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Hemisphere jet mass distribution at finite N_c

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ABSTRACT

We perform the leading logarithmic resummation of nonglobal logarithms for the single-hemisphere jet mass distribution in e^+e^- annihilation including the finite-N_c corrections. The result is compared with the previous all-order result in the large-N_c limit as well as fixed-order perturbative calculations.

1. Introduction

The hemisphere jet mass distribution is an event shape variable in e^+e^--annihilation defined as the distribution of invariant mass in a single hemisphere whose axis coincides with the thrust axis. As is usually the case with all event shapes, it receives logarithmically enhanced perturbative corrections when the shape variable becomes small. However, unlike other event shapes for which systematic resummation methods are available (see [1,2] and references therein), the resummation of logarithms for the hemisphere jet mass distribution has turned out to be thorny and so far remained unsatisfactory even at the leading logarithmic level. This is due to the presence of the so-called nonglobal logarithms [3] which arise from the energy-ordered radiation of soft gluons in a restricted region of phase space.

The difficulty of resumming nonglobal logarithms stems from the fact that one has to keep track of the distribution of an arbitrary number of secondary soft gluons emitted at large angle. (For a recent review, see [4].) The original work by Dasgupta and Salam [3] employed a Monte Carlo algorithm, valid to leading logarithmic accuracy and in the large-N_c approximation, to actually generate soft gluon cascades on a computer. Later, Banfi, Marchesini and Smye (BMS) [5] reduced the problem, still at large-N_c, to solving a nonlinear integro-differential equation. This latter approach paved the way for the inclusion of the finite-N_c corrections in the resummation [6] which has been recently put on a firmer ground [7], and the first quantitative finite-N_c result can be found in [8]. However, so far only one particular observable ('interjet energy flow') has been computed and the full impact of the finite-N_c resummation is yet to be uncovered.

In this work, we apply the method developed in [8] to the hemisphere jet mass distribution and numerically carry out the resummation of nonglobal logarithms at finite-N_c, thereby achieving the full leading logarithmic accuracy for this observable.[1] In the next section we define the observable and introduce the BMS equation which resums the nonglobal logarithms in the large-N_c limit. In Section 3, we discuss the resummation strategy at finite-N_c. It turns out that a naive application of the previous method is plagued by large numerical errors, and we shall propose a refined method which cures this problem. In Section 4, we present the numerical result and compare it with the previous all-order result at large-N_c [3] as well as the recent fixed-order calculations [9,10].

2. Hemisphere jet mass distribution

Consider a two-jet event in e^+e^--annihilation with the center-of-mass energy Q. Without loss of generality, we assume that the quark jet is right-moving with momentum $p^\mu = \frac{Q}{2}(1,0,0,1) \equiv \frac{Q}{2}(1,\boldsymbol{n}_R)$ and the antiquark jet is left-moving with $\bar{p}^\mu = \frac{Q}{2}(1,0,0,-1) \equiv \frac{Q}{2}(1,\boldsymbol{n}_L)$. Suppose soft gluons with momentum $k_i^\mu = \omega_i(1,\boldsymbol{n}_i)$ ($\boldsymbol{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$) are emitted in the

[1] Recently, the resummation of nonglobal logarithms to next-to-leading logarithmic order has been discussed [7,11]. In particular, Ref. [7] explicitly derived the full NLL evolution kernel at finite N_c. See, also, an earlier suggestion [12] that the NLL resummation of nonglobal logarithms should be related to the NLL BFKL resummation via a conformal transformation.

