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Existence and mean approximation of fixed points of generalized hybrid non-self mappings in Hilbert spaces and some examples

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Abstract

In this paper we prove a fixed point theorem and mean convergence theorems of Baillon's type for widely more generalized hybrid non-self mappings in a Hilbert space. Moreover we give some examples of widely more generalized hybrid non-self mapping.

1 Introduction

Let $H$ be a real Hilbert space and let $C$ be a non-empty subset of $H$. In 2010, Kocourek, Takahashi and Yao [14] defined a class of nonlinear mappings in a Hilbert space. A mapping $T$ from $C$ into $H$ is said to be generalized hybrid if there exist real numbers $\alpha$ and $\beta$ such that

$$\alpha\Vert Tx - Ty\Vert^2 + (1 - \alpha)\Vert x - Ty\Vert^2 \leq \beta\Vert Tx - y\Vert^2 + (1 - \beta)\Vert x - y\Vert^2$$

for any $x, y \in C$. We call such a mapping $(\alpha, \beta)$-generalized hybrid mapping. We observe that the class of the mappings covers the classes of well-known mappings. For example, an $(\alpha, \beta)$-generalized hybrid mapping is nonexpansive [19] for $\alpha = 1$ and $\beta = 0$, that is, $\Vert Tx - Ty\Vert \leq \Vert x - y\Vert$ for any $x, y \in C$. It is nonspreading [16] for $\alpha = 2$ and $\beta = 1$, that is, $2\Vert Tx - Ty\Vert^2 \leq \Vert Tx - y\Vert^2 + \Vert Ty - x\Vert^2$ for any $x, y \in C$. It is also hybrid [20] for $\alpha = \frac{3}{2}$ and $\beta = \frac{1}{2}$, that is, $3\Vert Tx - Ty\Vert^2 \leq \Vert x - y\Vert^2 + \Vert Tx - y\Vert^2 + \Vert Ty - x\Vert^2$ for any $x, y \in C$. They proved fixed point theorems for such mappings; see also Kohsaka and Takahashi [15] and Iemoto and Takahashi [9]. Moreover they defined a more broad class of nonlinear mappings than the class of generalized hybrid mappings. A mapping $T$ from $C$ into $H$ is said to be super hybrid if there exist real numbers $\alpha, \beta$ and $\gamma$ such that

$$\alpha\Vert Tx - Ty\Vert^2 + (1 - \alpha + \gamma)\Vert x - Ty\Vert^2 \leq (\beta + (\beta - \alpha)\gamma)\Vert Tx - y\Vert^2 + (1 - \beta - (\beta - \alpha - 1)\gamma)\Vert x - y\Vert^2 + (\alpha - \beta)\gamma\Vert x - Tx\Vert^2 + \gamma\Vert y - Ty\Vert^2$$
for any $x, y \in C$. A generalized hybrid mapping with a fixed point is quasinonexpansive. However, a super hybrid mapping is not quasi-nonexpansive generally even if it has a fixed point. Very recently, the authors [11] also defined a class of nonlinear mappings in a Hilbert space which covers the class of contractive mappings and the class of generalized hybrid mappings defined by Kocourek, Takahashi and Yao [14]. A mapping $T$ from $C$ into $H$ is said to be widely generalized hybrid if there exist real numbers $\alpha, \beta, \gamma, \delta, \varepsilon$ and $\zeta$ such that

$$
\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \\
+ \max\{\varepsilon\|x - Tx\|^2, \zeta\|y - Ty\|^2\} \leq 0
$$

for any $x, y \in C$. Moreover the authors [12] defined a class of nonlinear mappings in a Hilbert space which covers the class of super hybrid mappings and the class of widely generalized hybrid mappings. A mapping $T$ from $C$ into $H$ is said to be widely more generalized hybrid if there exist real numbers $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$ and $\eta$ such that

$$
\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \\
+ \varepsilon\|x - Tx\|^2 + \zeta\|y - Ty\|^2 + \eta\|(x - Tx) - (y - Ty)\|^2 \leq 0
$$

for any $x, y \in C$. Then we prove fixed point theorems for such new mappings in a Hilbert space. Moreover we prove nonlinear ergodic theorems of Baillon’s type in a Hilbert space. It seems that the results are new and useful. For example, using our fixed point theorems, we can directly prove Browder and Petryshyn’s fixed point theorem [5] for strictly pseudocontractive mappings and Kocourek, Takahashi and Yao’s fixed point theorem [14] for super hybrid mappings. On the other hand, Hojo, Takahashi and Yao [8] defined a more broad class of nonlinear mappings than the class of generalized hybrid mappings. A mapping $T$ from $C$ into $H$ is said to be extended hybrid if there exist real numbers $\alpha, \beta$ and $\gamma$ such that

