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Multiple point principle of the Standard Model with scalar singlet dark matter and right-handed neutrinos

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We consider the multiple point principle of the Standard Model with scalar singlet dark matter (DM) and three heavy right-handed neutrinos at the scale where the beta function β_λ of the effective Higgs self-coupling $\lambda_{\rm eff}$ becomes zero. We do a two-loop analysis and find that the top quark mass M_t and the Higgs portal coupling κ are strongly related to each other. One of the good points of this model is that a larger $M_t(\gtrsim 171\,{\rm GeV})$ is allowed. This fact is consistent with the recent experimental value $M_t = 173.34 \pm 0.76\,{\rm GeV}$ [ATLAS, CDF, CMS, and D0 Collaborations, arXiv:1403.4427 [hep-ex]], which corresponds to the DM mass $769\,{\rm GeV} \leq m_{\rm DM} \leq 1053\,{\rm GeV}$.

Subject Index B32, B37

1. Introduction

The discovery of the Higgs-like particle and its mass [1,2] is a very meaningful result for the Standard Model (SM). It suggests that the Higgs potential can be stable up to the Planck scale $M_{\rm pl}$ and also that both the Higgs self-coupling λ and its beta function β_λ become very small around the Planck scale. This fact has attracted much attention, and there are many works trying to find its physical meaning [3–20].

One interesting and meaningful study is to consider how the physics beyond the SM affects such a criticality. For example, recently there has been a two-loop analysis of the Higgs portal Z_2 scalar model [21]. In this model, the SM singlet scalar is a dark matter (DM) candidate, and it is found that its mass can be predicted to be $400\,{\rm GeV} < m_{\rm DM} < 470\,{\rm GeV}$ from the requirement that λ and β_λ simultaneously become zero at 10^{17} GeV; this is usually called the multiple point principle (MPP) [3–5].

In this paper, we study the MPP of the next minimal extension of the SM, namely, besides the Higgs portal Z_2 scalar, we include SM singlet heavy right-handed neutrinos [19,22,23]. The MPP of this model at the (reduced) Planck scale $M_{\rm pl}$ has already been investigated in Ref. [19]. There, by using the two-loop beta functions and the tree-level Higgs potential, they concluded that $m_{\rm DM}$ and the heavy Majorana mass $M_{\rm R}$ of the right-handed neutrino should be

$$8.5\,(8.0) \times 10^2\,{\rm GeV} \leq m_{\rm DM} \leq 1.4\,(1.2) \times 10^3\,{\rm GeV}, \tag{1}$$

$$6.3\,(5.5) \times 10^{13}\,{\rm GeV} \leq M_{\rm R} \leq 1.6\,(1.2) \times 10^{14}\,{\rm GeV}, \tag{2}$$

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within $172.6\,\text{GeV} \le M_t \le 174.1\,\text{GeV}$. The different points in this paper are as follows:

(1) We consider the MPP at the scale where β_λ becomes zero. Namely, we do not fix the MPP scale at M_{pl}. As a result, the condition $\beta_\lambda = 0$ does not reduce the degrees of freedom of the parameters.
(2) In addition to the two-loop beta functions, we also calculate the one-loop effective potential.
(3) We fix M_{R} to 10^{13} GeV, and include the Yukawa coupling Y_{R} between the Z_2 scalar and the right-handed neutrinos.

Although, within the renormalizable Lagrangian, there are also two scalar couplings in this model (see Eq. (12)), we focus on λ (and β_λ) in this paper.[1] The existence of heavy right-handed neutrinos is naturally needed if we try to explain the light neutrino masses by the seesaw mechanism. Thus, this model is phenomenologically interesting because it can explain both DM and the light neutrino masses.

This paper is organized as follows. In Sect. 2, we review the MPP of the pure SM for the later discussion. In Sect. 3, we give a two-loop analysis of the SM with the scalar singlet DM and three right-handed neutrinos. In Sect. 4, a summary is given.

2. Preliminary: Multiple point principle of SM

In the SM, the one-loop effective potential in the Landau gauge is given by

$$V_{\text{eff}}(\phi,\mu) = V_{\text{tree}}(\phi,\mu) + V^{\text{SM}}_{1\,\text{loop}}(\phi,\mu), \tag{3}$$

where

$$V_{\text{tree}}(\phi,\mu) := e^{4\Gamma(\phi)}\frac{\lambda(\mu)}{4}\phi^4, \tag{4}$$

$$\begin{aligned} V_{1\,\text{loop}}(\phi) := e^{4\Gamma(\phi)}\Bigg\{ & -6\cdot\frac{M_t(\phi)^4}{64\pi^2}\left[\log\left(\frac{M_t^2(\phi)}{\mu^2}\right)-\frac{3}{2}+2\Gamma(\phi)\right] \\ & +3\cdot\frac{M_W(\phi)^4}{64\pi^2}\left[\log\left(\frac{M_W^2(\phi)}{\mu^2}\right)-\frac{5}{6}+2\Gamma(\phi)\right] \\ & +3\cdot\frac{M_Z(\phi)^4}{64\pi^2}\left[\log\left(\frac{M_Z^2(\phi)}{\mu^2}\right)-\frac{5}{6}+2\Gamma(\phi)\right]\Bigg\}, \end{aligned} \tag{5}$$

$$M_t(\phi) = \frac{y_t(\mu)}{\sqrt{2}}\phi, \quad M_W(\phi) = \frac{g_2(\mu)}{2}\phi, \quad M_Z = \frac{\sqrt{g_2^2(\mu)+g_{\text{Y}}^2(\mu)}}{2}\phi. \tag{6}$$

