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接触ラウンド手術と Lutz 捻りについて

(On contact round surgery and Lutz twist)

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0 Introduction

Classification and construction of contact structures and contact manifolds have been good and important problems in differential topology. In these few decades, 3-dimensional contact topology has developed drastically. Compared with this, contact topology in higher dimensions has not been studied very much. However, in recent few years, there are some remarkable movements on contact topology in higher dimensions. Now, it is expected that considering from the unified perspective would give both 3-dimensional and higher-dimensional contact topology good influences.

In this note, we discuss certain torsions of contact structures in general dimensions. We discuss some higher-dimensional generalizations of the Lutz twist and the Giroux torsion domain. All through this note, there exist observations from the view point of contact round surgery on the basement. This may be one of the unified perspectives to observe torsions of contact structures. Some of results in [Ad4] are introduced here.

Throughout this note, we see things through the perspective of contact round surgery. The contact round surgery is a notion introduced by the author in [Ad2]. The operation is defined for contact manifolds in any odd dimension. The round surgery as a modification of manifolds is introduced by Asimov [As] to study non-singular Morse-Smale flows. Not only for this purpose, round handle and round surgery have been used for several aspects. Especially, the round surgery seems to get along well with contact structures. There has been some attempts to apply the method to contact topology (see [Ad1], [Ad3]). As a useful tool for the study of contact structures in general dimensions, some applications of this method are expected.

1 Some torsions of contact structure

We discuss some torsions of contact structures in this note, through the perspective of contact round surgery. A *contact structure* is a completely non-integrable hyperplane field on an odd-dimensional manifold. In other words, it is always twisting. We discuss how they twist. In dimension 3, there are two important notions concerning torsion of contact structures, the overtwistedness and the Giroux torsion. A contact structure ξ on a 3-dimensional manifold M is said to be *overtwisted* if there exists an embedded disk $D \subset M$ which is tangent to ξ along the boundary ∂D, that is, $T_x D = \xi_x$ at any $x \in \partial D$. This implies that the plane field ξ turns upside down along the radius of the disk D. Such a disk is called an *overtwisted disk.* It is known that, if a contact structure is overtwisted, then the contact manifold can not be the boundary of a compact symplectic manifold, in a weak sense (see [Ge]). Similarly, a contact structure ξ is said to have the *Giroux torsion* at least $n \in \mathbb{N}$ if there exists a contact embedding $f_n\colon (T^2 \times [0,1], \zeta_n) \to (M, \xi)$, where $\zeta_n := \ker\{\cos(2n\pi r)d\theta + \sin(2n\pi r)d\phi\}$ with coordinates $(\phi, \theta, r) \in T^2 \times [0,1]$. Roughly, this implies that the plane field ξ twist $2n\pi$ between the two tori $T^2 \times \{0,1\}$. It is known that, if a contact structure has the Giroux torsion greater than 0, then the contact manifold can not be the boundary of a compact symplectic manifold in a strong sense (see [Ga]).

Some candidates of generalizations of the notions above has been proposed. Recently, a new notion of overtwisted disc in all dimensions was announced by Borman, Eliashberg, and Murphy [BoElMu]. It is stronger than other notions. First, our discussion is based on the following notions introduced by Massot, Niederkrüger, and Wendl. As a higher-dimensional generalization of an overtwisted disk, a notion of *bordered Legendrian open book* (bLob for short) is introduced (see [N], [MaNWn], [Gr]). Roughly speaking, it is an $(n + 1)$-dimensional open book embedded in a $(2n + 1)$-dimensional contact manifold (M, ξ) whose pages and boundary are Legendrian. It was proved by them that, if there exists a bordered Legendrian open book, then the contact manifold can not be the boundary of a compact symplectic manifold in a weak sense. In this sense, a contact manifold which admits a bordered Legendrian open book is said to be *PS-overtwisted.* In addition, as a higher-dimensional generalization of the Giroux π-torsion domain, a notion of *Giroux domain* is also introduced in [MaNWn]. Roughly, a Giroux domain is a contactization of a certain $2n$-dimensional symplectic manifold with contact-type boundary in a $(2n+1)$-dimensional contact manifold. It was proved in [MaNWn] that, if there exists a domain that consists of two Giroux domains glued together, then the contact manifold can not be the boundary of a compact symplectic manifold in a strong sense.