$$
\alpha(1 + \gamma)\|Tx - Ty\|^2 + (1 - \alpha(1 + \gamma))\|x - Ty\|^2 \\
\leq (\beta + \alpha\gamma)\|Tx - y\|^2 + (1 - (\beta + \alpha\gamma))\|x - y\|^2 \\
-(\alpha - \beta)\gamma\|x - Tx\|^2 - \gamma\|y - Ty\|^2
$$

for any $x, y \in C$. Moreover they proved a fixed point theorem for generalized hybrid non-self mappings by using the extended hybrid mapping.

In this paper, using an idea of [8], we prove a fixed point theorem for widely more generalized hybrid non-self mappings in Hilbert spaces. Moreover we prove mean convergence theorems of Baillon’s type for widely more generalized hybrid non-self mappings in a Hilbert space.

2 Preliminaries

Throughout this paper, we denote by $\mathbb{N}$ the set of positive integers and by $\mathbb{R}$ the set of real numbers. Let $H$ be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. We
denote the strong convergence and the weak convergence of \( \{x_n\} \) to \( x \in H \) by \( x_n \rightharpoonup x \) and \( x_n \rightarrow x \), respectively. Let \( A \) be a non-empty subset of \( H \). We denote by \( \overline{co}A \) the closure of the convex hull of \( A \). In a Hilbert space, it is known that
\[
\|\!(1 - \lambda)\!x + \lambda y\!\|^2 = (1 - \lambda)\|x\|^2 + \lambda\|y\|^2 - (1 - \lambda)\lambda\|x - y\|^2
\]
(2.1)
for any \( x, y \in H \) and for any \( \lambda \in \mathbb{R} \); see [19]. Moreover in a Hilbert space, we obtain that
\[
2\langle x - y, z - w \rangle = \|x - w\|^2 + \|y - z\|^2 - \|x - z\|^2 - \|y - w\|^2
\]
(2.2)
for any \( x, y, z, w \in H \). Let \( C \) be a non-empty subset of \( H \) and let \( T \) be a mapping from \( C \) into \( H \). We denote by \( F(T) \) the set of fixed points of \( T \). A mapping \( T \) from \( C \) into \( H \) with \( F(T) \neq \emptyset \) is said to be quasi-nonexpansive if \( \|x - Ty\| \leq \|x - y\| \) for any \( x \in F(T) \) and for any \( y \in C \). It is well-known that the set \( F(T) \) of fixed points of a quasi-nonexpansive mapping \( T \) is closed and convex; see Ito and Takahashi [10]. It is not difficult to prove such a result in a Hilbert space; see, for instance, [22]. Let \( C \) be a non-empty closed convex subset of \( H \) and \( x \in H \). Then, we know that there exists a unique nearest point \( z \in C \) such that \( \|x - z\| = \inf_{y \in C} \|x - y\| \). We denote such a correspondence by \( z = P_Cx \). The mapping \( P_C \) is said to be the metric projection from \( H \) onto \( C \). It is known that \( P_C \) is nonexpansive and
\[
\langle x - P_Cx, P_Cx - u \rangle \geq 0
\]
for any \( x \in H \) and for any \( u \in C \); see [19] for more details. For proving a mean convergence theorem, we also need the following lemma proved by Takahashi and Toyoda [21].

**Lemma 2.1.** Let \( C \) be a non-empty closed convex subset of \( H \). Let \( P_C \) be the metric projection from \( H \) onto \( C \). Let \( \{u_n\} \) be a sequence in \( H \). If \( \|u_{n+1} - u\| \leq \|u_n - u\| \) for any \( u \in C \) and for any \( n \in \mathbb{N} \), then \( \{P_Cu_n\} \) converges strongly to some \( u_0 \in C \).

