“Trade Liberalization, Division of Labor and Welfare under Oligopoly”

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Trade Liberalization, Division of Labor and Welfare under Oligopoly*

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Abstract

Incorporating explicitly division of labor into a two-country general oligopolistic equilibrium model, we examine the firm productivity effect of trade liberalization and its welfare implication. We show that a tariff reduction increases the firm productivity of the trading industries but decreases that of the non-trading industries. An expansion of the trading industries, in contrast, decreases the firm productivity of both the trading and non-trading industries. We then find that a tariff reduction necessarily reduces welfare while the welfare effect of expansion of trading industries is ambiguous.

Keywords: General oligopolistic equilibrium, division of labor, firm productivity, welfare.

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1 Introduction

It is well-recognized in international economics that trade liberalization improves welfare. Traditional theory of comparative advantage tells that the opening of trade leads to higher welfare by making the world resource allocation more efficient. In addition, the presence of imperfect competition and/or economies of scale allows us to discover new sources of trade gains. Among others, consumers enjoy a larger variety of products under monopolistic competition, and international trade promotes competition in an oligopoly, both of which yield welfare improvements.¹

The developments of theory and empirics of international trade in this century have uncovered further gains from trade. The large body of literature on firm heterogeneity finds that international trade is welfare-improving because it increases the share of the more efficient firms.² Besides, Chaney and Ossa (2013) offer a new explanation of gains from trade. By introducing division of labor in a vertical production process into Krugman’s (1979) model of monopolistic competition, they demonstrate that international trade modelled by an increase in market size is gainful by deepening division of labor.³ While Chaney and Ossa (2013) assume away firm heterogeneity for simplicity, it is naturally expected that the reallocation effect and the firm productivity effect jointly enhance welfare. Recent empirical studies find that exporting firms become more productive once they start to export, which the literature calls ‘learning-by-exporting.’ De Loecker (2007) uses the micro data of manufacturing firms in Slovenia, finding that exporting entrants improve firm productivity. Using the Japanese manufacturing data, Yashiro and Hirano (2009) show that the firm productivity growth of large trading firms is higher

¹See, for example, Feenstra (2015) for the latest account of classical and new trade theories of gains from trade, and their empirical tests.
²Chaney and Ossa (2013, p. 177) refer to this effect as a ‘reallocation effect.’ Melitz (2003) is undoubtedly the most influential work in this literature; see Melitz and Redding (2014) for a comprehensive survey of the firm heterogeneity literature.
³This effect is called a ‘firm productivity effect’ in Chaney and Ossa (2013, p.177).
than that of non-trading firms. Wagner (2012) surveys the recent empirical studies that international trade improves the firm productivity of exporting firms.

This paper is closely related to the above direction of researches, but we employ an approach that is quite different from the previous works. In order to seek the effect of trade liberalization on firm productivity from another perspective, we combine the formulation of division of labor by Chaney and Ossa (2013) with a general oligopolistic equilibrium (GOLE) model developed by Neary (2016). Kamei (2014) develops a closed economy version of this model, showing welfare losses from a competition policy, i.e. an increase in the number of oligopolistic firms. We extend his model to a two-country reciprocal market model of Brander and Krugman (1983), and examine two forms of trade liberalization. The first form is trade liberalization at the intensive margin, which is modelled by a reduction in import tariff. The second form is trade liberalization at the extensive margin, i.e. an expansion of the share of trading industries (shrink of the share of non-trading industries). We show that trade liberalization at the intensive margin raises the firm productivity of the trading industries, but lowers that of the non-trading industries. We then find that trade liberalization at the extensive margin lowers the firm productivity of all industries. After demonstrating these results, we establish the following results on the welfare effect of trade liberalization. A tariff reduction necessarily reduces welfare, and the welfare effect of an expansion of the trading industries is ambiguous. While both of these welfare effects seem counter-intuitive in view of the existing literature, we carefully interpret them, and consider why our results are so different from the previous ones. To this end, we compare our results with those of Bastos and Straume (2012) and Kreickemeier and Meland (2013), both addressing the welfare effect of the above two kinds of trade liberalization in

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4The first version of Neary (2016) was released in 2002. Colacicco (2015) reviews the fundamental working of the GOLE model and some of its applications.
the segmented market GOLE model.