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right hemisphere $0 \le \theta_i \le \frac{\pi}{2}$. The invariant mass in the right hemisphere is

$$M_R^2 = \left(p + \sum_i k_i\right)^2 \approx \sum_i 2p \cdot k_i = \sum_i Q\,\omega_i(1-\cos\theta_i). \tag{1}$$

We shall be interested in the probability

$$P_{LR}(\rho) = \frac{1}{\sigma}\int_0^\rho \frac{d\sigma}{d\rho'}d\rho', \tag{2}$$

that the rescaled invariant mass

$$\rho' = \frac{M_R^2}{Q^2} = \sum_i \frac{\omega_i(1-\cos\theta_i)}{Q} \tag{3}$$

is less than some value $\rho < 1$. When $\rho \ll 1$, one has to resum large logarithms $\ln^n 1/\rho$ in the perturbative calculation of $P_{LR}(\rho)$. As observed in [3], this resummation consists of two parts. One is the Sudakov double logarithms $(\alpha_s \ln^2 1/\rho)^n$ which can be resummed via exponentiation. The other is the nonglobal logarithms $(\alpha_s \ln 1/\rho)^n$ which arise from the fact that measurement is done only in a part of phase space (i.e., in the right hemisphere). The latter resummation affects various single-hemisphere event shapes in e^+e^- annihilation and DIS [3]. It is also relevant to the so-called soft function in the dijet mass distribution $d^2\sigma/dM_R dM_L$ in the asymmetric limit $M_L \gg M_R$ [13,14,9].

So far, the resummation of nonglobal logarithms for P_{LR} has been carried out only in the large-N_c limit [3], or to finite orders of perturbation theory at large-N_c [9] and finite-N_c [10]. As stated in the introduction, we shall perform the all-order leading logarithmic resummation at finite-N_c along the lines of [6,8]. To explain our approach, it is best to start with the BMS equation which resums both the Sudakov and nonglobal logarithms in the large-N_c limit [5]. Adapted to the hemisphere jet mass distribution [9], the equation reads

$$\partial_\tau P_{\alpha\beta} = N_c \int \frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{\alpha\beta}(\gamma)\Big(\Theta_L(\gamma)P_{\alpha\gamma}P_{\gamma\beta} - P_{\alpha\beta}\Big), \tag{4}$$

where

$$\mathcal{M}_{\alpha\beta}(\gamma) = \frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})} \tag{5}$$

is the soft gluon emission kernel and we defined

$$\tau = \frac{\alpha_s}{\pi}\ln\frac{1}{\rho}. \tag{6}$$

$\Theta_{L/R}(\gamma)$ is the 'step function' which restricts the angular integral $d\Omega_\gamma = d\cos\theta_\gamma d\phi_\gamma$ to the left/right hemisphere. [Below we also use a shorthand notation $\int_{L/R} d\Omega$ to represent this.] In (4), $P_{\alpha\beta} = P(\Omega_\alpha, \Omega_\beta)$ is the generalization of P_{LR} defined for arbitrary pairs of solid angle directions.

Taken at its face value, Eq. (4) is ill-defined. When α or β is in the right hemisphere, the $d\Omega_\gamma$ integral in the second term on the right-hand side (the virtual term) is divergent, and this is precisely the situation $(\alpha\beta) = (LR)$ we are eventually interested in. Physically, this collinear divergence should be cut off by the kinematical effect, yielding the Sudakov factor $e^{-\mathcal{O}(\alpha_s)\ln^2 1/\rho}$. However, since the Sudakov factor is well understood anyway, one can leave it out of consideration by defining

$$P_{\alpha\beta} \equiv \exp\left(-2C_F\tau\int_R \frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{\alpha\beta}(\gamma)\right) g_{\alpha\beta}. \tag{7}$$

($C_F = \frac{N_c^2-1}{2N_c}$ and $2C_F \approx N_c$ in the large-N_c limit.) Unlike (4), the equation satisfied by $g_{\alpha\beta}$ is well-defined and amenable to analytical and numerical approaches. In particular, Ref. [9] analytically calculated g_{LR} to five loops using the hidden $SL(2,\mathbb{R})$ symmetry of the BMS equation [15].

