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Vanishing Higgs potential in minimal dark matter models

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ABSTRACT

We consider the Standard Model with a new particle which is charged under $SU(2)_L$ with the hypercharge being zero. Such a particle is known as one of the dark matter (DM) candidates. We examine the realization of the multiple point criticality principle (MPP) in this class of models. Namely, we investigate whether the one-loop effective Higgs potential $V_{\text{eff}}(\phi)$ and its derivative $dV_{\text{eff}}(\phi)/d\phi$ can become simultaneously zero at around the string/Planck scale, based on the one/two-loop renormalization group equations. As a result, we find that only the $SU(2)_L$ triplet extensions can realize the MPP. More concretely, in the case of the triplet Majorana fermion, the MPP is realized at the scale $\phi = \mathcal{O}(10^{16}\text{ GeV})$ if the top mass M_t is around 172 GeV. On the other hand, for the real triplet scalar, the MPP can be satisfied for $10^{16}\text{ GeV} \lesssim \phi \lesssim 10^{17}$ GeV and 172 GeV $\gtrsim M_t \gtrsim 171$ GeV, depending on the coupling between the Higgs and DM.

The discovery of the Higgs particle [1,2] is very meaningful for the Standard Model (SM). The experimental value of the Higgs mass suggests that the Higgs potential can be stable up to the Planck scale M_{pl} and also that both the Higgs self coupling λ and its beta function β_λ become very small around M_{pl}. This fact attracts much attention, and there are many works which try to find its physical meaning [3–29] and implications for cosmology [30–55].

In [3,4], the Higgs mass was predicted to be around 130 GeV by the requirement that $\lambda(\mu)$ and $\beta_\lambda(\mu)$ simultaneously become zero around M_{pl}.[1] Namely, the minimum of the Higgs potential $V(\phi)$ around M_{pl} vanishes. Such a requirement is called the multiple point criticality principle (MPP), and there have been many suggestions [39,46,56–62,64] that this principle might be closely related to physics at the Planck scale. One of the good points of the principle is its predictability: The low-energy effective couplings are fixed so that the minimum of the potential takes zero around M_{pl}. See [39,62–65] for examples of the prediction.

By taking the fact that the MPP is realized in the SM into consideration, a natural question is whether the MPP can be also realized in the models beyond the SM. It is meaningful to consider the MPP of these models because we can understand whether the SM is actually special among them. One of the interesting extensions is adding a new weakly interacting fermion χ or scalar X, which is an $n_{\chi(X)}$ representation of $SU(2)_L$ with the hypercharge $Y_{\chi(X)}$. Such extensions are phenomenologically well studied because they have dark matter (DM) candidates when $Y_{\chi(X)} = 0$ [66–68]. In this paper, we focus on $Y_{\chi(X)} = 0$, that is, Majorana fermions and real scalars. We examine the realization of the MPP of these models, based on the one/two-loop renormalization group equations (RGEs). We use the effective Higgs self coupling λ_{eff} and its beta function $\beta_{\lambda_{\text{eff}}}$ defined from the one-loop effective Higgs potential $V_{\text{eff}}(\phi)$. Their definitions and the two-loop RGEs when we add a new fermion are presented in Appendix A. In the case of the new scalar (fermion), we only have to consider $n_X = 3$ ($n_\chi = 3, 5$) since the scalar couplings ($SU(2)_L$ coupling g_2) rapidly blow(s) up when $n_X \geq 4$ [69] ($n_\chi \geq 7$ [66]), and the theory does not valid up to M_{pl}. For the septet and nonet fermion cases, we discuss this point in Appendix B.

In the following discussion, we regard the top mass M_t as a free parameter, and the Higgs mass is varied within [70]

$$M_h = 125.09 \pm 0.32 \text{ GeV}. \tag{1}$$

As for the initial values of the $\overline{\text{MS}}$ SM couplings, we use the results of [19]. For illustration, the $Y_\chi \neq 0$ cases are also discussed in Appendix C.

First, we consider a new fermion. For $n_\chi = 3$ and 5, the mass M_χ is determined by the thermal relic abundance [67,68]:

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[1] It is interesting that the quadratic divergent bare Higgs mass also vanishes around this scale [13].