This paper is organized as follows. Section 2 presents a model. Sections 3 and 4 consider the effect of trade liberalization on the firm productivity and welfare, respectively. Section 5 concludes. The proof of a few propositions is left in Appendices.

2 Model

This section presents the model. Suppose two identical countries and a continuum of oligopolistic industries in a unit interval $[0, 1]$. Firms in industry $z \in [0, \tilde{z}]$ domestically serve and export while firms in industry $z \in [\tilde{z}, 1]$ just supply domestically. In what follows, we call the former industry a trading industry and the latter a non-trading industry. Both countries impose an ad-valorem tariff $t \geq 0$ on their imports of trading goods.

The representative consumer in the Home country solves the following utility maximization problem.

$$
\max_{\{x(z)\}} \int_0^1 \ln x(z)dz \quad \text{subject to} \quad \int_0^1 p(z)x(z)dz \leq I, \quad (1)
$$

where $x(z)$ and $p(z)$ are the consumption and price of Good $z$, and $I$ is (nominal) national income. Then, the first-order condition for utility maximization is $1/x(z) = \lambda p(z)$, where $\lambda$ is a Lagrangean multiplier, which represents marginal utility of income.

Following Neary (2016), we assume that all oligopolistic firms are ‘large’ in their product market, but ‘small’ in the economy as a whole. In other words, oligopolistic firms exercise market power in choosing output, but take economy-wide variables such as the wage rate $w$ and national income $I$ as given. This assumption allows us to set $\lambda = 1$ without loss of generality and express the demand function as $x(z) = 1/p(z)$. Substituting this into the direct utility function in (1), indirect utility or welfare $W$ is measured by

$$
W = -\int_0^1 \ln p(z)dz - \int_{\tilde{z}}^1 \ln p(z)dz - \int_{\tilde{z}}^1 \ln p(z)dz. \quad (2)
$$
The formulation of the production side is the same as that of Chaney and Ossa (2013) and Kamei (2014). We suppose division of labor under vertical specialization. In producing one unit of Good \( z \), a sequence of tasks are needed. The set of such tasks is given by a closed interval \([0, 2]\), and if task \( \omega \in [0, \omega_1] \) is completed, input \( \omega_1 \) is obtained. Then, input \( \omega_1 \) is transformed into another input \( \omega_2 (\omega_2 > \omega_1) \) by completing tasks \( \omega \in [\omega_1, \omega_2] \). In order for a production team to complete each task in \([\omega_1, \omega_2]\), the following amount of labor has to be employed.

\[
l(\omega_1, \omega_2) = \frac{1}{2} \int_{\omega_1}^{\omega_2} |c - \omega| \gamma d\omega, \quad \gamma > 0,
\]

where \( c \in [0, 2] \) is a core competency. Furthermore, fixed labor \( f > 0 \) is needed to launch each team. A team with a smaller range of tasks leads to a smaller labor requirement to produce a final good. Thus, if firms are rational, the core competences of teams are placed at equal intervals on the product line. Summarizing these assumptions, total cost of producing \( y \) units of output is derived as

\[
wT \left( f + y \int_0^T \omega^\gamma d\omega \right) = w \left( T f + \frac{yT^{-\gamma}}{1 + \gamma} \right), \quad (3)
\]

where \( w \) is the wage rate, and \( T \) is the number of teams. From Eq. (3), the firm cannot reduce production cost by organizing an infinite number of teams because there is a team fixed cost, \( f \), for organizing a team. If the number of teams is determined so as to minimize total cost (3), the optimal number of teams is obtained as

\[
T = \left[ \frac{\gamma y}{(1 + \gamma) f} \right]^{\frac{1}{1 + \gamma}}. \quad (4)
\]

Substituting (4) into (3), total cost \( TC \) becomes a function of total output as follows.

\[
TC = w \left[ \frac{(1 + \gamma) f}{\gamma} \right]^{\frac{\gamma}{1 + \gamma}} y^{\frac{1}{1 + \gamma}}. \quad (5)
\]
Note here that increasing returns to scale exhibit in all industries from the assumption that $\gamma > 0$.