Here, μ is the renormalization scale, $\Gamma(\phi)$ is the wave function renormalization, and $\lambda(\mu)$, $y_t(\mu)$, $g_2(\mu)$, and $g_Y(\mu)$ are the renormalized couplings.[2] By using these results, the effective Higgs self-coupling $\lambda_{\text{eff}}(\phi,\mu)$ can be defined as

$$V_{\text{eff}}(\phi,\mu) := \frac{\lambda_{\text{eff}}(\phi,\mu)}{4}\phi^4. \tag{7}$$

To minimize the contribution of $V^{\text{SM}}_{1\,\text{loop}}(\phi,\mu)$, we set $\phi = \mu$ in the following discussion.

[1] It is difficult to realize the MPP of the other scalar couplings simultaneously in addition to λ. This is discussed in Appendix B.

[2] For the beta functions of the SM, see, e.g., Refs. [21,24,29]. Alternatively, we can reproduce them by using the results in Appendix A.

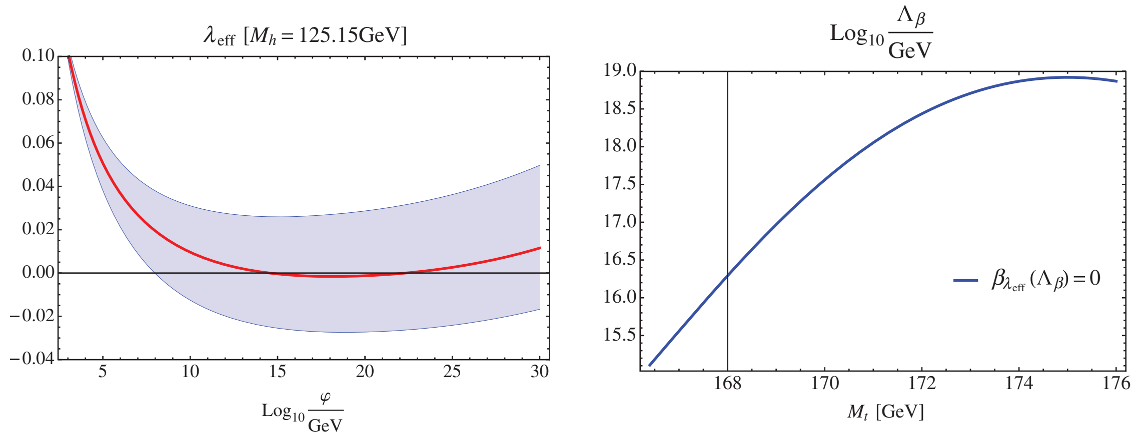

Fig. 1. Left: the running effective Higgs self-coupling $\lambda_{\rm eff}$ as a function of the Higgs field ϕ. The blue band corresponds to 95% confidence level (CL) deviation of the top quark pole mass M_t. Right: the scale Λ_β where $\beta_{\lambda_{\rm eff}}$ becomes zero as a function of M_t.

The left panel of Fig. 1 shows $\lambda_{\rm eff}(\phi)$ as a function of ϕ. For the initial values, we have used the numerical results of Ref. [24], and the Higgs mass is fixed at

$$M_h = 125.15\,\mathrm{GeV}. \tag{8}$$

We use Eq. (8) as a typical value in the following discussion. The band corresponds to 95% CL deviation of the top quark pole mass M_t. For the 1σ level, this is given by [25]

$$M_t = 171.2 \pm 2.4\,\mathrm{GeV}. \tag{9}$$

If we assume that all the other parameters of the SM except for M_t are fixed, we can find the scale Λ_β where $\beta_{\lambda_{\rm eff}}$ becomes zero as a function of M_t. Here, $\beta_{\lambda_{\rm eff}}$ means

$$\beta_{\lambda_{\rm eff}}(\phi) := \frac{d\lambda_{\rm eff}(\phi)}{d\log\phi}. \tag{10}$$

The right panel of Fig. 1 shows Λ_β as a function of M_t. The MPP requires that $\lambda_{\rm eff}(\Lambda_\beta)$ should become zero, and predicts

$$M_t = 170.9\,\mathrm{GeV}. \tag{11}$$

This is the MPP of the pure SM. In the next section, we discuss the MPP of the SM with the scalar singlet DM and three right-handed neutrinos.