3. Resummation at finite N_c

We now turn to the physically relevant case $N_c = 3$. Temporarily forgetting about the issue of the collinear divergence, we recapitulate the resummation strategy developed in [16,6,8]. First we make the formal identification

$$P_{\alpha\beta} \leftrightarrow \frac{1}{N_c}\,\mathrm{tr}\,U_\alpha U_\beta^\dagger, \tag{8}$$

where U_α is the Wilson line in the fundamental representation of $SU(N_c)$ from the origin to infinity in the Ω_α direction. The product in (8) represents the propagation of the $q\bar{q}$ jets ('dipole') in the eikonal approximation. As τ is increased, more and more soft gluons are emitted from the dipole and also from the secondary gluons. This can be simulated as a stochastic process in which the Wilson lines receive random kicks in the color $SU(N_c)$ space, and is described by the following Langevin equation in discretized 'time' τ [8][2]

$$U_\alpha(\tau+\varepsilon) = e^{iS_\alpha^{(2)}}e^{iA_\alpha}U_\alpha(\tau)e^{iB_\alpha}e^{iS_\alpha^{(1)}}, \tag{9}$$

where

$$S_\alpha^{(i)} = \sqrt{\frac{\varepsilon}{4\pi}}\int_R d\Omega_\gamma \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma}t^a\xi_{\gamma a}^{(i)k} \qquad (i=1,2), \tag{10}$$

$$A_\alpha = -\sqrt{\frac{\varepsilon}{4\pi}}\int_L d\Omega_\gamma \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma}U_\gamma t^a U_\gamma^\dagger \xi_{\gamma a}^{(1)k}, \tag{11}$$

$$B_\alpha = \sqrt{\frac{\varepsilon}{4\pi}}\int_L d\Omega_\gamma \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma}t^a\xi_{\gamma a}^{(1)k}, \tag{12}$$

and $\xi^{(1)}$, $\xi^{(2)}$ are the Gaussian noises

$$\langle \xi_{\gamma a}^{(i)k}(\tau)\xi_{\gamma' b}^{(j)l}(\tau')\rangle = \delta^{ij}\delta_{\tau,\tau'}\delta(\Omega_\gamma - \Omega_{\gamma'})\delta_{ab}\delta^{kl}. \tag{13}$$

This is equivalent to the following 'Fokker–Planck' equation to be compared with (4)

$$\partial_\tau\langle P_{\alpha\beta}\rangle_\xi = N_c\int\frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{\alpha\beta}(\gamma)\Big\{\Theta_L(\gamma)\big(\langle P_{\alpha\gamma}P_{\gamma\beta}\rangle_\xi - \langle P_{\alpha\beta}\rangle_\xi\big) - \frac{2C_F}{N_c}\Theta_R(\gamma)\langle P_{\alpha\beta}\rangle_\xi\Big\}, \tag{14}$$

where $\langle\ldots\rangle_\xi$ denotes averaging over the noises. In principle, $P_{LR}(\tau)$ at finite-N_c can be evaluated by computing $\frac{1}{N_c}\,\mathrm{tr}\,U_L U_R^\dagger$ for a given random walk trajectory with the initial condition $U_\alpha(\tau=0)=1$, and then averaging over many trajectories. In this calculation, it suffices to define U_α in the left hemisphere and at a single point $\alpha = R$ in the right hemisphere.

However, this strategy does not apply straightforwardly to our present problem. $\langle P_{LR}\rangle_\xi$ quickly goes to zero due to the collinear

[2] We write the evolution (9) in a slightly different, but equivalent form compared to Ref. [8]. It should be understood that various exponentials are meaningful only to $\mathcal{O}(\epsilon)$ [8], although in practice we keep all orders in $\sqrt{\epsilon}$ in order to preserve the unitarity of U_α.

divergence in the Sudakov factor.[3] One may try to regularize the divergence by introducing a cutoff δ and extract the finite part

$$\langle g_{LR}(\tau)\rangle_\xi = \lim_{\delta\to 0} \exp\left(2C_F\tau \int_R \frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{LR}(\gamma)\right)_\delta \langle P^\delta_{LR}(\tau)\rangle_\xi. \quad (15)$$

Unfortunately, this does not work in practice because $\langle P^\delta_{LR}\rangle_\xi$ becomes very small and the exponential factor becomes very large as $\delta \to 0$. It is difficult to numerically achieve the precise cancelation between the two factors.

