Enriques quotients of the universal cover of $E^{[n]}$ of an Enriques surface E

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1. Introduction

An Enriques surface E is a compact complex surface with $H^1(E, \mathcal{O}_E) = 0$, $H^2(E, \mathcal{O}_E) = 0$, and $\omega_E^{\otimes 2} \simeq \mathcal{O}_E$. Let $E^{[n]}$ be the Hilbert scheme of n points of E. We fix the universal cover $\pi : X \to E^{[n]}$ and its covering involution ρ. It is known that $\pi_1(E^{[n]}) = 2$ and X is a Calabi-Yau manifold.

Definition 1.1. *A variety Y is called an Enriques quotient of X if there is an Enriques surface E' and a free involution τ of X such that $Y = X/\langle\tau\rangle \cong E'^{[n]}$. Here we will call two Enriques quotients of X distinct if they are not isomorphic to each other.*

We count the number of distinct Enriques quotients of X.

2. Ohashi's result

When $n = 1$, Ohashi obtained the following theorem (see [1, Theorem 0.1]).

Theorem 2.1. *For any nonnegative integer l, there exists a $K3$ surface with exactly 2^{l+10} distinct Enriques quotients. In particular, there does not exist a universal bound for the number of distinct Enriques quotients of a $K3$ surface.*

3. Main Theorem 1

When $n \geqq 3$, the situation is totally different from Ohashi's result (see[2, Theorem 1.7]). We get the following Theorem.

Theorem 3.1. *If τ is a free involution of X such that $X/\langle\tau\rangle$ is an Enriques quotient of X, then $\tau = \rho$. In particular the number of distinct Enriques quotients of X is one.*

4. Stragety

i) We show that for $n \geq 3$, the covering involution of $\pi : X \to E^{[n]}$ acts on $H^2(X, \mathbb{C})$ as id and $H^{2n-1,1}(X, \mathbb{C})$ as $-$id. Remark $n = 2$, the covering involution of does not acts on $\mathrm{H}^2(X, \mathbb{C})$ as id.
ii) We show that for $n \geq 2$, if an automorphism φ of X acts on $H^2(X, \mathbb{C})$ as identity, then φ is a lift of a natural automorphism of $E^{[n]}$.

By using the above, we get Main Theorem 1.

5. Main Theorem 2

When $n = 2$, we get the follwing Theorem.

Theorem 5.1.
i) For two Enriques surfaces E and E',
if $E^{[2]} \cong E'^{[2]}$, then $E \cong E'$.
ii) $\mathrm{Aut}(E^{[2]}) \cong \mathrm{Aut}(E)$, i.e. all automorphisms of $\mathrm{Aut}(E^{[2]})$ are the natural automorphisms.

Remark 5.2.
When $n = 2$, we did not yet count the number of distinct Enriques quotients of X.

6. References

[1] H. Ohashi: On the number of Enriques quotients of a K3 surface. Publ. Res. Inst. Math. Sci. 43 (2007), no. 1, 181-200. 14J28.
[2] T. Hayashi: Universal covering calabi-yau manifolds of the Hilbert schemes of n points of Enriques surfaces. arXiv:1502.02231.