3. MPP of the SM with scalar singlet dark matter and right-handed neutrinos

We consider the following renormalizable Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm SM} &+ \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{m_{\rm DM}^2}{2}S^2 - \frac{\kappa}{2}S^2H^\dagger H - \frac{\lambda_{\rm DM}}{4!}S^4 + \sum_{j=1}^{3}\bar{\nu}_{{\rm R}j}i\gamma^\mu\partial_\mu\nu_{{\rm R}j} \\ &- \sum_{i,j}\left(y_{\nu ij}\bar{L}_iH^\dagger\nu_{{\rm R}j} + \mathrm{h.c}\right) - \sum_{i,j}\left(M_{{\rm R}ij} + \frac{Y_{{\rm R}ij}}{\sqrt{2}}S\right)\bar{\nu}_{{\rm R}i}^c\nu_{{\rm R}j}.\end{aligned} \tag{12}$$

Here, H is the Higgs field, S is the SM singlet real scalar field, $m_{\rm DM}$ is its mass, $\nu_{{\rm R}i}$ are right-handed neutrinos, $M_{{\rm R}ij}$ are their Majorana masses, and $(Y_{{\rm R}ij},\, y_{\nu ij})$ are the Yukawa couplings. For simplicity, we assume that $M_{\mathrm{R}ij}$, $Y_{\mathrm{R}ij}$, and $y_{\nu ij}$ are diagonalized, and also that they are respectively equal for the three generations. In this case, Eq. (12) becomes

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{m_{\mathrm{DM}}^2}{2}S^2 - \frac{\kappa}{2}S^2 H^\dagger H - \frac{\lambda_{\mathrm{DM}}}{4!}S^4 + \sum_{i=1}^{3}\bar{\nu}_{\mathrm{R}i} i\gamma^\mu \partial_\mu \nu_{\mathrm{R}i}$$
$$- y_\nu \sum_{i=1}^{3}\left(\bar{L}_i H^\dagger \nu_{\mathrm{R}i} + \mathrm{h.c}\right) - \sum_{i=1}^{3}\left(M_{\mathrm{R}} + \frac{Y_{\mathrm{R}}}{\sqrt{2}}S\right)\bar{\nu}^c_{\mathrm{R}i}\nu_{\mathrm{R}i}. \tag{13}$$

Thus, including the top mass M_t, there are seven unknown parameters,

$$M_t, \quad m_{\mathrm{DM}}, \quad \kappa, \quad \lambda_{\mathrm{DM}}, \quad y_\nu, \quad M_{\mathrm{R}}, \quad Y_{\mathrm{R}}, \tag{14}$$

in this model. In the following discussion, to distinguish the initial values of these parameters at $\mu = M_t$ from their running couplings, we use the subscript 0 for their initial values, like κ_0, except for M_t. Because S is the candidate for the DM, m_{DM} and κ must satisfy some relation such that they can explain the observed energy density [27]:

$$\Omega_{\mathrm{DM}}h^2 := \frac{\rho_{\mathrm{DM}}h^2}{\rho_{\mathrm{tot}}} = 0.1196 \pm 0.0031\ (68\%\ \mathrm{CL}). \tag{15}$$

For $m_{\mathrm{DM}} \gtrsim M_h$, this relation is approximately given by [28]

$$\log_{10}\kappa \simeq -3.63 + 1.04\log_{10}\left(\frac{m_{\mathrm{DM}}}{\mathrm{GeV}}\right). \tag{16}$$

Moreover, if we assume that the neutrino mass is 0.1 eV, y_ν and M_{R} must satisfy

$$-\frac{M_{\mathrm{R}}}{2}\left(1 - \sqrt{1 + \frac{2y_\nu^2 v_h^2}{M_{\mathrm{R}}^2}}\right) \simeq \frac{y_\nu^2 v_h^2}{2M_{\mathrm{R}}} = 0.1\ \mathrm{eV}, \tag{17}$$

where v_h is the Higgs expectation value. This is the usual relation of the seesaw mechanism. In the following discussion, we choose $M_{\mathrm{R}} = 10^{13}$ GeV, so y_ν is fixed by Eq. (17). As a result, four of the seven parameters remain free; they are

$$M_t, \quad \kappa, \quad \lambda_{\mathrm{DM}}, \quad \text{and} \quad Y_{\mathrm{R}}. \tag{18}$$

To discuss how the effective couplings behave at the high-energy scale, we must know the renormalization group equations (RGEs) of this model. Their results are presented in Appendix A. Here, note that the contributions from the heavy right-handed neutrinos should be taken into account at the scale where $\mu \geq M_{\mathrm{R}}$. The one-loop effective potential of the Higgs field is given by

$$V_{1\,\mathrm{loop}}(\phi,\mu) := \begin{cases} V^{\mathrm{SM}}_{1\,\mathrm{loop}}(\phi) + \dfrac{M_{\mathrm{DM}}(\phi)^4}{64\pi^2}\left[\log\left(\dfrac{M_{\mathrm{DM}}(\phi)^2}{\mu^2}\right) - \dfrac{3}{2}\right] - 6\cdot\dfrac{M_\nu^-(\phi)^4}{64\pi^2}\left[\log\left(\dfrac{M_\nu^-(\phi)^2}{\mu^2}\right) - \dfrac{3}{2}\right] \\ \quad (\text{for } \phi < M_{\mathrm{R}}), \\ V^{\mathrm{SM}}_{1\,\mathrm{loop}}(\phi) + \dfrac{M_{\mathrm{DM}}(\phi)^4}{64\pi^2}\left[\log\left(\dfrac{M_{\mathrm{DM}}(\phi)^2}{\mu^2}\right) - \dfrac{3}{2}\right] - 6\cdot\dfrac{M_\nu^-(\phi)^4}{64\pi^2}\left[\log\left(\dfrac{M_\nu^-(\phi)^2}{\mu^2}\right) - \dfrac{3}{2}\right] \\ \quad -6\cdot\dfrac{M_\nu^+(\phi)^4}{64\pi^2}\left[\log\left(\dfrac{M_\nu^+(\phi)^2}{\mu^2}\right) - \dfrac{3}{2}\right] \quad (\text{for } \phi > M_{\mathrm{R}}), \end{cases} \tag{19}$$

where

$$M_{\mathrm{DM}}(\phi) := \sqrt{e^{2\Gamma(\phi)}\frac{\kappa\phi^2}{2} + m_{\mathrm{DM}}^2}, \quad M_\nu^{\pm}(\phi) := \frac{M_{\mathrm{R}}}{2}\left(1 \pm \sqrt{1 + \frac{2y_\nu^2 e^{2\Gamma(\phi)}\phi^2}{M_{\mathrm{R}}^2}}\right). \tag{20}$$