As a matter of fact, the same problem was already noticed in the original Monte Carlo simulation at large-N_c [3]. There the authors subtracted the Sudakov contribution step-by-step, by modifying the emission probability as $\mathcal{M}_{\alpha\beta}(\gamma) \to \mathcal{M}_{\alpha\beta}(\gamma) - \Theta_R(\gamma)\mathcal{M}_{LR}(\gamma)$. Here we shall implement a similar subtraction directly in the evolution of U_α. The origin of the collinear divergence can be traced to the factors $e^{iS^{(i)}_\alpha}$ in (9). They give, after averaging over the noise,

$$\begin{aligned}
&\langle e^{iS^{(2)}_\alpha} e^{iS^{(1)}_\alpha} e^{-iS^{(1)}_\beta} e^{-iS^{(2)}_\beta}\rangle_\xi \\
&= \prod_i^{1,2} \Bigg\langle \exp\left(i\sqrt{\frac{\varepsilon}{4\pi}}\int_R d\Omega_\gamma \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma} t^a \xi^{(i)k}_{\gamma a}\right) \\
&\qquad \times \exp\left(-i\sqrt{\frac{\varepsilon}{4\pi}}\int_R d\Omega_\gamma \frac{(\boldsymbol{n}_\beta - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\beta\cdot\boldsymbol{n}_\gamma} t^a \xi^{(i)k}_{\gamma a}\right)\Bigg\rangle_\xi \\
&= \exp\left(-2C_F\varepsilon \int_R \frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{\alpha\beta}(\gamma)\right), \quad (16)
\end{aligned}$$

which is indeed the Sudakov factor (7) generated in a single step. (16) can be checked by using the identity [8]

$$\begin{aligned}
&\mathcal{M}_{\alpha\beta}(\gamma) = 2\mathcal{K}_{\alpha\beta}(\gamma) - \mathcal{K}_{\alpha\alpha}(\gamma) - \mathcal{K}_{\beta\beta}(\gamma), \\
&\mathcal{K}_{\alpha\beta}(\gamma) \equiv \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)\cdot(\boldsymbol{n}_\gamma - \boldsymbol{n}_\beta)}{2(1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma)(1-\boldsymbol{n}_\gamma\cdot\boldsymbol{n}_\beta)}. \quad (17)
\end{aligned}$$

In the special case $(\alpha\beta) = (LR)$, we have that $\mathcal{K}_{LR}(\gamma) \equiv 0$ and

$$\begin{aligned}
\int_R \frac{d\Omega_\gamma}{4\pi}\mathcal{M}_{LR}(\gamma) &= \int_R \frac{d\Omega_\gamma}{4\pi}\left(-\mathcal{K}_{LL}(\gamma) - \mathcal{K}_{RR}(\gamma)\right) \\
&= \frac{\ln 2}{2} + \int_R \frac{d\Omega_\gamma}{4\pi}\frac{1}{1-\boldsymbol{n}_R\cdot\boldsymbol{n}_\gamma}. \quad (18)
\end{aligned}$$

We see that the singularity entirely comes from the second order term in the expansion of $e^{iS^{(i)}_R}$.

It is thus tempting to remove the factor $e^{iS^{(i)}_\alpha}$ altogether and use a modified evolution equation $\widetilde{U}_\alpha(\tau+\varepsilon) = e^{iA_\alpha}\widetilde{U}_\alpha(\tau)e^{iB_\alpha}$. However, this also removes an essential part of the nonglobal logarithms. The reason is that the linear term in the expansion of $e^{iS_\alpha(\xi)} = 1 + iS_\alpha(\xi) + \cdots$ can give finite contributions when the Gaussian noise ξ is contracted with that implicit in $U_\gamma t^a U^\dagger_\gamma$ in (11). Physically, the factor $U_\gamma t^a U^\dagger_\gamma = U^{ab}_A t^b$ (with U_A being the Wilson line in the adjoint representation) represents the emission of real gluons which is restricted to the left hemisphere. These gluons together with the original $q\bar{q}$ pair form a QCD antenna which coherently emits the softest gluon in the right hemisphere, thereby producing nonglobal logarithms.

