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Birational rigidity of complete intersections

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Background
Recall the following definition:

Definition
A Mori fiber space $X/S$ is called birationally superrigid if any
birational map to the source of another Mori fiber space is isomor-
phism.

It implies that $X$ is non-rational and $\text{Bl}(X) = \text{Aut}(X)$.

Consider a complete intersection $X$ of type $X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i}$ of
dimension $\geq 3$ with only mild singularities, which is defined by $s$
hypersurfaces of degree $d_1, \ldots, d_s$ in a projective space $\mathbb{P} \Sigma_i^{1:d_i}$. It is
Fano of index 1 and rationally-connected.

For $s = 1$, after the works of Iskovskih-Manin, Pukhlikov and
Cheltsov, de Fernex proved:

Theorem (de Fernex ’13)
For $N \geq 4$, every smooth hypersurface $X = X_N \subset \mathbb{P}^N$ of degree
$N$ is birationally superrigid.

For $s \geq 2$, its birational superrigidity is known only when $X$ is one of the
following:

Known cases when $s \geq 2$
- a smooth complete intersection $X = X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i}$ of
dimension $\geq 12$ which satisfies so-called regularity conditions, except three infinite series $X_{2,2,2}, X_{2,2,2}$ and $X_{2,2,2}$ by
Pukhlikov,
- a smooth complete intersection $X = X_{2,4} \subset \mathbb{P}^2$ not containing
planes by Cheltsov.

No explicit examples which satisfy these conditions have been ob-
tained so far.

Birational superrigid complete intersections
We prove birational superrigidity of smooth and singular complete
intersections. For $s$ positive integers $d_1, \ldots, d_s$, set
$$c_s(d_1, \ldots, d_s) = \frac{2(\Sigma_i d_i + 1)}{\prod_i d_i} - 5s$$
in what follows.

Smooth complete intersections

Theorem A
Let $d_1, \ldots, d_s \geq 2$ be integers which satisfy
$$1 \leq c_s(d_1, \ldots, d_s).$$
Then every smooth complete intersection $X = X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i}$
is birationally superrigid.

We obtain explicit examples of birationally superrigid complete inter-
sections which are not hypersurfaces. Here we only give the simplest
ones among them.

Corollary
Every smooth complete intersection
$$X = X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i},$$
is birationally superrigid for $d \geq 55, 83, 111, 246$ respectively.

Singular complete intersections : Case 1
Recall that an isolated singularity is called a semi-homogeneous hy-
perurface singularity if its tangent cone is a hypersurface which is
smooth away from the vertex.

Theorem B
For positive integers $d_1, \ldots, d_s \geq 2$, every complete intersection
$X = X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i}$, with only semi-homogeneous hypersur-
face singularities of multiplicity at most $c_s(d_1, \ldots, d_s) - 2$ is birationally
superrigid.

Singular complete intersections : Case 2
For a complete intersection $X \subset \mathbb{P}^N$, denote by $X^v \subset (\mathbb{P}^N)^*$ the
dual variety of $X$. Set $d = \deg X$ and $\pi = \Sigma_{i=5,4} - \dim X(d_i - 1)^0 \cdots (d_i - 1)^{\rho_i}$. It is known that $X^v$ is a hypersurface and $\deg X^v = dx$ if $X$ is smooth.

Theorem C
Let $d_1, \ldots, d_s \geq 2$ be positive integers and $X = X_{d_1, \ldots, d_s} \subset \mathbb{P} \Sigma_i^{1:d_i}$ be a singular complete intersection with $t$ isolated hyper-
surface singularities. If $X^v$ is a hypersurface and
$$dx - \deg X^v \leq c_s(d_1, \ldots, d_s) + 2t - 5,$$
then $X$ is birationally superrigid.

Pukhlikov’s multiplicity bounds
The following is a key proposition, which was known when $s = 1$ by
Pukhlikov and $k = 1$ by Cheltsov.

Proposition
Let $X$ be a complete intersection in $\mathbb{P}^N$ defined by $s$ hypersurfaces
and $\alpha$ be an effective cycle on $X$ of pure codimension $k$ such that
$$\alpha \sim m \cdot \sigma(\mathbb{O}(1)^{k} \cap [X])$$
Assume either that $X$ is smooth or $|k| = \dim \text{Sing}(X) + 1 < N$. Then $\epsilon_s(\alpha) \leq m$ for every closed subvariety $S \subset X$ of dimension $k$s not meeting the singular locus of $X$.

Proof: We may assume that $\dim S = k$ and $S \subset [\alpha]$. Then we use the
method of multiple residual intersection, to construct a cycle $R$
of pure-dimensional $k$ on $X$ such that
$$\begin{align*}
(1) & \quad R \subset X^m, \\
(2) & \quad \alpha \text{ and } R \text{ intersect properly on } X, \text{ i.e. } \dim [\alpha] \cap [R] = 0, \\
(3) & \quad S \cap [R] \text{ contains at least } \deg R \text{ points.}
\end{align*}$$

Then
$$m \deg R = \alpha \cdot R \geq \sum_{S \cap [R]} i(t, \alpha \cdot R, X) \geq \sum_{S \cap [R]} \epsilon_s(\alpha) \cdot \epsilon_s(R) \geq \epsilon_s(\alpha) \cdot \deg R$$
The proof is done.