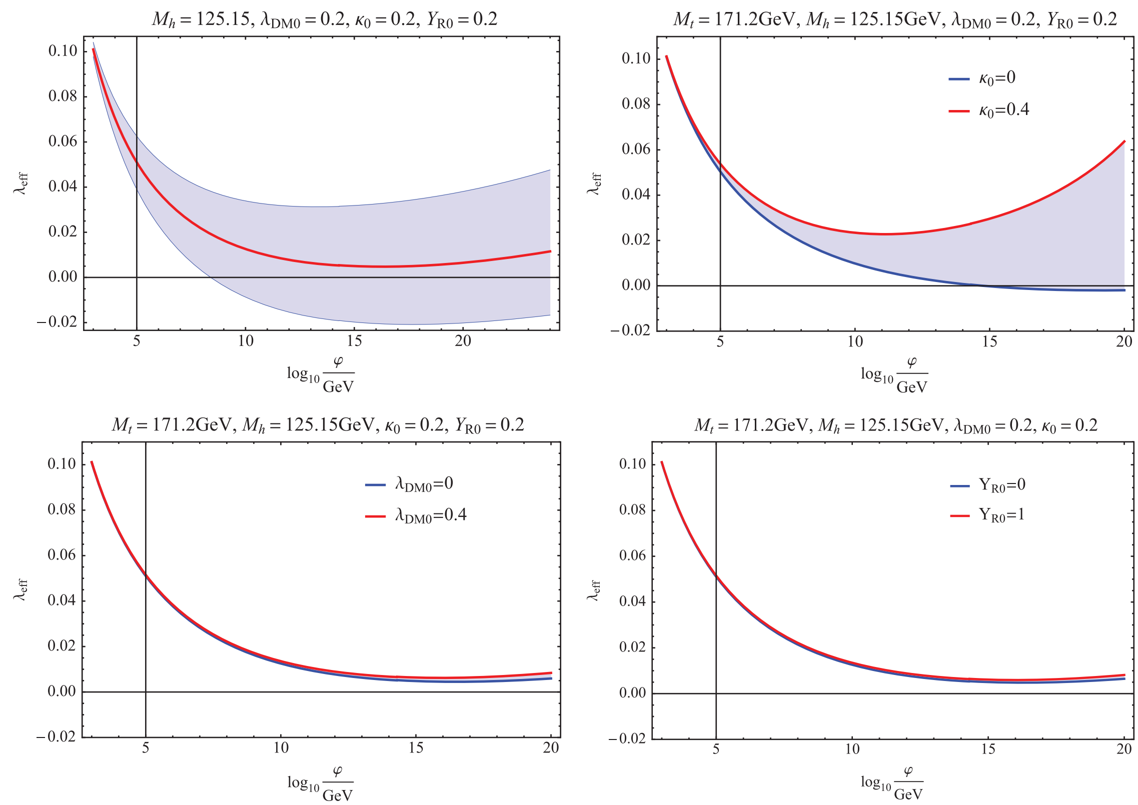

Fig. 2. The running effective Higgs self-coupling λ_{eff} as a function of ϕ. The upper left (right) panel shows the M_t (κ_0) dependence. For M_t, the blue band corresponds to 95% CL deviation from 171.2 GeV. The lower left (right) panel shows the λ_{DM0} (Y_{R0}) dependence.

In these expressions, we have put $S = 0$ because we now focus on the MPP of the Higgs sector.[3] Furthermore, we can neglect m_{DM} in Eq. (20) because its effect is very small when $\phi \gg m_{\text{DM}}$. As in Sect. 2, we set $\phi = \mu$, and define the effective Higgs self-coupling λ_{eff} as

$$\lambda_{\text{eff}}(\phi) := \frac{4}{\phi^4} V(\phi) = \frac{4}{\phi^4}\left(V_{\text{tree}}(\phi) + V_{1\,\text{loop}}(\phi)\right). \tag{21}$$

Figure 2 shows $\lambda_{\text{eff}}(\phi)$ for various values of the parameters. Here, the typical values are chosen to be

$$\lambda_{\text{DM0}} = 0.2, \quad \kappa_0 = 0.2, \quad Y_{\text{R0}} = 0.2. \tag{22}$$

One can see that λ_{eff} depends mainly on M_t and κ_0, and hardly on λ_{DM0} and Y_{R0}. This is because λ_{DM} does not appear in β_λ and Y_{R} appears at the two-loop level (see Eq. (A7) in Appendix A). Therefore, by fixing λ_{DM} and Y_{R}, we can relate M_t and κ_0 from the MPP.