To make the last statement more concrete, we follow the evolution (9) analytically up to $\tau = 2\varepsilon$ (two steps) and collect the non-Sudakov contributions. We find

$$\begin{aligned}
&\langle g_{LR}(\tau)\rangle_\xi \\
&\sim 2C_F N_c \tau^2 \int_L \frac{d\Omega_\gamma}{4\pi} \\
&\qquad \times \int_R \frac{d\Omega_\lambda}{4\pi}\left(\mathcal{K}_{LL}(\gamma) + \mathcal{K}_{RR}(\gamma)\right)\left(\mathcal{K}_{L\gamma}(\lambda) + \mathcal{K}_{\gamma R}(\lambda) - \mathcal{K}_{\gamma\gamma}(\lambda)\right) \\
&= -C_F N_c \tau^2 \int_L \frac{d\Omega_\gamma}{4\pi} \\
&\qquad \times \int_R \frac{d\Omega_\lambda}{4\pi}\mathcal{M}_{LR}(\gamma)\left(\mathcal{M}_{L\gamma}(\lambda) + \mathcal{M}_{\gamma R}(\lambda) - \mathcal{M}_{LR}(\lambda)\right) \\
&= -\pi^2 \frac{C_F N_c \tau^2}{12}, \quad (19)
\end{aligned}$$

in agreement with the lowest order (two-loop) result [3,9].[4] In the third line of (19), the factors $\mathcal{K}_{L\gamma}$ and $\mathcal{K}_{\gamma R}$ come from the linear terms in $e^{iS_L} \approx 1 + iS_L$ and $e^{iS_R} \approx 1 + iS_R$, respectively. They both seem to be essential for obtaining the correct result.

Importantly, however, the term $\mathcal{K}_{\gamma R}(\lambda)$ vanishes when integrating over the azimuthal angle ϕ_λ

$$\int_0^{2\pi} d\phi_\lambda \mathcal{K}_{\gamma R}(\lambda) = \int_0^{2\pi} d\phi_{\lambda\gamma} \frac{\cos\theta_\lambda - 1 - \cos\theta_\gamma + \cos\theta_{\lambda\gamma}}{2(1-\cos\theta_\lambda)(1-\cos\theta_{\lambda\gamma})} = 0, \quad (20)$$

where we used $\cos\theta_\lambda > 0 > \cos\theta_\gamma$. Moreover, by following the evolution (9) a few more steps, it is easy to convince oneself that the linear term iS_R does not produce nonglobal logarithms to all orders because this term always reduces to factors like $\mathcal{K}_{\gamma^{(n)}R}(\lambda)$ (after contracting with the n-th gluon emitted in the left hemisphere) and vanishes when integrating over ϕ_λ in the right hemisphere. This observation brings in a major simplification in our resummation strategy. We can neglect the factors $e^{iS^{(1,2)}_R}$ in (9) for $\alpha = R$ and use the modified Langevin equation

$$\widetilde{U}_R(\tau+\epsilon) = e^{iA_R}\widetilde{U}_R(\tau)e^{iB_R}. \quad (21)$$

As for U_α in the left hemisphere, we may continue to use the same evolution (9). Actually, we can make a slight improvement which speeds up the numerical simulation. The two independent noises $\xi^{(1,2)}$ defined in the right hemisphere always give identical contributions for the observable at hand. Therefore, we can eliminate one of them and use a modified equation

$$\begin{aligned}
U_\alpha(\tau+\varepsilon) &= \exp\left(i\sqrt{\frac{2\varepsilon}{4\pi}}\int_R d\Omega_\gamma \frac{(\boldsymbol{n}_\alpha - \boldsymbol{n}_\gamma)^k}{1-\boldsymbol{n}_\alpha\cdot\boldsymbol{n}_\gamma} t^a \xi^{(1)k}_{\gamma a}\right) \\
&\qquad \times e^{iA_\alpha}U_\alpha(\tau)e^{iB_\alpha}. \quad (22)
\end{aligned}$$

Note the factor of $\sqrt{2}$. One can check that (22) leads to the same equation (14) for the product of two Wilson lines.[5] Using these

[3] This problem was not encountered in [8] because there α and β were always confined in the unobserved part of phase space.