Let \( \ell^\infty \) be the Banach space of bounded sequences with supremum norm. Let \( \mu \) be an element of \( (\ell^\infty)^* \) (the dual space of \( \ell^\infty \)). Then, we denote by \( \mu(f) \) the value of \( \mu \) at \( f = (x_1, x_2, x_3, \ldots) \in \ell^\infty \). Sometimes, we denote by \( \mu_n(x_n) \) the value \( \mu(f) \). A linear functional \( \mu \) on \( \ell^\infty \) is said to be a mean if \( \mu(e) = \|\mu\| = 1 \), where \( e = (1, 1, 1, \ldots) \). A mean \( \mu \) is said to be a Banach limit on \( \ell^\infty \) if \( \mu_n(x_{n+1}) = \mu_n(x_n) \). We know that there exists a Banach limit on \( \ell^\infty \). If \( \mu \) is a Banach limit on \( \ell^\infty \), then for \( f = (x_1, x_2, x_3, \ldots) \in \ell^\infty \),
\[
\lim \inf_{n \rightarrow \infty} x_n \leq \mu_n(x_n) \leq \lim \sup_{n \rightarrow \infty} x_n.
\]
In particular, if \( f = (x_1, x_2, x_3, \ldots) \in \ell^\infty \) and \( x_n \rightarrow a \in \mathbb{R} \), then we obtain \( \mu(f) = \mu_n(x_n) = a \). See [18] for the proof of existence of a Banach limit and its other elementary properties. By means and the Riesz theorem, we have the following result; see [17] and [18].

**Lemma 2.2.** Let \( H \) be a Hilbert space, let \( \{x_n\} \) be a bounded sequence in \( H \) and let \( \mu \) be a mean on \( \ell^\infty \). Then there exists a unique point \( z_0 \in \overline{co}\{x_n \mid n \in \mathbb{N}\} \) such that
\[
\mu_n(x_n, y) = \langle z_0, y \rangle
\]
for any \( y \in H \).
Kawasaki and Takahashi [12] proved by Lemma 2.2 the following fixed point theorem.

**Theorem 2.1.** Let $H$ be a real Hilbert space, let $C$ be a non-empty closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into itself which satisfies the following condition (1) or (2):

1. $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \gamma + \epsilon + \eta > 0$ and $\zeta + \eta \geq 0$;
2. $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \beta + \zeta + \eta > 0$ and $\epsilon + \eta \geq 0$.

Then $T$ has a fixed point if and only if there exists $z \in C$ such that $\{T^nz \mid n = 0, 1, \ldots\}$ is bounded. In particular, a fixed point of $T$ is unique in the case of $\alpha + \beta + \gamma + \delta > 0$ on the conditions (1) and (2).

As a direct consequence of Theorem 2.1, we obtain the following.

**Theorem 2.2.** Let $H$ be a real Hilbert space, let $C$ be a bounded closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into itself which satisfies the following condition (1) or (2):

1. $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \gamma + \epsilon + \eta > 0$ and $\zeta + \eta \geq 0$;
2. $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \beta + \zeta + \eta > 0$ and $\epsilon + \eta \geq 0$.

Then $T$ has a fixed point. In particular, a fixed point of $T$ is unique in the case of $\alpha + \beta + \gamma + \delta > 0$ on the conditions (1) and (2).

## 3 Fixed point theorem

Let $H$ be a real Hilbert space and let $C$ be a non-empty subset of $H$. A mapping $T$ from $C$ into $H$ was said to be widely more generalized hybrid if there exist $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{R}$ such that

$$
\alpha \Vert Tx - Ty \Vert^2 + \beta \Vert x - Ty \Vert^2 + \gamma \Vert Tx - y \Vert^2 + \delta \Vert x - y \Vert^2
+ \epsilon \Vert x - Tx \Vert^2 + \zeta \Vert y - Ty \Vert^2 + \eta \Vert (x - Tx) - (y - Ty) \Vert^2 \leq 0
$$

(3.1)

for any $x, y \in C$; see Introduction. Such a mapping $T$ is said to be $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid; see [12]. An $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping is generalized hybrid in the sense of Kocourek, Takahashi and Yao [14] if $\alpha + \beta = -\gamma - \delta = 1$ and $\epsilon = \zeta = \eta = 0$. Moreover it is an extension of widely generalized hybrid mappings in the sense of Kawasaki and Takahashi [11]. By Theorem 2.2 we prove a fixed point theorem for widely more generalized hybrid non-self mappings in a Hilbert space.