By the same procedure of Sect. 2, we can calculate the scale Λ_β where $\beta_{\lambda_{\text{eff}}}$ becomes zero, and obtain $\lambda_{\text{eff}}(\Lambda_\beta)$ as a function of M_t and κ_0. Figure 3 shows the results. In the upper (lower) panels, Y_{R0} is fixed to 0.2 (0.7). The difference between the left and right panels is whether the tree- or one-loop-level potential is used. The parameter regions where $\lambda_{\text{eff}}(\Lambda_\beta) < 0$ and $\lambda_{\text{DM}}(\Lambda_\beta) < 0$ are filled, respectively, with blue and red. Both of them are excluded from the stability of the potentials. The MPP predicts that M_t and κ_0 should exist on the green contour. One of the good points of this

[3] Of course, we can consider the MPP of the DM sector. We study such a situation in Appendix B.

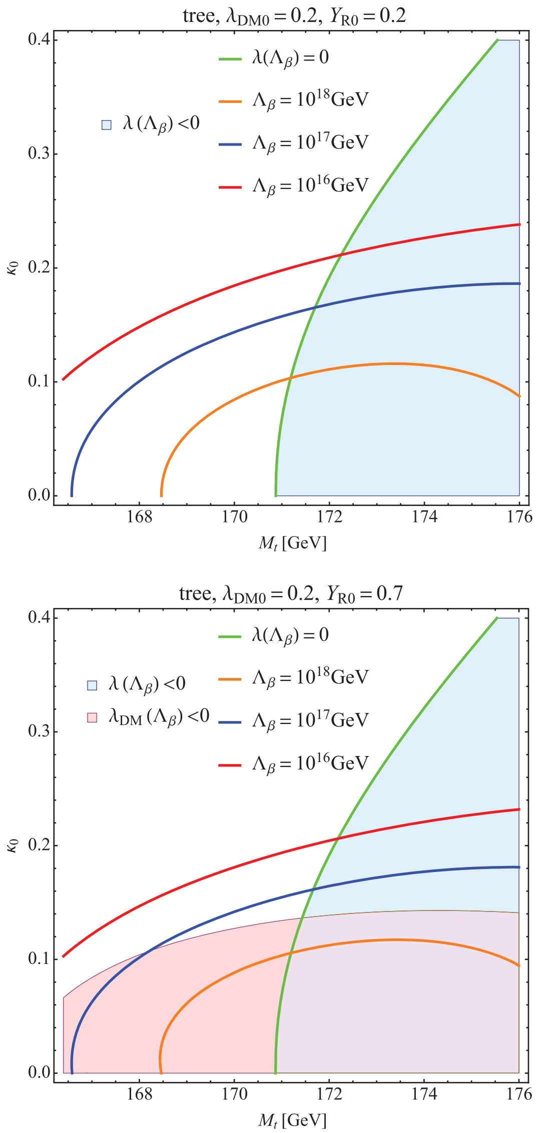

Fig. 3. The parameter dependences of $\lambda_{\rm eff}(\Lambda_\beta)$. Here, $\lambda_{\rm DM0}$ is fixed at 0.2, and $Y_{\rm R0}$ is fixed at 0.2 (0.7) in the upper (lower) panels. The left (right) panels show the calculations by using the tree- (one-loop-) level potential. The green lines show the prediction by the MPP. The contours that represent $\Lambda_\beta = 10^{16}$ GeV, 10^{17} GeV, and 10^{18} GeV are also shown by red, blue, and orange, respectively.

model is that a larger M_t is allowed, unlike in the SM. This is consistent with the recent experimental value [26],

$$M_t = 173.34 \pm 0.76\,\text{GeV}, \tag{23}$$

which corresponds to the DM mass (see Eq. (16)),

$$769\,\text{GeV} \leq m_{\rm DM} \leq 1053\,\text{GeV}. \tag{24}$$

Two comments are needed.

1. The contours that represent $\Lambda_\beta = 10^{16}$ GeV, 10^{17} GeV, and 10^{18} GeV are also shown in Fig. 3 by red, blue, and orange, respectively. Thus, a larger M_t (such as Eq. (23)) means that, in this model, the MPP of the Higgs potential occurs at the relatively low-energy scale ($\lesssim 10^{16}$ GeV).

2. As is seen from the lower panels of Fig. 3, we can also require $\lambda_{\rm DM}(\Lambda_\beta) = 0$ in addition to $\lambda_{\rm eff}(\lambda_\beta) = 0$. Because κ_0 and $Y_{\rm R0}$ appear in the one-loop part of $\beta_{\lambda_{\rm DM}}$, we can obtain a further relation between them by $\lambda_{\rm DM}(\Lambda_\beta) = 0$. Although one might think that the one remaining parameter can be determined by $\beta_{\lambda_{\rm DM}}(\Lambda_\beta) = 0$, we have checked that it is difficult to satisfy $\lambda_{\rm DM}(\Lambda_\beta) = \beta_{\lambda_{\rm DM}}(\Lambda_\beta) = 0$ simultaneously. See Appendix B for more details.