An overtwisted disc, introduced in [BoElMu], in a $(2n+1)$-dimensional contact manifold is a certain piecewise smooth $2n$-dimensional disc with a germ of contact structures along it. Roughly, it is a part of the boundary of the $(2n + 1)$-dimensional disk obtained as a product of the overtwisted disk in dimension 3 and a certain $(2n - 1)$-dimensional disk. It is proved in [BoElMu] that contact manifolds with overtwisted disc satisfy a parametric h-principle.

2 Lutz twist and preceding results

Some methods modifying contact structures so that they have the bordered Legendrian open books or the Giroux domains has been introduced. The original modification is the so-called Lutz twist in dimension 3 (see [L]). It is a modification of a contact structure on a 3-dimensional manifold along a knot transverse to the structure replacing the standard tubular neighborhood $\left(S^1 \times D^2(\sqrt{\varepsilon}), \zeta\right)$ with $\left(S^1 \times D^2(\sqrt{\varepsilon + n\pi}), \zeta\right)$, where $\zeta = \ker\left\{\cos r^2 d\theta + \sin r^2 d\phi\right\}$, $\varepsilon > 0$ sufficiently small, and $D^2(\sqrt{\varepsilon})$ is a disk with radius $\sqrt{\varepsilon}$. When $n = 1$ (resp. $n = 2$), it is called the *π-Lutz twist* (resp. *2π-Lutz twist*). The Lutz twist makes an S^1-family of overtwisted disks. The meridian disk $\{\phi = \text{const.}\}$ contains an overtwisted disk. There exists an important difference between the π- and 2π-Lutz twists. The π-Lutz twist contributes to the Euler class of a contact structure, while 2π-Lutz twist does not change the homotopy class of a contact structure as plane fields. In addition to that, a similar modification along a pre-Lagrangian torus is also called the Lutz twist. A pre-Lagrangian torus is an embedded torus in a contact 3-manifold whose characteristic foliation is linear with closed leaves. The *Lutz twist* along a pre-Lagrangian torus is a modification replacing $(T^2 \times [\delta - \varepsilon, \delta + \varepsilon], \tilde{\zeta})$ with $(T^2 \times [\delta - \varepsilon, \delta + \varepsilon + k\pi], \tilde{\zeta})$, where $\tilde{\zeta} = \ker\{\cos r d\phi + \sin r d\theta\}$ on $T^2 \times \mathbb{R}$, $k \in \mathbb{N}$, and $T^2 \times \{\delta\}$ is pre-Lagrangian. This operation makes the *Giroux π-torsion domain* $(T^2 \times [a, a + \pi], \tilde{\zeta})$.

There are some attempts to generalize the Lutz twists to higher dimensions.

One is due to Etnyre and Pancholi [EtPa]. They constructed a modification of a contact structure so that it has a family of bordered Legendrian open books. Their generalization takes an n-dimensional submanifold $B \times S^1$ of a $(2n + 1)$-dimensional contact manifold (M, ξ) as a generalization of a transverse knot, where B is a closed $(n - 1)$-dimensional isotropic submanifold with a trivial conformal symplectic normal bundle. Then it modifies the contact structure ξ along $B \times S^1$ so that it has an S^1-family of bordered Legendrian open books whose bindings are B. Moreover, the modified contact structure is homotopic to the original ξ as almost contact structures. In this sense, it is a generalization of the 2π-Lutz twist. The existence of a generalization of π-Lutz twist, that is, a modification that contributes to the Euler class of a contact structure is still open in [EtPa]. In order to avoid confusion, we call the generalization of the Lutz twist introduced in [EtPa] the *Etnyre-Pancholi twist* (the *EP-twist* for short) in this note.