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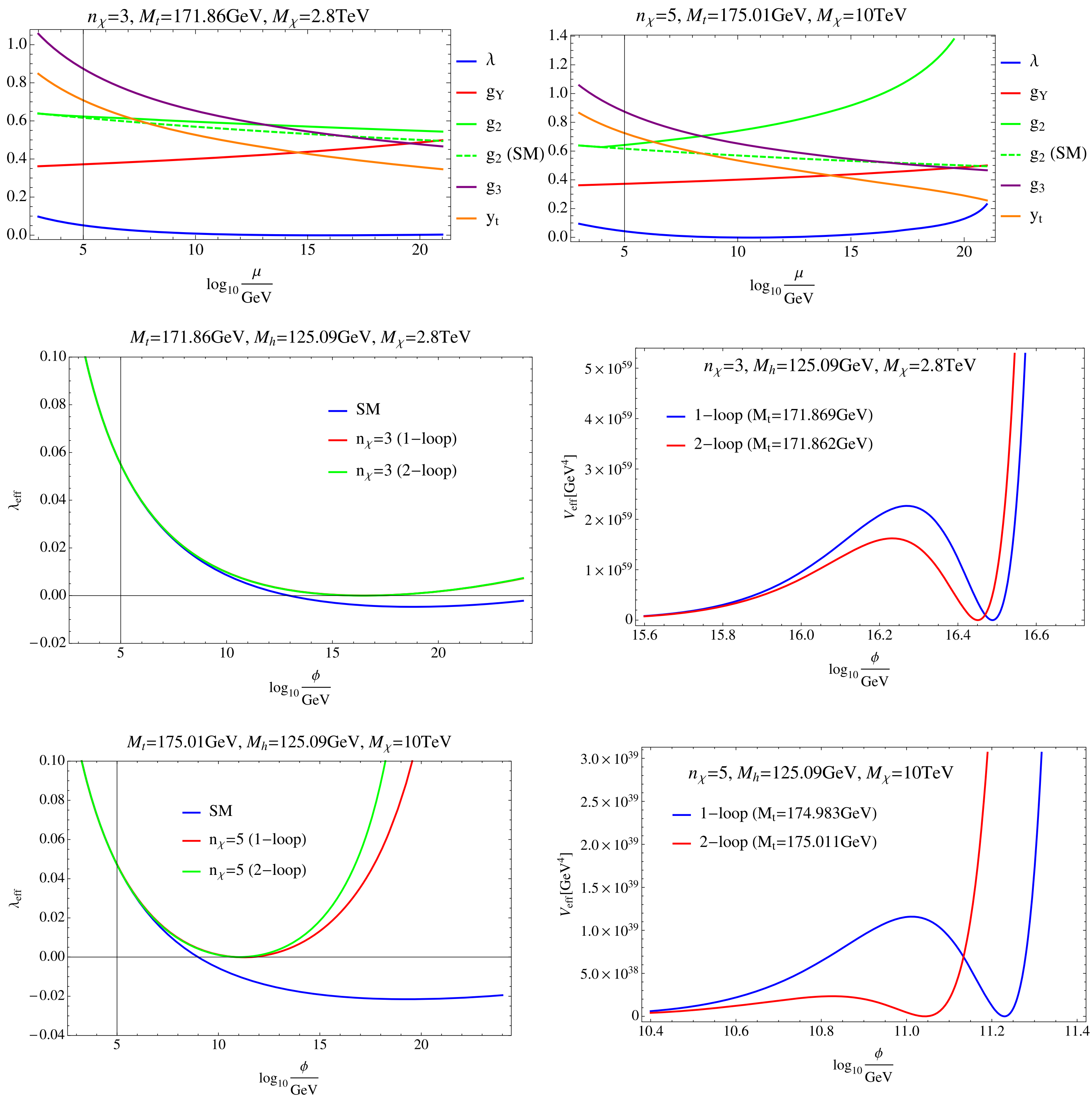

Fig. 1. Upper left (right): the runnings of the SM parameters when $n_\chi = 3$ (5). Here, the dashed green lines represent the SM running of g_2. Middle (Lower): the running of the effective Higgs self coupling λ_{eff} (left) and the one-loop effective Higgs potential $V_{\text{eff}}(\phi)$ (right) in the case of $n_\chi = 3$ (5). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

$$M_\chi \simeq \begin{cases} 2.8\ \text{TeV} & (\text{for } n_\chi = 3), \\ 10\ \text{TeV} & (\text{for } n_\chi = 5). \end{cases} \tag{2}$$

As a result, M_t and Λ_{MPP} are uniquely predicted because there is no additional free parameter. The results are

$$\begin{aligned} &171.7\ \text{GeV} \le M_t \le 172.0\ \text{GeV}, \\ &2.5 \times 10^{16}\ \text{GeV} \le \Lambda_{\text{MPP}} \le 3.2 \times 10^{16}\ \text{GeV}\ (\text{for } n_\chi = 3), \\ &174.8\ \text{GeV} \le M_t \le 175.2\ \text{GeV}, \\ &1.1 \times 10^{11}\ \text{GeV} \le \Lambda_{\text{MPP}} \le 1.2 \times 10^{11}\ \text{GeV}\ (\text{for } n_\chi = 5), \end{aligned} \tag{3}$$

depending on 124.77 GeV $\le M_h \le$ 125.41 GeV.[2] The upper panels of Fig. 1 show the runnings of the SM parameters where $M_h = 125.09$ GeV, and M_t is correspondingly fixed so that the MPP is realized. Here, we also show the SM running of g_2 by the dashed green line for comparison. Furthermore, in the middle and

