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A new parameter in attractor single-field inflation

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ABSTRACT

We revisit the notion of slow-roll in the context of general single-field inflation. As a generalization of slow-roll dynamics, we consider an inflaton ϕ in an attractor phase where the time derivative of ϕ is determined by a function of ϕ, $\dot{\phi} = \dot{\phi}(\phi)$. In other words, we consider the case when the number of e-folds N counted backward in time from the end of inflation is solely a function of ϕ, $N = N(\phi)$. In this case, it is found that we need a new independent parameter to properly describe the dynamics of the inflaton field in general, in addition to the standard parameters conventionally denoted by ϵ, η, c_s^2 and s. Two illustrative examples are presented to discuss the non-slow-roll dynamics of the inflaton field consistent with observations.

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1. Introduction

The primordial inflation [1] in the very early universe before the onset of the standard hot big bang evolution is now the leading candidate to explain otherwise extremely finely tuned initial conditions, such as the horizon and flatness problems. Furthermore, inflation can naturally provide a causal mechanism of producing primordial curvature perturbations that should have existed on super-horizon scales [2]. These primordial curvature perturbations are predicted to have a nearly scale invariant power spectrum and are statistically almost perfectly Gaussian. By recent observations including the Planck mission, these properties have been confirmed with very high accuracy [3–5].

While the inflationary picture itself is more and more supported and favored by recent observations, constructing a realistic and concrete model of inflation in the context of particle physics remains an open conundrum [6]. In this situation we should be open-minded and consider a wider, more general possibilities for inflation than the simplest model where a single, canonically normalized inflaton minimally coupled to Einstein gravity drives inflation. Such general theories may well predict verifiable new observational signatures such as a slight blue tilt for tensor perturbations [7] and suppression of the curvature perturbation on large scales [8]. We may have to take these possibilities more seriously, as the simplest possibilities including the $m^2\phi^2$ model seem to be not favored by the new Planck data [5].

A caution is in order when we study such general possibilities. We should keep in mind that many notions we have developed in the canonical models are not directly applicable to them. For example, the moment of horizon crossing which is crucial for standard single field inflation may not be as important as any other instants during inflation. This is because, contrary to the canonical model, the curvature perturbation may keep evolving on super-horizon scales until the end of inflation [9,10] by e.g. the existence of other relevant degrees of freedom, which may reflect the signatures of high energy physics [11]. In this article, we revisit the term “slow-roll” in the context of k-inflation type general $P(X,\phi)$ theory where $X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ [12].

The article is organized as follows. In Section 2 we extend the notion of slow-roll single-field inflation and consider the general, attractor phase inflation in the context of $P(X,\phi)$ theory. In describing the dynamics of the inflaton field, we introduce an independent new parameter p [see (10)] which identically vanishes in the canonical single-field model. The new parameter is slow-roll suppressed if the inflaton is slow-rolling. However, in the general case of attractor inflation where the inflaton is may not be slow-rolling, it may become of order unity. In Section 3 we present two examples to illustrate the possibility of the non-slow-roll dynamics consistent with the current observational constraints. We conclude the paper in Section 4.

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2. General attractor inflation

For $P(X,\phi)$ theory, the matter Lagrangian is given by

$$S_m = \int d^4x\sqrt{-g}P(X,\phi)\,. \tag{1}$$

This is the most general single scalar field action with their linear derivatives, which includes the standard canonical action $P = X - V$ and the Dirac–Born–Infeld type action. We assume that the inflaton is in an attractor phase, i.e., $\dot{\phi}$ is determined by a function of ϕ, but ϕ is not necessarily slowly evolving, as discussed in more detail below. Thus, in particular, we do not consider non-attractor inflation [14] where the dynamics depends both on ϕ and $\dot{\phi}$.