Another generalization is due to Massot, Niederkrüger, and Wendl [MaNWn]. Their results are inspired by works of Mori [Mo] on 5-dimensional sphere. Their generalization takes a certain $(2n - 1)$-dimensional contact submanifold N of a $(2n + 1)$-dimensional contact manifold (M, ξ) as a generalization of a transverse knot. Then it modifies the contact structure ξ along N so that it has a family of bordered Legendrian open books with an n-dimensional parameter space in N. The modified contact structure is homotopic to the original ξ as almost contact structures. They also generalize the 3-dimensional Lutz twist along a pre-Lagrangian 2-torus. The generalization takes the so-called ξ-round hypersurface $H = K^{2n-1} \times S^1$, which is roughly an S^1-family of closed contact submanifold of codimension 2, as a generalization of a pre-Lagrangian 2-torus in dimension 3. Then it modifies ξ along H so that it has the bordered Legendrian open book or the Giroux domains, that is, a generalization of the Giroux torsion domain. The generalization of this type is carefully studied by Kasuya [Ka]. In order to avoid confusion, we call the generalization of the Lutz twist introduced in [MaNWn] the *Massot-Niederkrüger-Wendl twist* (the *MNW-twist* for short) in this note.

3 Results

In this note, some of results in [Ad4] are introduced. Another generalization of the Lutz twist is proposed. We deal with generalization of the both 3-dimensional Lutz twists along a transverse knot and along a pre-Lagrangian torus. The basic ideas are the descriptions of the Lutz twists by contact round surgeries in dimension 3 (see [Ad3], [Ad1]).

First, we discuss a generalization of the Lutz twist along a transverse knot. The description of the 3-dimensional Lutz twist along a transverse knot by contact round surgeries begins with the contact round surgery of index 1. In other words, the tubular neighborhood of a transverse knot where the Lutz twist is operated is regarded as one connected component of the attaching region $\partial_- R_1^4 \cong \partial D^1 \times D^2 \times S^1$ of the 4-dimensional symplectic round handle $R_1^4 = D^1 \times D^2 \times S^1$ of index 1. We generalize this method to higher dimensions. The first operation that we operate for a $(2n+1)$-dimensional contact manifold is also the contact round surgery of index 1. This implies that the modified region is one of the connected components of the attaching region $\partial_- R_1^{2n+2} \cong \partial D^1 \times D^{2n} \times S^1$ of the $(2n+2)$-dimensional symplectic round handle $R_1^{2n+2} \cong D^1 \times D^{2n} \times S^1$ of index 1. It is regarded as a tubular neighborhood of a certain circle. The generalization of the Lutz twist proposed in this note is operated along a circle embedded into a contact manifold which is transverse to the contact structure. The first result is the following.

Theorem A. *Let* (M, ξ) *be a contact manifold of dimension* $(2n + 1)$*, and* $\Gamma \subset (M, \xi)$ *an embedded transverse circle. Then we can modify* ξ *in a small tubular neighborhood of* Γ *so that the modified contact structure* $\tilde{\xi}$ *admits an* S^1*-family of the bordered Legendrian open books each of which has* $(n - 1)$*-dimensional torus* T^{n-1} *as binding. In other words, the contact manifold* $(M, \tilde{\xi})$ *does not have any weak symplectic filling if* M *is closed.*

Further, this modification has two versions. One can be done so that $\tilde{\xi}$ *is homotopic to the original contact structure* ξ *as almost contact structures. The other can be done so that it contributes to the Euler class of the contact structure.*

In this note, we call these generalizations of the Lutz twist obtained in Theorem A simply the *generalized Lutz twist* along a transverse circle.

As we reviewed above, other higher-dimensional generalizations of the Lutz twist, the EP-twist and the MNW-twist, also preserve the homotopy class of a contact structure as almost contact structures. On the other hand, the existence of a generalization of π-Lutz twist was an open question (see [EtPa]). Theorem A gives a generalization that contributes to the Euler class of a contact structure. Then it is an answer to one of the questions in [EtPa].

From the viewpoint of the new notion of overtwisted disc introduced by Borman, Eliashberg, and Murphy [BoElMu], it is proved that the modification of a contact structure in Theorem A creates not only an S^1-family of bordered Legendrian open books but also an S^1-family of overtwisted discs in any dimensions. Then, instead of Theorem A, we can claim the following.