Having formalized the consumption and production sides, we now derive the equilibrium in the present model. In each oligopolistic industry, $n > 1$ firms play a quantity-setting Cournot game. Since the demand function of Good $z$ is $x(z) = 1/p(z)$, the profit of firm $i$ in the trading industry $\pi_i(z)$ is defined by

$$\pi_i(z) = p(z)y_i(z) + \frac{p(z)y_i^*(z)}{1 + t} - TC,$$

where $y_i(z)$ is supply for the domestic market, $y_i^*(z)$ is supply for the exporting market, and the inverse demand and total cost are given by

$$p(z) = \frac{1}{\sum_{i=1}^{n} y_i(z) + \sum_{i=1}^{n} y_i^*(z)}, \quad TC = w \left[ \frac{(1 + \gamma)f}{\gamma} \right] \frac{\gamma}{1 + \gamma} [y_i(z) + y_i^*(z)]^{1 + \gamma}. \quad (6)$$

Similarly, the profit of the non-trading industry is

$$\pi_i(z) = p(z)y_i(z) - TC,$$

where inverse demand and total cost are

$$p(z) = \frac{1}{\sum_{i=1}^{n} y_i(z)}, \quad TC = w \left[ \frac{(1 + \gamma)f}{\gamma} \right] \frac{\gamma}{1 + \gamma} [y_i(z)]^{1 + \gamma}. \quad (7)$$

Profit maximization gives the familiar first-order condition that marginal revenue is equal to marginal cost. Given the symmetry among $n$ firms and the assumption of identical countries, this condition takes the forms

$$\frac{(n - 1)y(z) + ny^*(z)}{n^2[y(z) + y^*(z)]^2} = \frac{w[y(z) + y^*(z)]^{-\frac{\gamma}{1 + \gamma}} f^{\frac{1}{1 + \gamma}}}{\gamma^{\frac{1}{1 + \gamma}} (1 + \gamma)^{\frac{1}{1 + \gamma}}},$$

$$\frac{ny(z) + (n - 1)y^*(z)}{n^2[y(z) + y^*(z)]^2} = \frac{(1 + t)w[y(z) + y^*(z)]^{-\frac{\gamma}{1 + \gamma}} f^{\frac{1}{1 + \gamma}}}{\gamma^{\frac{1}{1 + \gamma}} (1 + \gamma)^{\frac{1}{1 + \gamma}}},$$

for each trading industry. Summing these equations up and solving for total output $y(z) + y^*(z)$, we have

$$y(z) + y^*(z) = \left[ \frac{(2n - 1)\gamma^{\frac{1}{1 + \gamma}} (1 + \gamma)^{\frac{1}{1 + \gamma}}}{n^2(2 + t)w f^{\frac{1}{1 + \gamma}}} \right]^{1 + \gamma}. \quad (8)$$
It follows from (7) that aggregate labor demand in the trading industries becomes

\[ \int_0^{\tilde{z}} n \left[ \frac{(1 + \gamma)f}{\gamma} \right]^{\frac{1}{1+\gamma}} [y(z) + y^{*}(z)]^{\frac{1}{1+\gamma}} \, dz = \frac{\tilde{z}(2n - 1)(1 + \gamma)}{n(2 + t)w}. \] (9)

Making a parallel manipulation, the equilibrium output and aggregate labor demand in the non-trading industries are computed as follows.

