Stochastic Modeling of Hydrological Events for Better Water Management

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Abstract

Water availability in many countries declined with growing population over the past decades. Food production, protecting the natural ecosystem and human life require water resources management. Alteration of the hydrological cycles caused by climate change is also increasingly affecting water resources. Offering sustainable solutions to the water management problems require hydrological process studies. The goal of this thesis is to provide a mathematical and numerical modeling framework for assessing hydrological events, deducing optimal control strategies, and studying their real world applications for better water management. This thesis focuses in particular on stochastic modeling and optimal control of reservoirs as well as rainfed agriculture.

Irregular occurrence of dry spells in cropping rainy seasons leads to the vulnerability of subsistence rainfed agriculture. In the first part of this thesis, a fundamental concept is presented to comprehend alternation of dry and wet spells. A stochastic process model consisting of the Langevin equation, which is a stochastic differential equation governing a zero-reverting Ornstein-Uhlenbeck (OU) processes, is developed to model the alternation of dry and wet spells. Soil moisture and rainfall were monitored for five years at a study site which is located in the Guinea savanna agro ecological zone of Ghana with monomodal rainfall pattern. Values of model parameters are identified so that the observed data series are consistent with the transition probability of the zero-reverting OU process. A numerical scheme using the finite element method is proposed as well to compute the transition probability. The onsets of rainy seasons are modelled with the concept of first exit time. The model with appropriately identified parameter values is applicable to a variety of problems in subsistent rainfed agriculture.

As a first application of the model, the optimality of rainfed agriculture is investigated in the context of stochastic control theory considering the level of drought severity as the zero-reverting OU process. The optimal control problem is formulated in the context of dynamic programming

to deduce the optimal strategies for better water management. Computational methods are presented to numerically approximate the relevant equations. The occurrence of drought terminating growth of crops is modelled with the concept of first exit time. A Hamilton-Jacobi-Bellman (HJB) equation governing the optimal control is deduced to identify the set of cost functions optimizing rainfed agriculture in terms of an inverse problem. The model attempts to comprehend the rationale of situation in Sub-Saharan Africa, where subsistence rainfed agriculture is the predominant source of food and the production uncertainty is associated with the stochastic nature of rainfall. Data and information were collected in the coastal savanna agro-ecological zone of Ghana, to formulate the stochastic control problem, identify model parameters, solve the inverse problem, and then verify optimality of rainfed agriculture.

Another application is to rainwater harvesting (RWH), which is a technology where surface runoff is effectively collected during yielding rain periods. RWH is a likely viable option to increase water productivity in water-scarce countries. A novel type of hydraulic structure is proposed for RWH in an arid area to develop an irrigation scheme. The scheme consists of an irrigated farm, a reservoir, and an intake structure to divert ephemeral flood flows into the reservoir. Design, construction, and operation of the actual RWH system are presented with model parameters identified from observed data. Details of structure dimensions are designed with numerical and hydraulic model experiments. A computational scheme was used to numerically reproduce the whole flow fields during rainwater harvesting events with the maximum design discharge. Hydraulic model tests were conducted at an experimental station to tune up and verify the computational scheme. The experiments confirmed the actual structure showing desired performance. To operate the constructed RWH system, a dynamic programming approach is taken to deduce optimal operational strategies for the reservoir harvesting stochastic surface water flows to meet the demand from command areas. In order to outline mathematical issues intrinsic to dynamic programming, a primitive deterministic problem is firstly considered. The problem is to make a decision on the intake discharge from an irrigation tank, where the optimal strategies are obtained from the viscosity solution of a HJB equation. Then, the real life application is discussed with numerical solutions to a HJB equation. The zero-reverting OU process is applied to representing the stochastic surface water flows and the demand from command areas. Finally, the numerical solutions of the HJB equation yield optimal irrigation strategies represented in terms of rule curves prescribing water withdrawal limits.