With the above Lagrangian, it is known that the spectral index of the curvature perturbation is given by [12]

$$n_{\mathcal{R}} - 1 = -2\epsilon - \eta - s\,, \tag{2}$$

as well as the running of the spectral index [13]

$$\alpha_{\mathcal{R}} = -2\epsilon\eta - \frac{\dot{\eta}}{H} - \frac{\dot{s}}{H}\,, \tag{3}$$

where

$$\begin{aligned}
\epsilon &\equiv -\frac{\dot{H}}{H^2} = \frac{XP_X}{m_{\rm Pl}^2 H^2}\,,\\
\eta &\equiv \frac{\dot{\epsilon}}{H\epsilon} = -\frac{\ddot{H}}{H^3\epsilon} + 2\epsilon\,,\\
s &\equiv \frac{\dot{c}_s}{Hc_s}\,,
\end{aligned} \tag{4}$$

with the speed of sound c_s given by

$$c_s^{-2} = 1 + \frac{2XP_{XX}}{P_X}\,. \tag{5}$$

In deriving (2), it is assumed that H and c_s are slowly varying. The constrained value of $n_{\mathcal{R}} - 1 = 0.968 \pm 0.006$ [5] demands ϵ, η and s are all small, barring accidental cancellation among them. This situation is usually referred to as the "slow-roll" approximation. It is however quite misleading because the smallness of these parameters does not necessarily mean the inflaton is slowly evolving. This becomes more transparent if we consider the equation of motion for ϕ, which reads [12]

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3P_X\dot{\phi}\right) = \frac{d}{dt}(P_X\dot{\phi}) + 3HP_X\dot{\phi} = P_\phi\,. \tag{6}$$

In the canonical case where $P_X = 1$, the smallness of ϵ and η would imply the smallness of the $\ddot{\phi}$ term in comparison with $3H\dot{\phi}$ term, which is the usual slow-roll approximation. But the second derivative term may not be negligible in the general $P(X,\phi)$ theory a priori.

Let us take another point of view by considering the second order component of the comoving curvature perturbation $\mathcal{R}$. In the context of the δN formalism [9,15] where $N = N(\phi)$, for single field case we can find (see for detail Appendix A)

$$\delta N = \mathcal{R} = \mathcal{R}_l\left[1 + \frac{1}{2}(\epsilon + \delta)\mathcal{R}_l + \cdots\right]\,, \tag{7}$$

where $\mathcal{R}_l \equiv -H\delta\phi/\dot{\phi}$ is the linear component of $\mathcal{R}$ with $\delta\phi$ being evaluated on flat slices at horizon crossing, and

$$\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}}\,. \tag{8}$$

The notion of "slow-roll", i.e. slow evolution of the inflaton field is thus equivalent to requiring $|\delta| \ll 1$. Note that (7) is in fact the second order gauge transformation [16] and is independent of the structure of the matter sector, so should remain valid for $P(X,\phi)$ theory. Only when the kinetic sector is canonical we can use the relation $\dot{H} = -X/m_{\rm Pl}^2$ and find $\eta = 2(\epsilon + \delta)$, so that the smallness of the second order component of $\mathcal{R}$ in (7) is guaranteed. However, in $P(X,\phi)$ theory, $\dot{H} = -XP_X/m_{\rm Pl}^2$ so that in general we have

$$\eta = 2(\epsilon + \delta) + p\,, \tag{9}$$

where we have introduced a new parameter p defined by

$$p \equiv \frac{\dot{P}_X}{HP_X}\,. \tag{10}$$

Thus the coefficient in front of the second order component of $\mathcal{R}$ is not necessarily small.

Note that p may be expressed as

$$p = \delta\left(\frac{1}{c_s^2} - 1\right) + \frac{P_{X\phi}}{HP_X}\dot{\phi} = -3 - \delta + \frac{P_\phi}{H\dot{\phi}P_X}\,, \tag{11}$$

where for the second equality we have used the equation of motion (6). Equating these two expressions for p, we can eliminate HP_X and can write p as

$$p = \frac{\left(c_s^{-2} - 1\right)\delta + 2(3+\delta)q}{1 - 2q} \quad \text{where} \quad q \equiv \frac{XP_{X\phi}}{P_\phi}\,. \tag{12}$$

This is another useful formula. Since p is expressed in terms of the cross derivative $P_{X\phi}$, we can see the qualitative dependence of p on how close the theory is to the canonical form where $P_{X\phi} = 0$. Explicitly, we can express p as

$$p \approx \begin{cases} \left(c_s^{-2} - 1\right)\delta + \mathcal{O}(q) & \text{for } |q| \ll 1 \\ -3 - \delta + \mathcal{O}(q^{-1}) & \text{for } |q| \gg 1 \end{cases}\,. \tag{13}$$

Thus on general ground we expect that when $P(X,\phi)$ is highly non-canonical, we may have $|q| \gg 1$, and the slow-roll dynamics of the inflaton field is not guaranteed. In fact if $|q| \gg 1$, combined with (9), it is *required* that the non-slow-rollness must be as large as $\delta \approx 3$ with ϵ and η being kept small.