[2] These values of M_t are consistent with the recent analyses: $M_t = 173.34 \pm 0.76$ GeV [71] and $M_t = 172.38 \pm 0.10 \pm 0.65$ GeV [72] at 2σ level. However, the relation between these masses and the pole mass is not clear. In the following calculation of the bare Higgs mass, we use more conservative value of M_t determined by the $t\bar{t}$ total cross section [73].

lower panels, we show the corresponding $\lambda_{\rm eff}$ (left) and $V_{\rm eff}(\phi)$ (right). In these figures, the one-loop results are also shown. One can actually see that the potential and its derivative simultaneously become zero at a high energy scale, and that the only triplet can have the other vacuum near the string/Planck scale. We note that the two-loop effects are small.

Now let us consider a new scalar. As mentioned before, the remaining possibility is $n_X = 3$ [69]. The potential of the scalar fields is

$$V = -\frac{M_h^2}{2} H^\dagger H + \frac{M_X^2}{2} XX + \lambda \left(H^\dagger H\right)^2 + \lambda_{DM} (XX)^2 + \kappa \left(H^\dagger H\right)(XX). \tag{4}$$

Here, H is the SM Higgs doublet. The one-loop RGEs which are different from those of the SM are as follows[3]:

$$\frac{dg_2}{dt} = -\frac{g_2^3}{(4\pi)^2}\frac{17}{6}, \tag{5}$$

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(\lambda\left(24\lambda - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{3}{2}\kappa^2 + \frac{3}{4}g_Y^2 g_2^2 + \frac{9}{8}g_2^4 + \frac{3}{8}g_Y^4 - 6y_t^4\right), \tag{6}$$

$$\frac{d\lambda_{DM}}{dt} = \frac{1}{16\pi^2}\left(22\lambda_{DM}^2 + 2\kappa^2 - 24g_2^2\lambda_{DM} + 12g_2^4\right), \tag{7}$$

$$\frac{d\kappa}{dt} = \frac{1}{16\pi^2}\left(4\kappa^2 + 12\kappa\lambda + 10\kappa\lambda_{DM} + 6y_t^2\kappa - \frac{33}{2}g_2^2\kappa - \frac{3}{2}g_Y^2\kappa + 6g_2^4\right). \tag{8}$$

Furthermore, there is an additional contribution to $V_{\rm eff}(\phi)$:

$$\Delta V_{\text{1-loop}}(\phi) = \frac{3m_{DM}(\phi)^4}{64\pi^2}\left(\ln\left(\frac{m_{DM}(\phi)^2}{\phi^2}\right) - \frac{3}{2}\right), \tag{9}$$

where

$$m_{DM}(\phi) = \sqrt{M_X^2 + \kappa(\phi)e^{2\Gamma(\phi)}\phi^2}. \tag{10}$$

In this case, the thermal abundance of X depends on the value of κ. Here we use

$$M_X = 2.6 \text{ TeV and } 3.1 \text{ TeV} \tag{11}$$

for our calculation.[4] $M_X = 2.6$ TeV and $M_X = 3.1$ TeV correspond to $\kappa = 0$ and $\kappa = 1$, respectively [68]. The upper panels of Fig. 2 show the runnings of $\lambda_{\rm eff}$ when $M_X = 2.6$ TeV. Here, the blue band of the left panel corresponds to the change of κ at $\mu = M_X$ from 0 to 0.4. In the case of $\lambda_{DM}(M_X) = 0.4$ of the right panel, the rapid increase of $\lambda_{\rm eff}$ around 10^{16} GeV is due to the Landau pole of λ_{DM}. Namely, λ_{DM} becomes infinity below M_{pl}. The lower left (right) panel of Fig. 3 shows the contour plot of $\Lambda_{\rm MPP}$ (M_t) as a function of λ_{DM} and κ at $\mu = M_X$. The blue (red) contours correspond to $M_X = 2.6$ (3.1) TeV. One can see that $\Lambda_{\rm MPP}$ is close to the string/Planck scale when $\kappa(M_X) \lesssim 0.1$ and $M_t \lesssim 172$ GeV.

In order to discuss the Higgs potential around the cutoff scale Λ, it is meaningful to consider how the existence of a new particle changes the behavior of the bare Higgs mass m_B as a function of Λ.[5] This is because m_B would appear in the Higgs potential above Λ [31]. We now examine whether m_B vanishes around the string scale or not.[6] See [13] for the evaluation of m_B in the SM.