[4] At this order, we have to interpret $2\varepsilon^2 = \tau(\tau-\varepsilon) \approx \tau^2$ to correct an error in iteratively solving a discretized differential equation.

[5] We have checked numerically that (22) and (9) give equivalent results. The equivalence may not hold for more complicated observables.

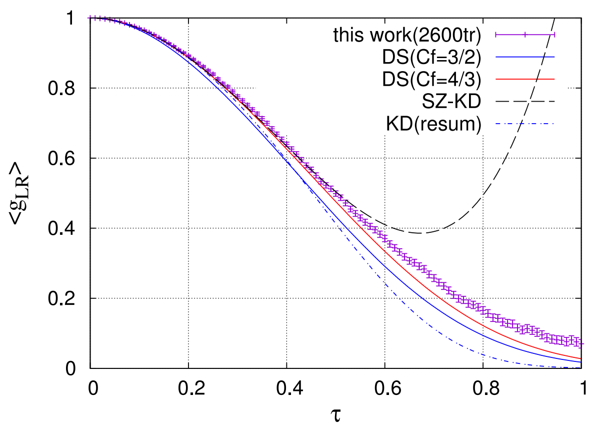

Fig. 1. $\langle g_{LR}(\tau)\rangle$ at $N_c = 3$ as a function of τ obtained by averaging over 2600 random walks. The error bars indicate the standard error. Data points are plotted every $0.01/\varepsilon = 200$ random walk steps. Various curves are explained in the text. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Langevin equations, we finally compute the average

$$\langle g_{LR}(\tau)\rangle_\xi = e^{\tau C_F \ln 2}\frac{1}{N_c}\langle \mathrm{tr}\, U_L(\tau)\widetilde{U}_R^\dagger(\tau)\rangle_\xi . \tag{23}$$

The multiplicative factor in front subtracts the finite part of the Sudakov factor (18) which is included in the evolution of U_L.

4. Numerical results and discussions

The numerical procedure is explained in Ref. [8] which we refer to for details. We discretize the solid angle $1 \geq \cos\theta \geq -1$ and $2\pi > \phi \geq 0$ into a lattice of 80×80 grid points and put a SU(3) matrix U_α at each grid point on the left hemisphere $\cos\theta < 0$. In addition, we define a SU(3) matrix $\widetilde{U}_R$ at a single point $\boldsymbol{n}_R$ in the right hemisphere. The Gaussian noise ξ is randomly generated at all grid points and at each time step.[6] We then evolve U_α and $\widetilde{U}_R$ according to (22) and (21), respectively, with $\varepsilon = 5 \times 10^{-5}$ and the initial conditions $U_\alpha = \widetilde{U}_R = 1$. As in the previous work [8], we observe large event-by-event fluctuations. In order to obtain a reasonably smooth curve, we typically have to average over $\mathcal{O}(10^3)$ random walks. Fig. 1 shows the result from 2600 random walks.[7] In the same figure, we make comparisons with the following results in the literature: The blue line is a parameterization of the all-order Monte Carlo result in the large-N_c limit by Dasgupta and Salam (DS) [3]

$$g^{DS}(\tau) = \exp\left(-C_F N_c \frac{\pi^2\tau^2}{12}\frac{1+(a\tau/2)^2}{1+(b\tau/2)^c}\right), \tag{24}$$

with $a = 0.85N_c$, $b = 0.86N_c$ and $c = 1.33$. Here we set $C_F \approx N_c/2 = 1.5$ which is what was actually used in [3]. The black dashed line is a combination of the fixed-order analytical results by Schwartz and Zhu (SZ) to five-loop at large-N_c [9] and Khelifa-Kerfa and Delenda (KD) to four-loop at finite N_c [10]

