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<th>Enriques quotients of the universal cover of $E[n]$ of an Enriques surface $E$</th>
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1. Introduction

An Enriques surface $E$ is a compact complex surface with $H^1(E, \mathcal{O}_E) = 0$, $H^2(E, \mathcal{O}_E) = 0$, and $\omega_E \simeq \mathcal{O}_E$. Let $E^{[n]}$ be the Hilbert scheme of $n$ points of $E$. We fix the universal cover $\pi: X \to E^{[n]}$ and its covering involution $\rho$. It is known that $\pi_1(E^{[n]}) = 2$ and $X$ is a Calabi-Yau manifold.

Definition 1.1. A variety $Y$ is called an Enriques quotient of $X$ if there is an Enriques surface $E_0$ and a free involution $\varphi$ of $X$ such that $Y = X/\varphi$. Here we will call two Enriques quotients of $X$ distinct if they are not isomorphic to each other.

We count the number of distinct Enriques quotients of $X$.

2. Ohashi’s result

When $n = 1$, Ohashi obtained the following theorem (see [1, Theorem 0.1]).

Theorem 2.1. For any nonnegative integer $l$, there exists a K3 surface with exactly $2^{2+10}$ distinct Enriques quotients. In particular, there does not exist a universal bound for the number of distinct Enriques quotients of a K3 surface.

3. Main Theorem 1

When $n \geq 3$, the situation is totally different from Ohashi’s result (see [2, Theorem 1.7]). We get the following Theorem.

Theorem 3.1. If $\tau$ is a free involution of $X$ such that $X/\tau$ is an Enriques quotient of $X$, then $\tau = \rho$. In particular the number of distinct Enriques quotients of $X$ is one.

4. Strategy

i) We show that for $n \geq 3$, the covering involution of $\pi: X \to E^{[n]}$ acts on $H^2(X, \mathbb{C})$ as $\text{id}$ and $H^{2n-1,1}(X, \mathbb{C})$ as $-\text{id}$. Remark $n = 2$, the covering involution of does not acts on $H^2(X, \mathbb{C})$ as $\text{id}$.

ii) We show that for $n \geq 2$, if an automorphism $\varphi$ of $X$ acts on $H^2(X, \mathbb{C})$ as identity, then $\varphi$ is a lift of a natural automorphism of $E^{[n]}$.

By using the above, we get Main Theorem 1.

5. Main Theorem 2

When $n = 2$, we get the following Theorem.

Theorem 5.1.

i) For two Enriques surfaces $E$ and $E'$, if $E^{[2]} \cong E'[2]$, then $E \cong E'$.

ii) $\text{Aut}(E^{[2]}) \cong \text{Aut}(E)$, i.e. all automorphisms of $\text{Aut}(E^{[2]})$ are the natural automorphisms.

Remark 5.2.

When $n = 2$, we did not yet count the number of distinct Enriques quotients of $X$.

6. References