Here, let us focus on $(n_{\chi(X)}, Y_{\chi(X)}) = (3, 0)$ at one-loop level. For χ, m_B is given by

$$\frac{m_B^2|_{\text{1-loop}}}{\Lambda^2/16\pi^2} = -\left(6\lambda + \frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2 - 6y_t^2\right), \tag{12}$$

where the couplings are evaluated at $\mu = \Lambda$. On the other hand, for X, $m_B^2|_{\text{1-loop}}$ becomes

$$\frac{m_B^2|_{\text{1-loop}}}{\Lambda^2/16\pi^2} = -\left(6\lambda + \frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2 - 6y_t^2 + \frac{3}{2}\kappa\right). \tag{13}$$

The left (right) panel of Fig. 3 shows m_B as a function of Λ when a new particle is fermion (scalar). Here, the green contour is the SM prediction when $M_t = 171.2$ GeV, and blue bands correspond to the 2σ deviation from it [73]:

$$M_t = 171.2 \pm 4.8 \text{ GeV} \quad (95\% \text{ CL}). \tag{14}$$

In the right panel, we change κ at $\mu = M_X$ from 0 to 0.4, and they are represented by a red band. Depending on the values of M_t and κ, one can see that the scale at which m_B becomes zero quite changes. In both of cases, m_B can take zero around the string scale.[7] In addition to the vanishing λ at around the string scale, this fact may suggest the MPP is realized at this scale.

In conclusion, we have studied the MPP of the SM with a weakly interacting new particle with its hypercharge being zero. When a new particle is a fermion, we have found that the top mass M_t and $\Lambda_{\rm MPP}$ can be uniquely predicted. On the other hand, when a new particle is scalar, there exists a new scalar coupling κ. Due to this coupling, we have found that $\Lambda_{\rm MPP}$ and M_t drastically change. In both of cases, only the triplets survive from the point of view that the other vacuum should exist around the string/Planck scale and that the theory is valid up to this scale. The analysis of this paper suggests that the SM and its triplet extensions are special in that the MPP can be realized around the string/Planck scale.

Acknowledgements

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[3] As one can see from the results of the fermion cases, the two-loop effects are small when we consider the MPP. This is why we consider the one-loop beta functions here.

[4] The mass of a new scalar suffers from fine-tuning problem. However, because our motivation in this paper is to distinguish the minimal dark matter models in the context of the MPP, we take Eq. (11) as the dark matter mass.

[5] Within field theory, the quadratic divergence does not appear after the renormalization. However, it can have the physical meaning if we consider the scale around the Planck/string one, because the SM couples with the gravity. In this paper, we assume that the physics around the Planck scale is described by string theory, which is the cutoff theory whose universal cutoff scale is given by the string scale. This is why we take the universal cutoff in the calculation of m_B^2. See [13] for the detail.

[6] The vanishing bare mass is so-called Veltman condition [74]. From the point of view of low energy field theory, $m_B^2 = 0$ is accidental and seems to require the fine-tuning at the Planck scale. We hope that $m_B^2 = 0$ comes from some mechanism related to the physics at the Planck scale.

[7] In order to obtain the correct electroweak symmetry breaking, we need to add small negative mass term to the Higgs potential, which is much small than Λ^2. However, in the case of the $SU(2)_L$ triplet scalar, it may be possible to realize the electroweak symmetry breaking by the Coleman–Weinberg mechanism. See Appendix D. We thank the referee for pointing this out.