Before closing this section, let us reconsider the curvature perturbation expanded to second order (7) in the context of non-Gaussianity. Conventionally a local non-Gaussianity is represented by the non-linear parameter $f_{\rm NL}$ [17] which appears in the expansion as

$$\mathcal{R} = \mathcal{R}_l + \frac{3}{5}f_{\rm NL}\mathcal{R}_l^2 + \cdots\,. \tag{14}$$

For the canonical case, using (9), (7) reads

$$\mathcal{R} = \mathcal{R}_l + \frac{\eta}{4}\mathcal{R}_l^2 + \cdots\,, \tag{15}$$

which implies

$$f_{\rm NL} = \frac{5}{12}\eta\,. \tag{16}$$

This is in fact a half of the consistency relation for the squeezed limit of the bispectrum [18]. The remaining half $5\epsilon/6$ comes from the intrinsic non-Gaussianity of $\mathcal{R}_l$, which we have not taken into account here. See Appendix B for detail. However, given that for most inflationary models $\epsilon \ll 1$, (16) contributes more importantly to $f_{\rm NL}$.

Now, following the same step, from (9) we obtain for $P(X,\phi)$ theory,

$$f_{\rm NL} = \frac{5}{12}\left(\eta - \frac{p}{2}\right)\,. \tag{17}$$

One might expect from (2) that p could be expressed in terms of c_s or s at an attractor stage where $N = N(\phi)$ in a *universal* manner.[1] However, actually it seems there is no universal relation between p and c_s or s. That is, essentially p is an independent parameter of the $P(X,\phi)$ theory. In the following, let us see this point more clearly in two simple examples.

3. Slow-roll versus non-slow-roll dynamics: examples

If we assume the slow-roll dynamics of the inflaton, i.e. $|\delta| \ll 1$, from (9) we have to *additionally* require

$$|p| \ll 1\,, \tag{18}$$

given the smallness of η. However this is an extra assumption not constrained by the current observations on $n_{\mathcal{R}}$, and in principle can be abandoned, à la general slow-roll [20] where the hierarchy between slow-roll parameters is not assumed. In this case $|\delta| = \mathcal{O}(1)$ can be canceled by $p \sim -\delta$, keeping small η so that there is no conflict with observations. Below we present two opposite examples: A trivial case where $|\delta| \ll 1$ and a non-trivial case where $|\delta| = \mathcal{O}(1)$.

3.1. Trivial example

In simple cases, η and δ go together, i.e. when one is small, so is the other. As a very simple example in this category, consider

$$P(X,\phi) = K(X) - V(\phi)\,. \tag{19}$$

This gives $P_{X\phi} = 0$, so p is very simple and is related to δ from (11) as

$$p = \left(\frac{1}{c_s^2} - 1\right)\delta = \frac{2XK_{XX}}{K_X}\delta\,, \tag{20}$$

where for the second equality we have used the expression for the speed of sound:

$$c_s^{-2} = 1 + \frac{2XK_{XX}}{K_X}\,. \tag{21}$$

Thus unless $c_s^2 \ll 1$, which is highly constrained from bounds on $f_{\rm NL}$ by Planck [4,5], we have $p = \mathcal{O}(\delta)$. Namely $|\eta| \ll 1$ demands $|\delta| \ll 1$, ensuring slow-roll dynamics. Notice that if $K \propto X^n$ ($n \neq 1$), c_s is constant and $s = 0$, hence the spectral index formula (2) as well as the running (3) are identical to the canonical case. But even in this case $f_{\rm NL}$ is different from the canonical case (16) because of the non-vanishing new term (20), though this difference is rather irrelevant since we still have $|f_{\rm NL}| \ll 1$.

3.2. Non-trivial example

As a non-trivial example where $|\eta| \ll 1$ and $|\delta| \gtrsim 1$, let us consider

$$P(X,\phi) = F(\phi)K(X) - V(\phi)$$
$$\text{with}\quad K(X) = \frac{X_0}{1+\gamma}\left[\left(\frac{X}{X_0}+1\right)^{1+\gamma} - 1\right]\,, \tag{22}$$

where X_0 is an arbitrary normalization. Note that $K \propto X^{\gamma+1}$ for $X \gg X_0$ while $K \propto X$ for $X \ll X_0$. Thus the system reduces to the canonical form when $X \ll X_0$ by appropriately redefining the inflaton field.