Theorem A-1. *Let* (M, ξ) *be a contact manifold of dimension* $(2n + 1)$*, and* $\Gamma \subset (M, \xi)$ *an embedded transverse circle. Then we can modify* ξ *in a small tubular neighborhood of* Γ *so that the modified contact structure* $\tilde{\xi}$ *admits an* S^1*-family of overtwisted discs.*

The key ideas for Theorem A are the following. One of the important object is a generalization of the Lutz tube. Recall that a contact round surgery description of the 3-dimensional Lutz twist needs the open Lutz tube:

$$(S^1 \times \mathbb{R}^2, \zeta_0), \qquad \zeta_0 = \ker\left\{\cos r^2 d\phi + \sin r^2 d\theta\right\},$$

where (ϕ, r, θ) is the cylindrical coordinates of $S^1 \times \mathbb{R}^2$ (see [Ad3]). As a generalization, we take the confoliation ζ on $S^1 \times \mathbb{R}^{2n}$ defined as

$$\zeta = \ker\left\{\prod_{i=1}^{n}(\cos r_i^2)d\phi + \sum_{i=1}^{n}(\sin r_i^2)d\theta_i\right\},$$

where (ϕ, r_i, θ_i) are coordinates of $S^1 \times \mathbb{R}^{2n}$. Like there exists an overtwisted disk in $(S^1 \times \mathbb{R}^2, \zeta_0)$, a bordered Legendrian open book exists in $\left(S^1 \times \mathbb{R}^{2n}, \zeta\right)$. The important observation is that there is no difference, outer or inner, between two boundary components of the toric annulus $S^1 \times \left\{D^2(\sqrt{2\pi}) \setminus \operatorname{int} D^2(\sqrt{\pi})\right\} = T^2 \times \left[\sqrt{\pi}, \sqrt{2\pi}\right] \subset (S^1 \times \mathbb{R}^2, \zeta_0)$. However, in higher-dimensions, it is not true. As we observe in [Ad3], from the view point of round surgery, the Lutz twist is not the simple replacement of $S^1 \times D^2(\sqrt{\varepsilon})$ with $S^1 \times D^2(\sqrt{\varepsilon + \pi}) \subset (S^1 \times \mathbb{R}^2, \zeta_0)$. It is the replacement of $S^1 \times D^2(\sqrt{\varepsilon})$ with the toric annulus $T^2 \times \left[\sqrt{\pi}, \sqrt{2\pi}\right]$ and the blowing down along the "outer" end $T^2 \times \{\sqrt{2\pi}\}$. In order to generalize this operations to higher-dimensions, we regard the toric annulus $T^2 \times \left[\sqrt{\pi}, \sqrt{2\pi}\right]$ as two toric annuli $T^2 \times \left[\sqrt{\pi}, \sqrt{3\pi/2}\right]$ glued along $T^2 \times \{\sqrt{3\pi/2}\}$. We generalize these observations to higher-dimensions. We introduce the double $\mathrm{DU}(\sqrt{\pi}) := U(\sqrt{\pi}) \cup U(\sqrt{\pi})$ of

$$U(\sqrt{\pi}) = \{0 \le r_i \le \sqrt{\pi}, i = 1, \dots, n\} \subset \left(S^1 \times \mathbb{R}^{2n}, \zeta\right)$$

as a fundamental unit. From the confoliation ζ' on $\mathrm{DU}(\sqrt{\pi})$ obtained from ζ, a contact structure $\tilde{\zeta}$ on $\mathrm{DU}(\sqrt{\pi})$ is obtained. Removing the standard tubular neighborhood of the transverse core $S^1 \times \{0\} \subset U(\sqrt{\pi}) \subset \left(S^1 \times \mathbb{R}^{2n}, \zeta\right)$ from $\mathrm{DU}(\sqrt{\pi})$, we obtain a contact manifold diffeomorphic to $S^1 \times D^{2n}$. We call it the *model π-Lutz tube*, and let $\left(\mathrm{LU}(\pi), \tilde{\zeta}\right)$ denote it (see Figure 1).

As a corollary of Theorem A, we obtain the following result. Recall that the generalized Lutz twist is operated along an embedded circle transverse to the contact structure. We claim that any circle γ embedded in a contact manifold (M, ξ) can be approximated by a circle $\bar{\gamma}$ transverse to ξ. Therefore, we can apply the generalized Lutz twist anywhere we like. This implies the following.

