho, and Yigal Meir. Time-dependent transport through a mesoscopic structure. *Phys. Rev. B*, 48:8487–8490, Sep 1993.
- [34] B. H. Wu and J. C. Cao. Noise of quantum dots in the ac Kondo regime: Slave-boson mean-field method and Floquet theorem. *Phys. Rev. B*, 77:233307, Jun 2008.
- [35] Gergely Zaránd and Jan von Delft. Analytical calculation of the finite-size crossover spectrum of the anisotropic two-channel Kondo model. *Phys. Rev. B*, 61:6918–6933, Mar 2000.