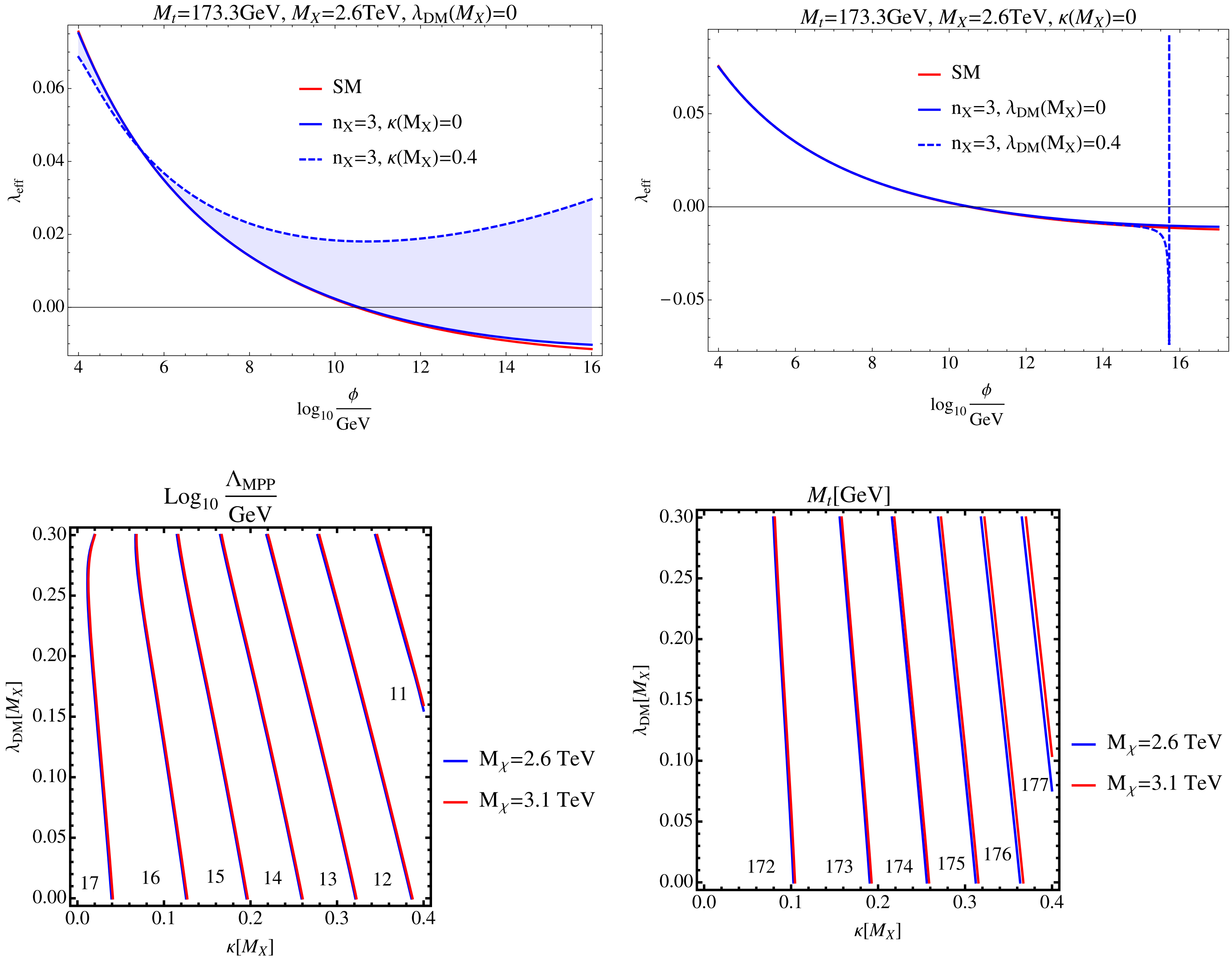

Fig. 2. Upper: the running of λ_{eff} in the case of $n_X = 3$. Here, the blue band of the left panel corresponds to the change of κ at $\mu = M_X$ from 0 to 0.4. Lower: Λ_{MPP} (left) and M_t (right) as a function of κ and λ_{DM} at $\mu = M_X$. The blue (red) contours correspond to $M_X = 2.6$ (3.1) TeV. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