\[ y(z) = \left[ \frac{(n - 1) \gamma^{\frac{1}{1+\gamma}} (1 + \gamma)^{\frac{1}{1+\gamma}}}{n^2 w f^{\frac{1}{1+\gamma}}} \right]^{1+\gamma}. \] (10)

\[ \int_{\tilde{z}}^{1} n \left[ \frac{(1 + \gamma)f}{\gamma} \right]^{\frac{1}{1+\gamma}} [y(z)]^{\frac{1}{1+\gamma}} \, dz = \frac{(1 - \tilde{z})(n - 1)(1 + \gamma)}{nw}. \] (11)

Accordingly, denoting the labor endowment by \( L \), the labor market-clearing condition is

\[ \frac{\tilde{z}(2n - 1)(1 + \gamma)}{n(2 + t)w} + \frac{(1 - \tilde{z})(n - 1)(1 + \gamma)}{nw} = L, \]

which leads to the equilibrium wage rate:

\[ w = \frac{(1 + \gamma)[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]}{nL(2 + t)}. \] (12)

Utilizing Eqs. (8), (10) and (12), the optimal number of teams in the trading and non-trading industries is respectively derived as

\[ T_1 = \frac{(2n - 1)\gamma L}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]} \] (13)

\[ T_2 = \frac{(n - 1)(2 + t)\gamma L}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]}. \] (14)

where \( T_1 \) and \( T_2 \) are the optimal number of teams in each trading and non-trading industry, respectively. These expressions of the optimal number of firms, which serve as a proxy of firm productivity, will prove useful in understanding the effect of trade liberalization. This completes the description of the model. The subsequent sections address the effects of trade liberalization on the firm productivity and welfare.
3 Trade Liberalization and Firm Productivity

This section investigates the effect of trade liberalization on the firm productivity, namely, the optimal number of teams derived just above. As already noted in Introduction, we consider two types of trade liberalization. The first is a reduction in import tariff which we call trade liberalization at the intensive margin. The second is an increase in $\tilde{z}$, namely, an exogenous expansion of the share of the trading industries, and we call it trade liberalization at the extensive margin. Before addressing the firm productivity effect of trade liberalization, we briefly examine the effect of these trade liberalization policies on the equilibrium wage rate.

Proposition 1. Trade liberalization at the intensive margin and the extensive margin raises the equilibrium wage rate.

Proof. Differentiating (12) with respect to $t$ and $\tilde{z}$, we have

$$\frac{\partial w}{\partial t} = -\frac{\tilde{z}(2n - 1)(1 + \gamma)}{nL(2 + t)^2} < 0, \quad \frac{\partial w}{\partial \tilde{z}} = \frac{(1 + \gamma)(1 - (n - 1)t)}{nL(2 + t)}.$$

As proved in Appendix 1, the prohibitive tariff is $t = 1/(n - 1)$, and hence $\partial w/\partial \tilde{z} > 0$ for any $t \in [0, 1/(n - 1)]$. These signs establish the proposition. 

The intuition behind this result is straightforward. From Eq. (8), a tariff reduction induces an increase in total output and labor demand in the trading industries, thereby raising the wage rate. By contrast, the effect of

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5This definition of firm productivity follows Chaney and Ossa (2013) and Kamei (2014).
6Bastos and Straume (2012) and Kreickemeier and Meland (2013) also analyze the impacts of these two scenarios of trade liberalization.
7These terminologies of trade liberalization follow those of Kreickemeier and Meland (2013). Bastos and Straume (2012) refer to the former as 'product market integration' and the latter to 'increased trade openness,' respectively.
increasing $\tilde{z}$ on the wage rate seems ambiguous because labor demand in the trading industries increases but labor demand in the non-trading industries decreases. However, this type of trade liberalization also raises the wage rate since an increase in labor demand in the trading industries outweighs a decrease in labor demand in the non-trading industries.\footnote{This is algebraically confirmed by noting that total output of the trading industries (Eq. (8)) is larger than that of the non-trading industries (Eq. (10)).} It is instructive to mention that the same result as Proposition 1 holds even in the absence of increasing returns to scale; see Propositions 2 and 4 in Kreickemeier and Meland (2013).