**Theorem 3.1.** Let $H$ be a real Hilbert space, let $C$ be a non-empty bounded closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$ which satisfies the following condition (1) or (2):
(1) \( \alpha + \beta + \gamma + \delta \geq 0, \alpha + \gamma + \epsilon + \eta > 0, \) and there exists \( \lambda \in \mathbb{R} \) such that \( \lambda \neq 1 \) and 
\((\alpha + \beta)\lambda + \zeta + \eta \geq 0; \)

(2) \( \alpha + \beta + \gamma + \delta \geq 0, \alpha + \beta + \zeta + \eta > 0, \) and there exists \( \lambda \in \mathbb{R} \) such that \( \lambda \neq 1 \) and 
\((\alpha + \gamma)\lambda + \epsilon + \eta \geq 0. \)

Suppose that for any \( x \in C, \) there exist \( m \in \mathbb{R} \) and \( y \in C \) such that \( 0 \leq (1 - \lambda)m \leq 1 \) and 
\( Tx = x + m(y - x). \) Then \( T \) has a fixed point. In particular, a fixed point of \( T \) is unique
in the case of \( \alpha + \beta + \gamma + \delta > 0 \) on the conditions (1) and (2).

Example 3.1. Let \( H = \mathbb{R}, \) let \( C = [0, \frac{\pi}{2}] \), let \( Tx = (1 + 2x) \cos x - 2x^2 \) and let \( \alpha = 1, \)
\( \beta = \gamma = 11, \delta = -22, \epsilon = \zeta = -12 \) and \( \eta = 1. \) Then \( T \) is an \((\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)\)-widely more
generalized hybrid mapping from \( C \) into \( H, \) \( \alpha + \beta + \gamma + \delta = 1 \geq 0 \) and \( \alpha + \gamma + \epsilon + \eta = 1 > 0. \)
Let \( \lambda = \frac{2+3\pi}{3(1+\pi)} \) and let \( m = 1+\pi. \) Then \( 0 \leq (1-\lambda)m = \frac{1}{3} < 1 \) and \( \alpha + \beta + \gamma + \delta = 1 \geq 0. \)
Let \( y = x + \frac{(1+2x)(\cos x-2x)}{1+\pi} \) for any \( x \in C. \) Then \( Tx = x + m(y - x) \) and \( y \in C. \) Therefore
by Theorem 3.1 \( T \) has a unique fixed point.

4 Nonlinear ergodic theorems

In this section, using the technique developed by Takahashi [17], we prove mean convergence
theorems of Baillon's type in a Hilbert space. Before proving the results, we need
the following lemmas.

Lemma 4.1. Let \( H \) be a real Hilbert space, let \( C \) be a non-empty closed convex subset of
\( H \) and let \( T \) be an \((\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)\)-widely more generalized hybrid mapping from \( C \) into
\( H \) which has a fixed point and satisfies the condition:

\[ \alpha + \gamma + \epsilon + \eta > 0, \text{ or } \alpha + \beta + \zeta + \eta > 0. \]

Then \( F(T) \) is closed.

Lemma 4.2. Let \( H \) be a real Hilbert space, let \( C \) be a non-empty closed convex subset of
\( H \) and let \( T \) be an \((\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)\)-widely more generalized hybrid mapping from \( C \) into
\( H \) which has a fixed point and satisfies the condition (1) or (2):

(1) \( \alpha + \beta + \gamma + \delta \geq 0 \) and \( \alpha + \gamma + \epsilon + \eta > 0; \)

(2) \( \alpha + \beta + \gamma + \delta \geq 0 \) and \( \alpha + \beta + \zeta + \eta > 0. \)

Then \( F(T) \) is convex.

Lemma 4.3. Let \( H \) be a real Hilbert space, let \( C \) be a non-empty closed convex subset of
\( H \) and let \( T \) be an \((\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)\)-widely more generalized hybrid mapping from \( C \) into
\( H \) which has a fixed point and satisfies the condition (1) or (2):

(1) \( \alpha + \beta + \gamma + \delta \geq 0, \alpha + \gamma > 0 \) and \( \epsilon + \eta \geq 0; \)
(2) $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \beta > 0$ and $\zeta + \eta \geq 0$.

Then $T$ is quasi-nonexpansive.

Moreover we obtain the following.

Lemma 4.4. Let $H$ be a real Hilbert space, let $C$ be a non-empty closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$ which has a fixed point and satisfies the condition (1) or (2):

(1) $\alpha + \beta + \gamma + \delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha + \gamma)\lambda + \epsilon + \eta < \alpha + \gamma + \epsilon + \eta$;

(2) $\alpha + \beta + \gamma + \delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha + \beta)\lambda + \zeta + \eta < \alpha + \beta + \zeta + \eta$.

Then $(1 - \lambda)T + \lambda I$ is quasi-nonexpansive.

Now we first obtain the following mean convergence theorem for widely more generalized hybrid mappings in a Hilbert space.