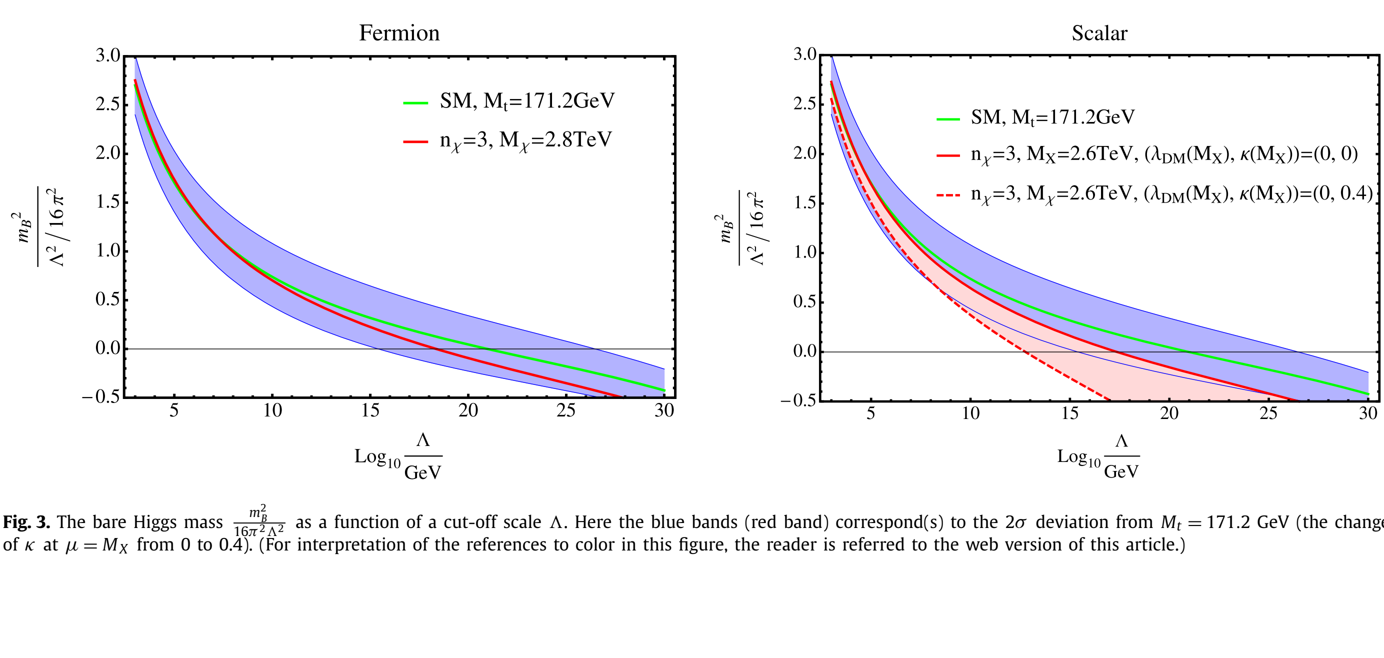

Fig. 3. The bare Higgs mass $\frac{m_B^2}{16\pi^2\Lambda^2}$ as a function of a cut-off scale Λ. Here the blue bands (red band) correspond(s) to the 2σ deviation from $M_t = 171.2$ GeV (the change of κ at $\mu = M_X$ from 0 to 0.4). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Appendix A. Two-loop renormalization group equations and one-loop effective Higgs potential

The two-loop RGEs of the SM with a new fermion which is a n_χ representation of $SU(2)_L$ with the hypercharge Y_χ are follows[8]:

$$\frac{d\Gamma}{dt} = \frac{1}{(4\pi)^2}\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 - 3y_t^2\right), \quad (15)$$

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{(4\pi)^2}\left(\frac{41}{6} + \eta n_\chi \frac{4}{3}Y_\chi^2\right) + \frac{g_Y^3}{(4\pi)^4}\left\{\left(\frac{199}{18} + 4\eta n_\chi Y_\chi^4\right)g_Y^2 + \left(\frac{9}{2} + 4\eta Y_\chi^2 C_n\right)g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}y_t^2\right\}, \quad (16)$$

$$\frac{dg_2}{dt} = \frac{g_2^3}{(4\pi)^2}\left(-\frac{19}{6} + \eta\frac{4}{3}S_n\right) + \frac{g_2^3}{(4\pi)^4}\left\{\left(\frac{3}{2} + \eta 4Y_\chi^2 S_n\right)g_Y^2 + \left(\frac{35}{6} + \eta\frac{40}{3}S_n + \eta 4C_n S_n\right)g_2^2 + 12g_3^2 - \frac{3}{2}y_t^2\right\},$$

$$\frac{dg_3}{dt} = -\frac{7}{(4\pi)^2}g_3^3 + \frac{g_3^3}{(4\pi)^4}\left(\frac{11}{6}g_Y^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2y_t^2\right), \quad (17)$$

$$\frac{dy_t}{dt} = \frac{y_t}{(4\pi)^2}\left(\frac{9}{2}y_t^2 + 3y_\nu^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_Y^2\right) + \frac{y_t}{(4\pi)^4}\left\{\left(\frac{1187}{216} + \frac{29}{27}\eta n_\chi Y_\chi^2\right)g_Y^4 + \left(-\frac{23}{4} + \eta S_n\right)g_2^4 - \frac{3}{4}g_2^2 g_Y^2 + \frac{19}{9}g_3^2 g_Y^2 + 9g_3^2 g_2^2 - 108g_3^4 + \left(\frac{131}{16}g_Y^2 + \frac{225}{16}g_2^2 + 36g_3^2\right)y_t^2 + 6\lambda^2 - 12\lambda y_t^2 - 12y_t^4\right\}, \quad (18)$$

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\Bigg(\lambda\left(24\lambda - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{3}{4}g_Y^2 g_2^2 + \frac{9}{8}g_2^4 + \frac{3}{8}g_Y^4 - 6y_t^4\Bigg) + \frac{1}{(4\pi)^4}\Bigg\{-312\lambda^3 + 36\lambda^2\left(g_Y^2 + 3g_2^2\right) + \lambda\left(\frac{629}{24}g_Y^4 + \frac{10}{3}Y_\chi^2 g_Y^4 - \frac{73}{8}g_2^4 + 10\eta S_n g_2^4 + \frac{39}{4}g_2^2 g_Y^2\right) + \left(\frac{305}{16} - 4\eta S_n\right)g_2^6 - \left(\frac{289}{48} + \frac{4}{3}\eta S_n\right)g_2^4 g_Y^2 - \left(\frac{559}{48} + \frac{4}{3}\eta n_\chi Y_\chi^2\right)g_2^2 g_Y^4 - \left(\frac{379}{48} + \frac{4}{3}n_\chi Y_\chi^2\right)g_Y^6 + \left(\frac{85}{6}g_Y^2 + \frac{45}{2}g_2^2 + 80g_3^2\right)\lambda y_t^2 + g_Y^2 y_t^2\left(\frac{21}{2}g_2^2 - \frac{19}{4}g_Y^2\right) - \frac{9}{4}g_2^4 y_t^2 - \frac{8}{3}g_Y^2 y_t^4 - 32g_3^2 y_t^4 - 144\lambda^2 y_t^2 - 3\lambda y_t^4 + 30y_t^6\Bigg\}. \quad (19)$$

[8] Our calculations are based on [75–78].