4. Summary

We have discussed the MPP of the SM with scalar singlet DM and right-handed neutrinos. We have found that $\lambda_{\rm eff}$ and $\beta_{\lambda_{\rm eff}}$ can simultaneously become zero within a reasonable parameter region. The MPP predicts a strong relation between the portal coupling κ and the top mass M_t. Unlike the pure SM, a larger M_t is allowed in this model, which is favorable for the recent experimental values [25,26]:

$$M_t = 173.34 \pm 0.76\,\text{GeV}. \tag{25}$$

Although we have found that the MPP can be satisfied for the Higgs potential, it is difficult to realize the exact flatness of the scalar potential at some high-energy scale Λ:

$$\lambda(\Lambda) = \beta_\lambda(\Lambda) = \lambda_{\rm DM}(\Lambda) = \beta_{\lambda_{\rm DM}}(\Lambda) = \kappa(\Lambda) = \beta_\kappa(\Lambda) = 0; \tag{26}$$

see Appendix B for details. It would be interesting to consider a generalization of this model in such a way that the MPP can be realized for whole scalar fields.

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We thank Hikaru Kawai, Yuta Hamada, and Koji Tsumura for valuable discussions.

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Appendix A. Two-loop renormalization group equations

The two-loop RGEs where the Lagrangian is given by Eq. (13) are as follows:[4]

$$\frac{dg_Y}{dt} = \frac{1}{(4\pi)^2}\frac{41}{6}g_Y^3 + \frac{g_Y^3}{(4\pi)^4}\left(\frac{199}{18}g_Y^2 + \frac{9}{2}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}y_t^2 - \frac{3}{2}y_\nu^2\right), \tag{A1}$$

$$\frac{dg_2}{dt} = -\frac{1}{(4\pi)^2}\frac{19}{6}g_2^3 + \frac{g_2^3}{(4\pi)^4}\left(\frac{3}{2}g_Y^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}\left(y_t^2 + y_\nu^2\right)\right), \tag{A2}$$

$$\frac{dg_3}{dt} = -\frac{7}{(4\pi)^2}g_3^3 + \frac{g_3^3}{(4\pi)^4}\left(\frac{11}{6}g_Y^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2y_t^2\right), \tag{A3}$$

[4] The calculations in this appendix are based on Refs. [30–32], and our results are in agreement with the recent result [23] when there is only one right-handed neutrino and $Y_{\rm R} = 0$.

$$\frac{dy_t}{dt} = \frac{y_t}{(4\pi)^2}\left(\frac{9}{2}y_t^2 + 3y_\nu^2 - \frac{17}{12}g_Y^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)$$
$$+ \frac{y_t}{(4\pi)^4}\left\{-12y_t^4 - \frac{27}{4}y_\nu^4 - \frac{27}{4}y_t^2y_\nu^2 - \frac{9}{8}Y_{\mathrm{R}}^2y_\nu^2 + 6\lambda^2 + \frac{1}{4}\kappa^2 - 12\lambda y_t^2\right.$$
$$+ g_Y^2\left(\frac{131}{16}y_t^2 + \frac{15}{8}y_\nu^2\right) + g_2^2\left(\frac{225}{16}y_t^2 + \frac{45}{8}y_\nu^2\right) + 36g_3^2y_t^2 + \frac{1187}{216}g_Y^4$$
$$\left. - \frac{23}{4}g_2^4 - 108g_3^4 - \frac{3}{4}g_Y^2g_2^2 + 9g_2^2g_3^2 + \frac{19}{9}g_3^2g_Y^2\right\}, \tag{A4}$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{(4\pi)^2}\left(\frac{9}{2}y_\nu^2 + 3y_t^2 + \frac{1}{4}Y_{\mathrm{R}}^2 - \frac{3}{4}g_Y^2 - \frac{9}{4}g_2^2\right)$$
$$+ \frac{y_\nu}{(4\pi)^4}\left\{-12y_\nu^4 - \frac{27}{4}y_t^4 - \frac{19}{32}Y_{\mathrm{R}}^4 - y_\nu^2\left(\frac{27}{4}y_t^2 + \frac{21}{16}Y_{\mathrm{R}}^2\right) + 6\lambda^2 + \frac{1}{4}\kappa^2\right.$$
$$- 12\lambda y_\nu^2 - \kappa Y_{\mathrm{R}}^2 + g_Y^2\left(\frac{123}{16}y_\nu^2 + \frac{85}{24}y_t^2 + \frac{9}{16}Y_{\mathrm{R}}^2\right) + g_2^2\left(\frac{225}{16}y_\nu^2 + \frac{45}{8}y_t^2 + \frac{27}{16}Y_{\mathrm{R}}^2\right)$$
$$\left. + 20g_3^2y_t^2 + \frac{35}{24}g_Y^4 - \frac{23}{4}g_2^4 - \frac{9}{4}g_Y^2g_2^2\right\}, \tag{A5}$$

$$\frac{dY_{\mathrm{R}}}{dt} = \frac{Y_{\mathrm{R}}}{(4\pi)^2}\left(3Y_{\mathrm{R}}^2 + 2y_\nu^2\right) + \frac{Y_{\mathrm{R}}}{(4\pi)^4}\left\{-\frac{81}{16}Y_{\mathrm{R}}^4 - \frac{27}{4}Y_{\mathrm{R}}^2y_\nu^2 - 9y_t^2y_\nu^2 - \frac{27}{2}y_\nu^4\right.$$
$$\left. - \lambda_{\mathrm{DM}}Y_{\mathrm{R}}^2 - 8\kappa y_\nu^2 - \frac{1}{4}g_Y^2y_\nu^2 - \frac{3}{4}g_2^2y_\nu^2\right\}, \tag{A6}$$