At this stage, one may guess that the effect of two types of trade liberalization on the firm productivity is qualitatively the same. But, we will show that this is not the case. The following result concerns the effect of tariff reduction on the firm productivity.

**Proposition 2.** Trade liberalization at the intensive margin raises the firm productivity in the trading industries, but lowers that in the non-trading industries.

**Proof.** Differentiating (13) and (14) with respect to $t$, we have

\[
\frac{dT_1}{dt} = -\frac{(1 - \tilde{z})(2n - 1)(n - 1)\gamma L}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]^2} < 0
\]

\[
\frac{dT_2}{dt} = \frac{\tilde{z}(2n - 1)(n - 1)\gamma L}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]^2} > 0.
\]

These inequalities imply the proposition. ||

The effect of tariff reduction on the equilibrium wage plays a crucial role for this result. As shown in Proposition 1, the wage rate rises as a result of tariff reduction. The resultant higher wage rate gives rise to a higher marginal cost in the non-trading industries. Consequently, total output of each firm
decreases in the non-trading industries, and so does the firm productivity. However, in the trading industries, the first-order effect of tariff reduction (the term $2 + t$ in the right-hand side of Eq. (8)) dominates the second-order effect through the wage increase. Thus, firms in the trading industries will expand total output, which, in turn, raises the firm productivity.

While the above result provides a theoretical rationale for the well-known evidence that trade liberalization improves the productivity the trading industries, the same is not true of trade liberalization at the extensive margin. This is stated in:

**Proposition 3.** Trade liberalization at the extensive margin lowers the firm productivity in both the trading and the non-trading industries.

**Proof.** Differentiating (13) and (14) with respect to $\tilde{z}$ yields

$$
\frac{dT_1}{d\tilde{z}} = -\frac{(2n - 1)\gamma L[1 - (n - 1)t]}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]^2} < 0
$$

$$
\frac{dT_2}{d\tilde{z}} = -\frac{(n - 1)(2 + t)\gamma L[1 - (n - 1)t]}{n(1 + \gamma)f[\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)]^2} < 0,
$$

which proves the proposition. ||

We know from Proposition 1 that an increase in $\tilde{z}$ leads to a higher wage rate. Since total output and the firm productivity in all industries are negatively related to $w$, this liberalization policy reduces total output and the firm productivity in all industries. Moreover, Eqs. (13) and (14) allow us to see that the number of teams decreases more in trading industries than in non-trading industries. When the increased wage raises the production cost and price, consumers do not decrease the consumption of all goods equally. The reason is that consumers minimize the reduction of consumption of non-trading goods to maintain the utility level.\(^9\)

\(^9\)This is because marginal utility of trading goods is lower than that of non-trading
We close this section by comparing our results on the firm productivity effect of trade liberalization with those in Chaney and Ossa (2013) who incorporate the division of labor under vertical specialization into monopolistic competition of Krugman (1979). They show the market size expansion due to economic integration reduces all the prices, and raises the firm productivity. In contrast, Proposition 2 above suggests that trade liberalization in the form of tariff reduction necessarily decreases the prices of traded goods, but increases those of non-traded goods. This asymmetric impacts on product prices lead to an increase in the firm productivity of trading industries but a decrease in that of non-trading industries. Furthermore, Proposition 3 tells that an expansion of the proportion of trading industries increases the prices of all goods, and decreases the firm productivity of the all industries.

4 Welfare

This section turns to the welfare effects of trade liberalization. Let us begin by addressing the welfare effect of trade liberalization at the intensive margin, i.e. tariff reduction. This is formally stated in:

Proposition 4. Trade liberalization at the intensive margin necessarily reduces welfare.