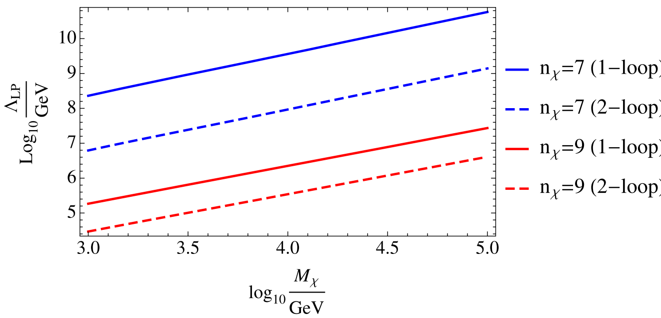

Fig. 4. The scale Λ_{LP} of the Landau pole as a function of M_χ when $n_\chi = 7$ (blue) and 9 (red). Here, the two-loop results are represented by dashed lines. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Here, $t = \ln\mu$ with μ being the renormalization scale, Γ is the wave function renormalization of the Higgs, C_n and S_n are the Casimir and Dynkin index, and $\eta = 1, \frac{1}{2}$ for Dirac and Weyl fermion. The two-loop RGEs of g_Y and g_2 are agreement with [51] by putting $y_t = 0$.

The one-loop effective Higgs potential is

$$V_{\rm eff}(\mu,\phi) = -\frac{M_h^2}{4}\phi^2 + \frac{\lambda(\mu)}{4}\phi^4 + V_{\text{1-loop}}(\mu,\phi), \quad (20)$$

where

$$V_{\text{1-loop}}(\mu,\phi) := e^{4\Gamma(\mu)}\left\{-12\cdot\frac{M_t(\phi,\mu)^4}{64\pi^2}\left[\log\left(\frac{M_t(\phi,\mu)^2}{\mu^2}\right) - \frac{3}{2} + 2\Gamma(\mu)\right] + 6\cdot\frac{M_W(\phi,\mu)^4}{64\pi^2}\left[\log\left(\frac{M_W(\phi,\mu)^2}{\mu^2}\right) - \frac{5}{6} + 2\Gamma(\mu)\right] + 3\cdot\frac{M_Z(\phi,\mu)^4}{64\pi^2}\left[\log\left(\frac{M_Z(\phi,\mu)^2}{\mu^2}\right) - \frac{5}{6} + 2\Gamma(\mu)\right]\right\}, \quad (21)$$

and

$$M_t(\phi,\mu) = \frac{y_t(\mu)}{\sqrt{2}}\phi\,,\ M_W(\phi,\mu) = \frac{g_2(\mu)}{2}\phi\,,\quad M_Z(\phi,\mu) = \frac{\sqrt{g_2(\mu)^2 + g_Y(\mu)^2}}{2}\phi. \quad (22)$$

In Eq. (21), we have neglected the contribution from the Higgs quartic term because it is small when we consider the MPP. In principle, μ should be determined as a function of ϕ so that $V_{\text{1-loop}}$ is minimized. However, in this paper, μ is taken to be ϕ for simplicity. It is known that this is a good approximation [44]. From $V_{\rm eff}$, we define $\lambda_{\rm eff}$ and $\beta_{\lambda_{\rm eff}}$ as follows:

$$\lambda_{\rm eff}(\phi) := \frac{4V_{\rm eff}(\phi)}{\phi^4}\,,\ \beta_{\lambda_{\rm eff}}(\phi) := \frac{d\lambda_{\rm eff}(\phi)}{d\ln\phi}. \quad (23)$$

Appendix B. Landau pole in septet and nonet fermion

As mentioned in the introduction, in cases of $n_\chi = 7$ and 9, there exists a scale Λ_{LP} at which g_2 becomes infinity below M_{pl}, which is well known as the Landau Pole. Therefore, these theories

