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<td>Title</td>
<td>THE FULL TRANSFORMATION SEMIGROUP OF FINITE RANK AND AMALGAMATION BASES FOR FINITE SEMIGROUPS (Logics, Algebras and Languages in Computer Science)</td>
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<tr>
<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>数理解析研究所講究録 2014年9月号 97-99</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2014-09</td>
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<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/223302">http://hdl.handle.net/2433/223302</a></td>
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<td>定結時</td>
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<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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THE FULL TRANSFORMATION SEMIGROUP OF FINITE RANK AND AMALGAMATION BASES FOR FINITE SEMIGROUPS*

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In this paper, we prove that the full transformation semigroup of finite rank is an amalgamation bases for finite semigroups.

1 Semigroup amalgamation bases

Definition Let $\mathcal{A}$ be the class of finite semigroups.
An amalgam $[S, T; U]$ of $\mathcal{A}$ is called to be weakly embeddable in $\mathcal{A}$ if there exist a semigroup $K$ belonging to $\mathcal{A}$ and monomorphisms $\xi_1 : S \to K$, $\xi_2 : T \to K$ such that the restrictions to $U$ of $\xi_1$ and $\xi_2$ are equal to each other (that is, $\xi_1(S) \cap \xi_2(T) \supseteq \xi_1(U)$).

An amalgam $[S, T; U]$ of $\mathcal{A}$ is called to be strongly embeddable in $\mathcal{A}$ if $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$.

A semigroup $U$ in $\mathcal{A}$ is amalgamation base [resp. weak amalgamation base] if any amalgam with a core $U$ in $\mathcal{A}$ is strongly embeddable [resp. weakly embeddable] in $\mathcal{A}$.

Result 1 [[3], Theorem 12] Any finite semigroup $U$ is an amalgamation base for finite semigroups if and only if $U$ is a weak amalgamation base for finite semigroups.

Result 2[[5], Theorem 1] If a finite semigroup $U$ is an amalgamation base for finite semigroups, then all $\mathcal{J}$-classes of $U$ form a chain.

Definition Let $U$ be a semigroup with zero, $0$, and $a, b \in S$.
The set $\{s \in U \mid sa = 0\}$ is called the left annihilator of $a$ in $S$ and is denoted by $Ann_l(a)$.
In this case, we say that $U$ satisfies the condition $Ann_l$ if $Ann_l(a) = Ann_l(b)$ implies $aU = bU$.
The right annihilator and the condition $Ann_r$ are defined by left-right duality.

Result 3 [[9], Theorem 1.6] Let $U$ be a finite regular semigroup whose all the $\mathcal{J}$-classes form a chain.
Suppose that there is a chain of principal ideals such that $U_i$ is a maximal subgroup and each $U_i/U_{i+1}$ is a completely 0-simple semigroups satisfying the conditions $Ann_l$ and $Ann_r$ ($1 \leq i \leq n - 1$). Then $U$ is an amalgamation base for finite semigroups.

*This is an abertact and the paper will appear elsewhere.
Consider $T(X)$, where the composition is from right to left.
The following result is a characterization of semigroups which is amalgamation baseses for finite semigroups.

**Result 4.** [[6], Lemma 1 and Corollary] Let $U$ be a finite semigroup. Then the following are equivalent:

1. $U$ is an amalgamation base for finite semigroups;
2. For any two embeddings $\phi_1, \phi_2$ of $U$ into the full transformation semigroup $T(X)$, there exist a finite set $Y$ and two embeddings $\delta_1, \delta_2 : T(X) \rightarrow T(Y)$ such that $Y$ contains $X$ as a subset and $\delta_1 \phi_1$ and $\delta_2 \phi_2$ coincide on $U$;
3. For any finite semigroups $S, T$, any finite faithful left [right] $S$-set $X$ and any finite faithful left [right] $T$-set $Y$, there exist a finite faithful left [right] $S$-set $X' \supseteq X$ and a finite faithful left [right] $T$-set $Y' \supseteq Y$ such that the $U$-sets $X', Y'$ are $U$-isomorphic to each other.

### 2 The main theorem

Consider $T^{op}(X)$, where the composition is from left to right.

Let $|X| = n$ and $I_k = \{ f \in T^{op}(X) \mid |(X)f| \leq k \}$. Then $T^{op}(X) = I_n \supset I_{n-1} \supset \cdots \supset I_2 \supset I_1$ is a chain of ideals of $T^{op}(X)$ and each factor semigroup $I_i/I_{i-1}$ $(i \geq 2)$ is a completely 0-simple semigroup satisfying the conditions $Ann_l$ and $Ann_r$.

Let $R_n$ denote the set $I_1$ of constant maps on $X$ and $S_n$ the set of bijective maps on $X$. Then in $T^{op}(X)$, then $S_n \cup R_n$ is a subsemigroup of $T^{op}(X)$.

**Proposition.** The semigroup $S_n \cup R_n$ is an amalgamation bases for finite semigroups.

By using Proposition and an analogue of Result 3, we obtain

**The main theorem.** The full transformation semigroup of finite rank is an amalgamation bases for finite semigroups.

### References