$$\begin{aligned} g^{SZ-KD}(\tau) = {} & 1 - C_F N_c \frac{\pi^2}{12}\tau^2 + \frac{C_F N_c^2 \zeta_3}{6}\tau^3 \\ & - \frac{1}{24}\left(\frac{25}{8}C_F N_c^3 \zeta_4 - \frac{13}{5}C_F^2 N_c^2 \zeta_2^2\right)\tau^4 \\ & + \frac{1}{120}\left(-8C_F^2 N_c^3 \zeta_2\zeta_3 + \frac{17}{2}C_F N_c^4 \zeta_5\right)\tau^5, \end{aligned} \tag{25}$$

where $C_F = 4/3$. Actually, the complete finite-N_c result at $\mathcal{O}(\tau^5)$ is not available, and the above formula is a well-motivated guess [10] which reduces to the known result in the large-N_c limit. Finally, the blue dash-dotted line is the following 'resummed' expression suggested by KD based on their four-loop result

$$\begin{aligned} g^{KD(resum)}(\tau) = \exp\Bigg(& -C_F N_c \frac{\pi^2\tau^2}{12} + \frac{C_F N_c^2 \zeta_3 \tau^3}{6} \\ & - \frac{\pi^4}{135}\left(\frac{25}{8}C_F N_c^3 + C_F^2 N_c^2\right)\frac{\tau^4}{16}\Bigg), \end{aligned} \tag{26}$$

with $C_F = 4/3$. Note, however, that at the moment it is not known whether the nonglobal logarithms actually exponentiate to all orders.

We see that our result agrees very well with the most-advanced fixed-order result (25) up to $\tau \lesssim 0.5$. Beyond this, the perturbative result quickly deviates and eventually blows up. It has been observed that higher loop contributions alternate in sign and converge rather poorly [17]. In addition, fixed-order results are numerically sensitive to the $1/N_c$-suppressed corrections when $\tau \sim \mathcal{O}(1)$. This can be partly remedied in the resummed formula (26). On the other hand, the all-order large-N_c result (24) stays close to our curve up to $\tau = 1$. In fact, the difference can be partly accounted for by choosing $C_F = 4/3$ in (24), which is what was actually suggested by DS as the likely functional form at finite-N_c and has been used for phenomenological purposes [18,19]. This is shown by the red line in Fig. 1. To correct the remaining difference, we independently determined a, b, c in (24) with $C_F = 4/3$ and obtained

$$a = 0.62 \pm 0.06, \qquad b = 0.06 \pm 0.03, \qquad c = 0.37 \pm 0.04. \tag{27}$$

In conclusion, we have completed the resummation project for the single-hemisphere jet mass distribution initiated in [3] by including the finite-N_c corrections to all orders. We find that the finite-N_c effect is numerically small, and this is consistent with the previous finding in [8]. However, it should be kept in mind that the observables calculated at finite-N_c so far are defined in e^+e^- annihilation where the two outgoing jets are represented by the product of two Wilson lines $\mathrm{tr}\, U_\alpha U_\beta^\dagger$. In hadron–hadron collisions, or in processes including hard gluons, one needs to consider the evolution of more complicated objects such as $\mathrm{tr}(U_\alpha U_\beta^\dagger U_\gamma U_\delta^\dagger)$ and $\mathrm{tr}(U_\alpha U_\beta^\dagger)\,\mathrm{tr}(U_\gamma U_\delta^\dagger)$ (cf., [20]). The finite-N_c effects in the resummation of nonglobal logarithms for these observables have not been studied so far.

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[6] In [8], the authors inadvertently assumed that the noise ξ (at each time step) is independent of ϕ at the degenerate points $\cos\theta = \pm 1$. Fortunately, this was innocuous for the observable considered in [8]. However, for the present observable this causes a systematic error in the evaluation of the Sudakov integral (the first term of (18)) already at small-τ because the integration region $\int d\Omega_R$ includes the point $\cos\theta = 1$ (which was not the case in [8]). In the present simulations, we fixed this problem by generating ξ at different values of ϕ independently also at $\cos\theta = \pm 1$.

[7] We also performed simulations with different discretization parameters (160×40 and 80×40 lattices, and $\varepsilon = 10^{-4}$) and found that the results are consistent with each other.

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