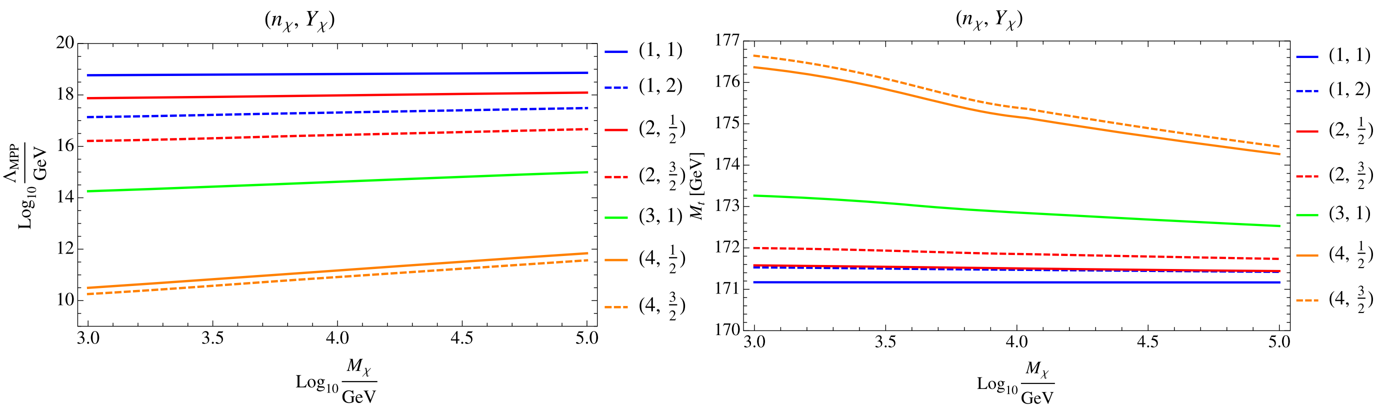

Fig. 5. Left (Right): Λ_{MPP} (M_t) as a function of M_χ.

are not favored from the point of view of perturbativity (triviality) up to the string/Planck scale. For completeness, we give numerical results of the Landau pole in Fig. 4. Here, the two-loop results are shown by dashed lines. As is known, the one-loop Landau pole can be analytically solved:

$$\Lambda_{LP}|_{\text{1-loop}} = M_\chi \exp\left(\frac{8\pi^2}{\left(-\frac{19}{6}+\frac{1}{2}\frac{4}{3}S_n\right)g_2(M_\chi)^2}\right), \tag{24}$$

where S_n is the Dynkin index. From Fig. 4, one can see that the two-loop effect is relatively important.

Appendix C. New Fermion with $Y_\chi \neq 0$

Here, we consider a new fermion with $Y_\chi \neq 0$. As well as the real $n_\chi = 7$ and 9 cases, the Landau pole of g_2 exists below M_{pl} when $n_\chi \geq 5$ [51]. So, let us here focus on $n_\chi \leq 4$.[9] Here, we leave M_χ as a free parameter.[10] The left (right) panel of Fig. 5 shows Λ_{MPP} (M_t) as a function of M_χ for each (n_χ, Y_χ).

Appendix D. Electroweak symmetry breaking by Coleman–Weinberg mechanism

Here, we discuss a possibility to realize the electroweak symmetry breaking by the Coleman–Weinberg mechanism in the case of the $SU(2)$ triplet scalar. The one-loop effective Higgs potential is

$$\begin{aligned} V(\mu,\phi) = {} & \frac{\lambda(\mu)}{4}e^{4\Gamma(\mu)}\phi^4 + \frac{3m_{DM}(\phi)^4}{64\pi^2}\left(\ln\left(\frac{m_{DM}(\phi)^2}{\mu^2}\right)-\frac{3}{2}\right) \\ & + e^{4\Gamma(\mu)}\frac{(3\lambda(\mu)\phi^2)^2}{64\pi^2}\left[\log\left(\frac{3\lambda(\mu)e^{2\Gamma(\mu)}\phi^2}{\mu^2}\right)-\frac{3}{2}\right] \\ & + \Delta V_{\text{1-loop}}(\mu,\phi), \end{aligned} \tag{25}$$

where $m_{DM}(\phi)$ and $\Delta V_{\text{1-loop}}(\mu,\phi)$ are given by Eq. (10) and Eq. (21) respectively, and we have assumed that the quadratic term vanishes at the tree-level. In the following, we choose $\mu = m_{DM}(\phi)$. Then, ϕ develops the vacuum expectation value v because the negative quadratic term appears from the second term in Eq. (25). The resultant vacuum expectation value is

$$v = \frac{3M_X}{4\pi}\sqrt{\frac{\kappa}{2\lambda}} \simeq 240\ \text{GeV}\left(\frac{M_X}{2.6\ \text{TeV}}\right)\times\sqrt{\frac{\kappa}{0.04}}, \tag{26}$$

where we have neglected the 1-loop correction to the quartic term. It is interesting that the successful electroweak symmetry breaking is realized for $\kappa \simeq 0.04$ which is also favored by the MPP around the Planck scale.

[9] For $n_\chi = 3$ and 4, the LP of the $U(1)_Y$ gauge coupling g_Y also appears below M_{pl} respectively when $Y_\chi = 2$ and $= 3/2$. This is why we only show $Y_\chi = 1$ when $n_\chi = 3$ in Fig. 5.

[10] Furthermore, when $n_\chi = 1$, 2 and 3, there are additional Yukawa couplings among the SM leptons (L_i, E_{Ri}), the Higgs H and χ. However, we can neglect these effects because the lepton masses are small.

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