$$\frac{d\lambda}{dt} = \frac{1}{(4\pi)^2}\left(\lambda\left(24\lambda - 9g_2^2 - 3g_Y^2 + 12y_\nu^2 + 12y_t^2\right) + \frac{3}{4}g_2^2g_Y^2 + \frac{9g_2^4}{8}\right.$$
$$\left. + \frac{3g_Y^4}{8} + \frac{\kappa^2}{2} - 6y_\nu^4 - 6y_t^4\right) + \frac{1}{(4\pi)^4}\left\{-2\kappa^3 - 5\kappa^2\lambda - 312\lambda^3 + 36\lambda^2\left(g_Y^2 + 3g_2^2\right)\right.$$
$$+ \lambda\left(\frac{629}{24}g_Y^4 + \frac{39}{4}g_2^2g_Y^2 - \frac{73}{8}g_2^4\right) + \frac{305}{16}g_2^6 - \frac{289}{48}g_Y^2g_2^4 - \frac{559}{48}g_Y^4g_2^2 + \frac{379}{48}g_Y^6$$
$$- 32g_3^2y_t^4 - \frac{8}{3}g_Y^2y_t^4 - \frac{9}{4}g_2^4\left(y_t^2 + y_\nu^2\right) + \lambda y_t^2\left(\frac{85}{6}g_Y^2 + \frac{45}{2}g_2^2 + 80g_3^2\right)$$
$$+ \lambda y_\nu^2\left(\frac{15}{2}g_Y^2 + \frac{45}{2}g_2^2\right) + g_Y^2y_t^2\left(-\frac{19}{4}g_Y^2 + \frac{21}{2}g_2^2\right) - g_Y^2y_\nu^2\left(\frac{3}{4}g_Y^2 + \frac{3}{2}g_2^2\right)$$
$$\left. - 144\lambda^2\left(y_t^2 + y_\nu^2\right) - 3\lambda\left(y_t^4 + y_\nu^4 + \frac{3}{2}Y_{\mathrm{R}}^2y_\nu^2\right) + 30\left(y_t^6 + y_\nu^6 + \frac{1}{5}Y_{\mathrm{R}}^2y_\nu^4\right) - \frac{3}{2}Y_{\mathrm{R}}^2\kappa^2\right\}, \tag{A7}$$

$$\frac{d\lambda_{\mathrm{DM}}}{dt} = \frac{1}{(4\pi)^2}\left(3\lambda_{\mathrm{DM}}^2 + 12\kappa^2 + 6\lambda_{\mathrm{DM}}Y_{\mathrm{R}}^2 - 18Y_{\mathrm{R}}^4\right)$$
$$+ \frac{1}{(4\pi)^4}\left\{-\frac{17}{3}\lambda_{\mathrm{DM}}^3 - 20\kappa^2\lambda_{\mathrm{DM}} - 48\kappa^3 - 72\left(y_t^2 + y_\nu^2\right)\kappa^2 + 24\left(g_Y^2 + 3g_2^2\right)\kappa^2\right.$$
$$\left. + 72Y_{\mathrm{R}}^4\left(Y_{\mathrm{R}}^2 + y_\nu^2\right) + \lambda_{\mathrm{DM}}Y_{\mathrm{R}}^2\left(\frac{21}{2}Y_{\mathrm{R}}^2 - 18y_\nu^2\right) - 9Y_{\mathrm{R}}^2\lambda_{\mathrm{DM}}^2\right\}, \tag{A8}$$

$$\frac{d\kappa}{dt} = \frac{1}{(4\pi)^2}\left(4\kappa^2 + 12\kappa\lambda + \kappa\lambda_{\rm DM} + 3\kappa\left(2y_t^2 + 2y_\nu^2 + Y_{\rm R}^2\right) - \frac{3}{2}\kappa\left(g_Y^2 + 3g_2^2\right) - 12Y_{\rm R}^2 y_\nu^2\right)$$
$$+ \frac{\kappa}{(4\pi)^4}\left\{-\frac{21}{2}\kappa^2 - 72\kappa\lambda - 60\lambda^2 - 6\kappa\lambda_{\rm DM} - \frac{5}{6}\lambda_{\rm DM}^2 - \left(y_t^2 + y_\nu^2\right)(12\kappa + 72\lambda)\right.$$
$$- 3Y_{\rm R}^2(2\kappa + \lambda_{\rm DM}) - \frac{27}{2}y_t^4 - \frac{27}{2}y_\nu^4 - \frac{3}{4}Y_{\rm R}^4 + \frac{51}{4}Y_{\rm R}^2 y_\nu^2 + g_Y^2(\kappa + 24\lambda) + 3g_2^2(\kappa + 24\lambda)$$
$$\left. + y_t^2\left(\frac{85}{12}g_Y^2 + \frac{45}{4}g_2^2 + 40g_3^2\right) + y_\nu^2\left(\frac{15}{4}g_Y^2 + \frac{45}{4}g_2^2\right) + \frac{557}{48}g_Y^4 - \frac{145}{16}g_2^4 + \frac{15}{8}g_Y^2 g_2^2\right\}$$
$$+ \frac{Y_{\rm R}^2 y_\nu^2}{(4\pi)^4}\left\{\frac{3}{2}\left(g_Y^2 + 3g_2^2\right) + 27Y_{\rm R}^2 + 66y_\nu^2\right\}, \qquad \text{(A9)}$$

$$\frac{d\Gamma}{dt} = \frac{1}{(4\pi)^2}\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 - 3y_t^2 - 3y_\nu^2\right). \qquad \text{(A10)}$$

Appendix B. Is an exact flat potential possible?