Taking the derivatives of (22), we find

$$P_X = F(\phi)K_X \quad \text{with} \quad K_X = \left(\frac{X}{X_0}+1\right)^{\gamma}\,, \tag{23}$$

$$P_{XX} = F(\phi)K_{XX} \quad \text{with} \quad K_{XX} = \frac{\gamma}{X_0}\left(\frac{X}{X_0}+1\right)^{\gamma-1}\,, \tag{24}$$

and from (5),

$$c_s^{-2} = 1 + \frac{2XK_{XX}}{K_X} = 1 + 2\gamma\frac{(X/X_0+1)^{\gamma-1}X/X_0}{(X/X_0+1)^{\gamma}}\,. \tag{25}$$

Note that for $X \gg X_0$, we have a simple result,

$$c_s^{-2} \approx 1 + 2\gamma\,. \tag{26}$$

Note also that with γ being a constant, $s \approx 0$ in this limit.

In the following, let us concentrate on this regime. To make the analysis simpler, we assume the time dependence of ϕ as

$$\phi \sim e^{\alpha N}\,, \tag{27}$$

where we are interested in the case when α is not small, $\alpha \gtrsim \mathcal{O}(1)$. The consistency of this assumption will be discussed later. Accordingly we find

$$\dot\phi = \alpha H\phi\,,$$
$$X = \frac{\alpha^2H^2}{2}\phi^2\,,$$
$$\frac{dX}{dN} = 2(\alpha-\epsilon)X\,. \tag{28}$$

What we want to see is whether $|\delta| \gtrsim 1$ while $|\eta| \ll 1$ is possible. For this purpose, let us express η in the form,

$$\eta = 2\epsilon + \frac{1}{F}\frac{dF}{dN} + \frac{1}{XK_X}\frac{d(XK_X)}{dN}\,, \tag{29}$$

where we have used (9) and (10). Note that δ is expressed as

$$\delta = \frac{1}{2}\frac{\dot X}{HX} = \frac{1}{2X}\frac{dX}{dN} = \alpha - \epsilon\,. \tag{30}$$

We see that given $\epsilon \ll 1$ the last two terms in (29) should nearly cancel each other to ensure small η.

With (27) and $X \gg X_0$, we find

$$\frac{1}{XK_X}\frac{d(XK_X)}{dN} \approx 2(\alpha-\epsilon)(1+\gamma)\,. \tag{31}$$

Now let us set

$$\frac{1}{F}\frac{dF}{dN} = -2(\alpha-\epsilon)(1+\gamma) + \xi\,. \tag{32}$$

For the last two terms in (29) to nearly cancel each other, we must have $|\xi| \ll 1$. This implies

$$F \approx F_0\left(\frac{\phi}{\phi_0}\right)^{-2(1+\gamma)}\,, \tag{33}$$

where we have ignored the corrections of $\mathcal{O}(\epsilon)$.

In the limit $X \gg X_0$, ϵ is given by

$$\epsilon \approx \frac{FX^{1+\gamma}}{m_{\rm Pl}^2H^2X_0^{\gamma}} \approx \frac{F_0\alpha^{2(1+\gamma)}}{2^{1+\gamma}}\frac{\phi_0^{2(\gamma+1)}H^{2\gamma}}{m_{\rm Pl}^2X_0^{\gamma}}\,. \tag{34}$$

[1] Note that in general $f_{\rm NL}$ can be comparable to or larger than $\mathcal{O}(1)$ at a non-attractor stage, breaking the consistency relation, even for canonical models [14], along with other possible peculiar signatures [19].

Thus by appropriately choosing the normalization constants, we can readily make $\epsilon \ll 1$. Turning to the equation of motion for ϕ, (6), we obtain

$$\frac{F_0}{X_0^\gamma}\frac{(\alpha H)^{2(1+\gamma)}}{2^\gamma}\left[3+\frac{1+2\gamma}{2(1+\gamma)}\xi\right]\frac{dN}{d\phi}=-V_\phi\,. \tag{35}$$

Then, ignoring ξ as well as the time variation of H, we can recover the advocated behavior (27) with a logarithmic potential,

$$V(\phi)=V_0+V_1\log\left(\frac{\phi}{\phi_0}\right), \tag{36}$$

upon appropriately choosing V_1.