Proof. See Appendix 2. ||

This result, which has also been shown in Bastos and Straume (2012) and Kreickemeier and Meland (2013), is somewhat surprising in view of the gains-from-trade result in a partial equilibrium model of Brander and Krugman goods.

\[10\text{Note that all firms sell domestically and export in Chaney and Ossa (2013) since Krugman (1979) assumes so.}\]

\[11\text{The same survives our model; an exogenous increase in } L \text{ improves the firm productivity of all industries by lowering the wage rate and marginal costs.}\]
(1983). The reason for this seemingly counter-intuitive result is as follows. When a tariff is reduced, marginal cost and the product price of traded goods fall since the first-order effect of tariff reduction is stronger than the second-order effect of wage increase. By contrast, the wage increase induced by tariff reduction leads to higher marginal cost in the non-trading industries. Hence, tariff reduction expands the price variability across goods, which results in welfare losses.

In contrast to the above case, it is regrettably impossible to obtain a clear result on the welfare effect of an increase in $\tilde{z}$. That is,

**Proposition 5.** *It is ambiguous whether trade liberalization at the extensive margin improves welfare.*

*Proof.* See Appendix 3. ||

Proposition 3 convinces us that an increase in $\tilde{z}$ decreases the firm productivity of all industries, which has a negative effect on welfare. In addition to this effect, the price variability among goods plays a crucial role for the welfare effect. One of the sources of the price variability is the proportion of trading industries $\tilde{z}$ as Kreickemeier and Meland (2013) highlight. When $\tilde{z}$ is sufficiently high, incremental $\tilde{z}$ reduces the price variability among goods, and positively affects welfare. This is confirmed as follows. Relating Proposition 3 to the product prices, an increase in $\tilde{z}$ raises the prices of all goods, but the change in prices is smaller as $\tilde{z}$ is large. Therefore, if $\tilde{z}$ is sufficiently large, trade liberalization at the extensive margin reduces the price variability across goods. If this effect dominates the firm productivity effect explained earlier, increased $\tilde{z}$ turns out welfare-improving.
5 Concluding Remarks

This paper has explored the effect of trade liberalization on the firm productivity and welfare by allowing for division of labor in a two-country GOLE framework. We have shown that trade liberalization at the intensive margin, i.e. a tariff reduction, improves the trading industries’ productivity but lowers the non-trading industries’ productivity. Furthermore, the productivity of both trading and non-trading industries declines as a result of trade liberalization at the extensive margin which is modelled by an expansion of trading industries. We have then demonstrated that trade liberalization at the intensive margin necessarily becomes welfare-reducing whereas it is ambiguous whether trade liberalization at the extensive margin improves welfare.

Our results hopefully provide a new insight on the effect of trade liberalization, but further research is needed. First, we guess that our losses-from-trade result is sensitive to the functional forms we assume: logarithmic utility function and exponential form of total cost. It is conjectured that other functional forms, e.g. a quadratic utility function, may dampen or reverse our conclusion of losses from trade. Second, following Chaney and Ossa (2013), we have assumed away firm heterogeneity a la Melitz (2003). If the reallocation effect in Melitz (2013) as well as the firm productivity effect is allowed, the prediction of this paper may be modified. Third, an empirical study is called for so as to qualify our theoretical results. These agenda are left as a future direction of researches.

Appendix 1: Derivation of the Prohibitive Tariff

Solving the system of first-order conditions for profit maximization, the Cournot equilibrium outputs are solved as

\[ y(z) = \frac{(1 + nt)A}{2n - 1}, \quad y^*(z) = \frac{[1 - (n - 1)t]A}{2n - 1} \]
\[ A = \frac{w \left[ y(z) + y^*(z) \right]^{-\frac{\gamma}{1+\gamma}} f^{1+\gamma}}{\gamma^{1+\gamma}(1+\gamma)^{1+\gamma}}, \]

where \( y(z) + y^*(z) \) in \( A \) is a function of primitive parameters only; see Eq. (8). Thus, the prohibitive tariff becomes \( t = 1/(n-1) \) by setting \( y^*(z) = 0 \).