Theorem 4.1. Let $H$ be a real Hilbert space, let $C$ be a non-empty closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$ which has a fixed point and satisfies the condition (1) or (2):

(1) $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \gamma > 0$ and $\epsilon + \eta \geq 0$;

(2) $\alpha + \beta + \gamma + \delta \geq 0$, $\alpha + \beta > 0$ and $\zeta + \eta \geq 0$.

Then for any $x \in C(T;0) = \{z \mid T^n z \in C$ for any $n \in \mathbb{N} \cup \{0\}\}$,

$$S_n x = \frac{1}{n} \sum_{k=0}^{n-1} T^k x$$

is weakly convergent to a fixed point $p$ of $T$, where $P$ is the metric projection from $H$ onto $F(T)$ and $p = \lim_{n \to \infty} P T^n x$.

Moreover we obtain the following.

Theorem 4.2. Let $H$ be a real Hilbert space, let $C$ be a non-empty closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$ which has a fixed point and satisfies the condition (1) or (2):

(1) $\alpha + \beta + \gamma + \delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha + \gamma)\lambda + \epsilon + \eta < \alpha + \gamma + \epsilon + \eta$;

(2) $\alpha + \beta + \gamma + \delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha + \beta)\lambda + \zeta + \eta < \alpha + \beta + \zeta + \eta$.

Then for any $x \in C(T;\lambda) = \{z \mid ((1 - \lambda)T + \lambda I)^n x \in C$ for any $n \in \mathbb{N} \cup \{0\}\}$,

$$S_n x = \frac{1}{n} \sum_{k=0}^{n-1} ((1 - \lambda)T + \lambda I)^k x$$

is weakly convergent to a fixed point $p$ of $T$, where $P$ is the metric projection from $H$ onto $F(T)$ and $p = \lim_{n \to \infty} P((1 - \lambda)T + \lambda I)^n x$. 
Theorem 4.3. Let $H$ be a real Hilbert space, let $C$ be a non-empty bounded closed convex subset of $H$ and let $T$ be an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$ which satisfies the following condition (1) or (2):

(1) $\alpha+\beta+\gamma+\delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha+\gamma)\lambda+\epsilon+\eta < \alpha+\gamma+\epsilon+\eta$;
(2) $\alpha+\beta+\gamma+\delta \geq 0$, and there exists $\lambda \in \mathbb{R}$ such that $0 \leq (\alpha+\beta)\lambda+\zeta+\eta < \alpha+\beta+\zeta+\eta$.

Suppose that for any $x \in C$, there exist $m \in \mathbb{R}$ and $y \in C$ such that $0 \leq (1-\lambda)m \leq 1$ and $Tx = x + m(y-x)$. Then for any $x \in C$,

$$S_n x = \frac{1}{n} \sum_{k=0}^{n-1} ((1-\lambda)T + \lambda I)^k x$$

is weakly convergent to a fixed point $p$ of $T$, where $P$ is the metric projection from $H$ onto $F(T)$ and $p = \lim_{n \to \infty} P((1-\lambda)T + \lambda I)^n x$.

Example 4.1. Let $H = \mathbb{R}$, let $C = \left[0, \frac{\pi}{2} \right]$, let $Tx = (1 + 2x) \cos x - 2x^2$ and let $\alpha = 1$, $\beta = \gamma = 11$, $\delta = -22$, $\epsilon = \zeta = -12$ and $\eta = 1$. Then $T$ is an $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$-widely more generalized hybrid mapping from $C$ into $H$, $\alpha+\beta+\gamma+\delta = 1 \geq 0$ and $\alpha+\gamma+\epsilon+\eta = 1 > 0$. Let $\lambda = \frac{2+3\pi}{3(1+\pi)}$ and $m = 1 + \pi$. Then $0 \leq (1-\lambda)m = \frac{3}{3} < 1$ and $0 \leq (\alpha+\gamma)\lambda+\epsilon+\eta = \frac{\pi-3}{1+\pi} < 1 = \alpha+\gamma+\epsilon+\eta$. Let $y = x + \frac{(1+2x)(\cos x-x)}{1+\pi}$ for any $x \in C$. Then $Tx = x + m(y-x)$ and $y \in C$. Therefore by Theorem 4.3 for any $x \in C$,

$$S_n x = \frac{1}{n} \sum_{k=0}^{n-1} ((1-\lambda)T + \lambda I)^k x$$

is weakly convergent to a fixed point $p$ of $T$, where $P$ is the metric projection from $H$ onto $F(T)$ and $p = \lim_{n \to \infty} P((1-\lambda)T + \lambda I)^n x$.

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