Notice that even for $X \gg X_0$ where we can make simplifications, p is not related to the speed of sound c_s (26). Instead we find

$$p=\frac{\dot{F}}{HF}+\frac{\dot{K}_X}{HK_X}\approx -2\alpha(1+\gamma)+2\alpha\gamma\approx -2\delta\,, \tag{37}$$

where for the last equality we have used $\alpha \approx \delta$. We see that $\gamma \approx \left(c_s^{-2}-1\right)/2$ disappears from the final result, implying that p is related to neither c_s nor s. From (9) we find

$$\eta=2(\epsilon+\delta)+p\approx 2(\epsilon+\delta)-2\delta\approx 2\epsilon \ll 1\,, \tag{38}$$

as required, so that there is no conflict with the observational constraints on $n_{\mathcal{R}}$. Further, in this regime the running is $\alpha_{\mathcal{R}} \approx -8\epsilon^2$ which can be well accommodated within the current observational bounds. On the other hand we find

$$f_{\rm NL}=\frac{5}{12}\left(\eta-\frac{p}{2}\right)\approx\frac{5}{12}(\eta+\delta)\approx\frac{5}{12}\alpha \gtrsim \mathcal{O}(1)\,. \tag{39}$$

Note that we have

$$\frac{XP_{X\phi}}{P_\phi}\approx -\frac{\alpha(1+\gamma)}{3-\alpha}\,, \tag{40}$$

so that upon choosing $\alpha \approx \delta \approx 3$ the dynamics of the inflaton becomes maximally non-slow-roll, corresponding the general case of $|q| \gg 1$ given in (13).

4. Conclusion

We have reconsidered the notion of slow-roll in the context of general $P(X,\phi)$ theory in terms of the parameters by which observable quantities are described. While in the standard single-field inflation the attractor phase corresponds to the slow-roll regime, they are not equivalent in general. Accordingly we have found that we need a new, independent parameter p defined by (10) to properly describe the dynamics of the inflaton field. We have presented two illustrative examples in order to clarify the role of the new parameter in the non-slow-roll dynamics of the inflaton field. In one of the examples, we have shown that we may indeed have a highly non-slow-roll stage of inflation without violating the current observational constraints. In other words, in near-future observations where the precision and accuracy will become much better, this new parameter can be used to perform a new observational test to constrain viable models of inflation.

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Appendix A. Non-linear $\mathcal{R}$ on attractor

On totally general ground, once the trajectory is in an attractor phase we can write

$$\mathcal{R}=\delta N=\frac{\partial N}{\partial\phi}\delta\phi+\frac{1}{2}\frac{\partial^2 N}{\partial\phi^2}\delta\phi^2+\cdots\,, \tag{41}$$

where $\mathcal{R}$ is the comoving curvature perturbation evaluated at some final time, and $\delta\phi$ is the field fluctuation on the initial flat slice. We can find the expansion coefficients explicitly as follows.

First, we note $dN=-Hdt$ where the minus sign is due to the fact that $N(\phi)$ is defined as the number of e-folds counted *backward in time* from the end of inflation to an initial time when the value of the inflation field was ϕ. Therefore we have

$$\frac{\partial N}{\partial\phi}=\frac{dt}{d\phi}\frac{\partial N}{\partial t}=-\frac{H}{\dot\phi}\,. \tag{42}$$

Once we have the first coefficient, it is straightforward to compute the other ones. The second order coefficient is

$$\frac{\partial}{\partial\phi}\left(\frac{\partial N}{\partial\phi}\right)=\frac{1}{\dot\phi}\frac{d}{dt}\left(\frac{\partial N}{\partial\phi}\right)=-\frac{\dot H}{\dot\phi^2}+\frac{H\ddot\phi}{\dot\phi^3}\,. \tag{43}$$

Plugging these coefficients into (41), we find

$$\begin{aligned}\mathcal{R}&=-\frac{H}{\dot\phi}\delta\phi+\frac{1}{2}\left(-\frac{\dot H}{\dot\phi^2}+\frac{H\ddot\phi}{\dot\phi^3}\right)\delta\phi^2+\cdots\\&=-\frac{H}{\dot\phi}\delta\phi+\frac{1}{2}\left(-\frac{\dot H}{H^2}+\frac{\ddot\phi}{H\dot\phi}\right)\left(-\frac{H}{\dot\phi}\delta\phi\right)^2+\cdots\,.\end{aligned} \tag{44}$$

Note that we have *not* assumed any particular form for the matter sector. Thus, using the definitions of ϵ in (4) and δ in (8) in the main text, and identifying $\mathcal{R}_l \equiv -H\delta\phi/\dot\phi$, (44) becomes

$$\mathcal{R}=\mathcal{R}_l\left[1+\frac{1}{2}(\epsilon+\delta)\mathcal{R}_l+\cdots\right]. \tag{45}$$

This is (7) used in the main text.