**Appendix 2: Proof of Proposition 4**

Noting that the product price and output in all the trading and non-trading industries become the same, welfare defined in (2) can be simplified to

\[ W = -\tilde{z} \ln p_1 - (1 - \tilde{z}) \ln p_2, \tag{15} \]

where \( p_1 \) and \( p_2 \) are the price of each trading and non-trading good, and are given as follows.

\[
\begin{align*}
p_1 &= n^{-1} \left[ \frac{(2n-1)\gamma^{\frac{\gamma}{1+\gamma}}(1+\gamma)^{\frac{1}{1+\gamma}}}{n^2(2+t)wf^{\frac{1}{1+\gamma}}} \right]^{-(1+\gamma)} \tag{16} \\
p_2 &= n^{-1} \left[ \frac{(n-1)\gamma^{\frac{\gamma}{1+\gamma}}(1+\gamma)^{\frac{1}{1+\gamma}}}{n^2w^{\frac{1}{1+\gamma}}} \right]^{-(1+\gamma)} , \tag{17}
\end{align*}
\]

where \( w \) is defined in (12). By making lengthy manipulations, a small increase in \( t \) affects \( W \) as follows.

\[
\frac{dW}{dt} = -\tilde{z} \frac{d \ln p_1}{dt} - (1 - \tilde{z}) \frac{d \ln p_2}{dt} \\
= -\tilde{z} \frac{dp_1/dt}{p_1} - (1 - \tilde{z}) \frac{dp_2/dt}{p_2} \\
= -\frac{\tilde{z}(1 - \tilde{z})(1 + \gamma)(n-1)}{\tilde{z}(2n-1) + (1 - \tilde{z})(n-1)(2+t)} + \frac{(1 - \tilde{z})\tilde{z}(1 + \gamma)(2n-1)}{(2+t)\tilde{z}(2n-1) + (1 - \tilde{z})(n-1)(2+t)} \\
= \frac{\tilde{z}(1 - \tilde{z})(1 + \gamma)[1 - (n-1)t]}{(2+t)\tilde{z}(2n-1) + (1 - \tilde{z})(n-1)(2+t)} > 0.
\]

The last inequality uses the assumption of \( 0 \leq t \leq 1/(n-1) \). Thus, any tariff reduction worsens welfare.
Appendix 3: Proof of Proposition 5

Differentiating (15) with respect to \( \tilde{z} \) and using the comparative statics result in Proposition 1 yield

\[
\frac{dW}{d\tilde{z}} = -\ln p_1 + \ln p_2 - \tilde{z} \frac{d\ln p_1}{d\tilde{z}} - (1 - \tilde{z}) \frac{d\ln p_2}{d\tilde{z}}
\]

\[
= \ln \left( \frac{p_2}{p_1} \right) - \tilde{z} \frac{d p_1/d\tilde{z}}{p_1} - (1 - \tilde{z}) \frac{d p_2/d\tilde{z}}{p_2}
\]

\[
= (1 + \gamma) \ln \left[ \frac{2n - 1}{(n - 1)(2 + t)} \right] - \tilde{z} \frac{(1 + \gamma)[1 - (n - 1)t]}{\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)}
\]

\[
- (1 - \tilde{z}) \frac{(1 + \gamma)[1 - (n - 1)t]}{\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)}
\]

\[
= (1 + \gamma) \ln \left[ \frac{2n - 1}{(n - 1)(2 + t)} \right] - \tilde{z} \frac{(1 + \gamma)[1 - (n - 1)t]}{\tilde{z}(2n - 1) + (1 - \tilde{z})(n - 1)(2 + t)}. \]

The sign of the right-hand side is unclear because the first term is positive for \( 0 < t < 1/(n - 1) \) but the second term is negative.

References