One question is whether the MPP can be realized exactly, namely, if

$$\lambda(\Lambda_\beta) = \beta_\lambda(\Lambda_\beta) = \lambda_{\rm DM}(\Lambda_\beta) = \beta_{\lambda_{\rm DM}}(\Lambda_\beta) = \kappa(\Lambda_\beta) = \beta_\kappa(\Lambda_\beta) = 0 \qquad \text{(B1)}$$

is possible or not. Here, for simplicity, we also define Λ_β as the scale where β_λ becomes zero. To discuss this possibility, it is qualitatively enough to consider one-loop RGEs. One can easily understand that it is impossible to realize Eq. (B1) as follows: even if $\lambda(\Lambda_\beta)$, $\beta_\lambda(\Lambda_\beta)$, $\lambda_{\rm DM}(\Lambda_\beta)$, and $\beta_{\lambda_{\rm DM}}(\Lambda_\beta)$ become simultaneously zero, we cannot make $\kappa(\Lambda_\beta)$ zero because the one-loop part of $\beta_{\lambda_{\rm DM}}$ at Λ_β becomes

$$\beta_{\lambda_{\rm DM}}|_{\Lambda_\beta} = \frac{1}{(4\pi)^2}\left(12\kappa^2 - 18Y_{\rm R}^4\right), \qquad \text{(B2)}$$

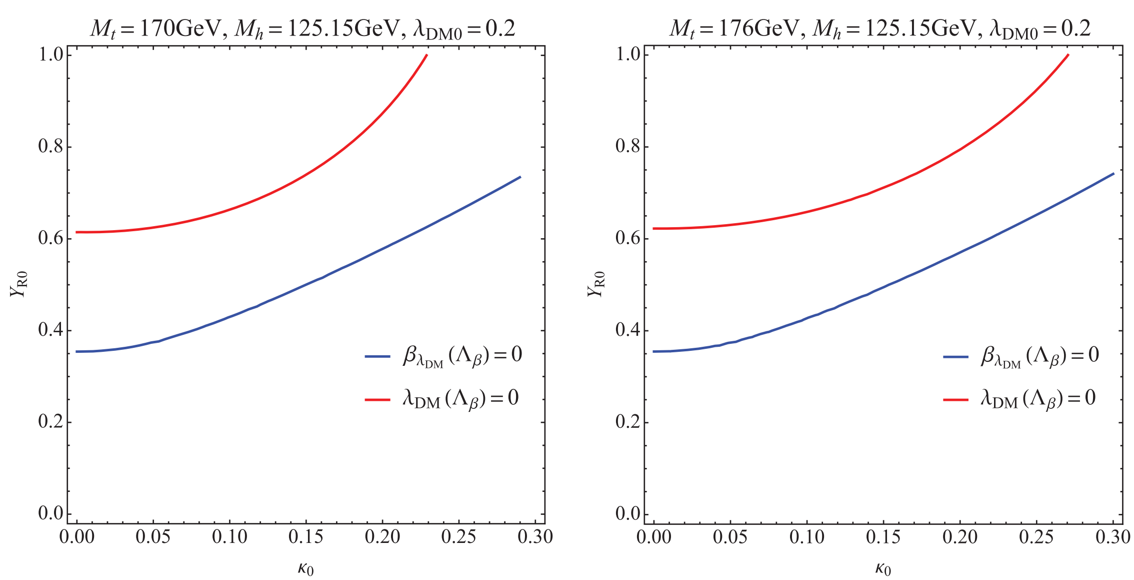

Fig. B1. The blue (red) lines show the contours where $\beta_{\lambda_{\rm DM}}(\lambda_{\rm DM})(\Lambda_\beta) = 0$. The left (right) panel is the $M_t = 170$ (176) GeV case. Here, note that, if $\kappa_0 \gtrsim 0.3$, Λ_β becomes less than $M_{\rm R} = 10^{13}$ GeV, and there is no solution of $\beta_{\lambda_{\rm DM}}(\Lambda_\beta) = 0$ because the one-loop part of $\beta_{\lambda_{\rm DM}}$ is always positive when $\mu \leq M_{\rm R}$.

and we need $\kappa(\Lambda_\beta) \neq 0$ to satisfy $\beta_{\lambda_{\rm DM}}(\Lambda_\beta) = 0$.[5] Furthermore, it is also difficult even to satisfy $\lambda_{\rm DM}(\Lambda_\beta) = \beta_{\lambda_{\rm DM}}(\Lambda_\beta) = 0$ simultaneously; see Fig. B1. This shows the contours such that $\lambda_{\rm DM}(\Lambda_\beta)$ and $\beta_{\lambda_{\rm DM}}(\Lambda_\beta)$ become zero, respectively. Here, we have used two-loop RGEs. One can see that the two contours do not intersect.

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[5] Typically, a non-zero positive $Y_{\rm R0}$ is needed to make $\beta_{\lambda_{\rm DM}}$ negative. As a result, $Y_{\rm R}(\Lambda_\beta)$ is non-zero because the one-loop part of $\beta_{Y_{\rm R}}$ is always positive.

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