Appendix B. Intrinsic non-Gaussianity of $\mathcal{R}$

We consider the cubic order action of $\delta\phi$ on flat slices [18],

$$\begin{aligned}S_3=\int d^4x a^3\Bigg[&-\frac{\dot\phi}{4m_{\rm Pl}^2H}\delta\phi\dot{\delta\phi}^2-\frac{\dot\phi}{4m_{\rm Pl}^2H}\delta\phi\frac{(\Delta[\delta\phi])^2}{a^2}\\&-\dot{\delta\phi}\frac{\delta\phi^{,i}\chi_{,i}}{a^2}+\cdots\Bigg],\end{aligned} \tag{46}$$

where we have only presented the leading order terms in slow-roll, and χ is the scalar component of the shift vector given by

$$\begin{aligned}\frac{1}{a^2}\Delta[\chi]&=\epsilon\frac{d}{dt}\left(-\frac{H}{\dot\phi}\delta\phi\right)\\&=-\frac{\dot\phi}{2m_{\rm Pl}^2H}\dot{\delta\phi}+\text{higher order in slow-roll}\,.\end{aligned} \tag{47}$$

Using the Bunch–Davies mode function in the de Sitter approximation with the conformal time $\tau=-1/(aH)$,

$$\delta\phi_k(\tau) = -\frac{\dot{\phi}}{H}\mathcal{R}_k(\tau) = -\frac{\dot{\phi}}{H}\frac{iH}{\sqrt{4\epsilon k^3}m_{\rm Pl}}(1+ik\tau)e^{-ik\tau}, \tag{48}$$

we find the three-point correlation function of $\delta\phi$ from (46) as

$$\begin{aligned}\left\langle\delta\phi_{\boldsymbol{k}_1}\delta\phi_{\boldsymbol{k}_2}\delta\phi_{\boldsymbol{k}_3}\right\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_{\delta\phi}(k_1,k_2,k_3)\\ &= (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{-H^4}{\sqrt{2\epsilon}m_{\rm Pl}}\frac{1}{(k_1k_2k_3)^3}\frac{\epsilon}{4}\\ &\quad\times\left[-\frac{k_1^3+k_2^3+k_3^3}{2}+\frac{k_1\left(k_2^2+k_3^2\right)+2\text{ perm}}{2}\right.\\ &\quad\left.+\frac{4\left(k_1^2k_2^2+2\text{ perm}\right)}{k_1+k_2+k_3}\right].\end{aligned} \tag{49}$$

Since $\delta\phi = -(\dot{\phi}/H)\mathcal{R}$ at leading order, the intrinsic bispectrum for the comoving curvature perturbation is given by

$$\begin{aligned}B_{\mathcal{R}}(k_1,k_2,k_3) &= -\frac{H^3}{\dot{\phi}^3}B_{\delta\phi}(k_1,k_2,k_3)\\ &= \frac{H^4}{16\epsilon m_{\rm Pl}^4}\frac{1}{(k_1k_2k_3)^3}\left[-\frac{k_1^3+k_2^3+k_3^3}{2}\right.\\ &\quad+\frac{k_1\left(k_2^2+k_3^2\right)+2\text{ perm}}{2}\\ &\quad\left.+\frac{4\left(k_1^2k_2^2+2\text{ perm}\right)}{k_1+k_2+k_3}\right].\end{aligned} \tag{50}$$

Taking the squeezed limit, say, $k_3 \to 0$, one can read off the non-linear parameter $f_{\rm NL}$ and find

$$f_{\rm NL} = \frac{5}{6}\epsilon\,. \tag{51}$$

Thus, we see that using the δN formalism alone, we cannot fully find the consistency relation. From the beginning the δN formalism captures only the super-horizon evolution, which gives a half of the consistency relation $f_{\rm NL} = 5\eta/12$. The remaining half, $f_{\rm NL} = 5\epsilon/6$, is due to the intrinsic non-Gaussianity, which we can find from the cubic order action.

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