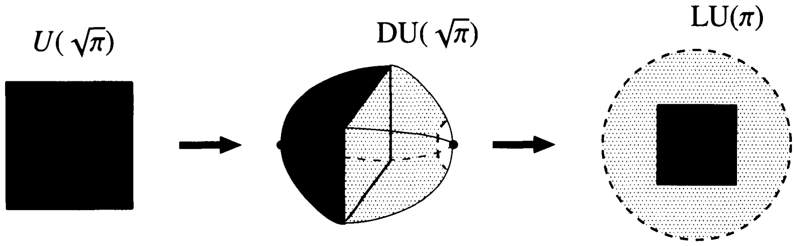

Figure 1: Generalized Lutz tube (mod S^1).

Corollary B. *If an odd-dimensional manifold M has a contact structure, it admits a PS-overtwisted contact structure.*

This result was firstly proved by Niederkrüger and van Koert [NvK], and then by Presas [Pr]. It can also be proved by using the EP-twist [EtPa]. However, as the MNW-twist [MaNWn] requires stricter conditions, it can not be applied everywhere.

Concerning the overtwistedness introduced in [BoElMu], we have the following result as a corollary of Theorem A-1. The argument is the same as above.

Corollary B-1. *If an odd-dimensional manifold M has a contact structure, it admits an overtwisted contact structure.*

Furthermore, we obtain the following result as a corollary of Theorem A. It was claimed by Etnyre and Pancholi [EtPa], and Niederkrüger and Presas [NPr] that there exist at least three distinct contact structures on $\mathbb{R}^{2n+1}$, $n \geq 1$. One is the standard contact structure $\ker\{d\phi + \sum_{i=1}^{n} r_i^2 d\theta_i\}$. Another is PS-overtwisted but standard at infinity, that is, it is standard outside a compact subset. The other is PS-overtwisted at infinity, that is, for any relatively compact open subset, there exists a bordered Legendrian open book outside the subset. By the same argument as Theorem A, a contact structure which is PS-overtwisted at infinity can be constructed. Then this implies the following.

Corollary C. *There are at least three distinct contact structures on $\mathbb{R}^{2n+1}$, $n \geq 1$.*

A contact structures PS-overtwisted at infinity is constructed as follows. The generalized Lutz twist in Theorem A is defined along an embedded transverse circle. In a similar way, we can define the generalized Lutz twist along the ϕ-axis in $\mathbb{R}^{2n+1}$ with the standard contact structure $\ker\{d\phi + \sum_{i=1}^{n} r_i^2 d\theta_i\}$. Then we naturally obtain a contact structure on $\mathbb{R}^{2n+1}$ with an $\mathbb{R}$-family of bordered Legendrian open books. It is PS-overtwisted at infinity.

Next, we discuss higher-dimensional generalizations of the 3-dimensional Lutz twist along a pre-Lagrangian 2-torus. It is defined as a modification of a contact structure along the so-called ξ-round hyper surface $H = K^{2n-1} \times S^1$ like the EP-twist. A ξ-round hypersurface is introduced in [MaNWn] as a generalization of a pre-Lagrangian 2-dimensional torus in a contact 3-manifold. It is regarded as higher-dimensional generalizations of pre-Lagrangian 2-torus in a contact 3-manifold.

The result for a generalization of the Lutz twist along a ξ-round hypersurface modeled on the standard contact sphere is as follows. Let $\eta_0 = \ker\left\{\sum_{i=1}^{n} r_i^2 d\theta_i\right\}|_{TS^{2n-1}}$ be the standard contact structure on a sphere $S^{2n-1} \subset \mathbb{R}^{2n}$, where $(r_1, \theta_1, \dots, r_n, \theta_n)$ are the coordinates of $\mathbb{R}^{2n}$.

Theorem D. *Let (M, ξ) be a contact manifold of dimension $(2n+1)$, $n > 1$, and $H = S^{2n-1} \times S^1 \subset (M, \xi)$ an embedded ξ-round hypersurface modeled on the standard contact sphere (S^{2n-1}, η_0). Then we can modify ξ in a small tubular neighborhood of H so that the modified contact structure $\tilde{\xi}$ admits an S^1-family of bordered Legendrian open books whose bindings are T^{n-1}. In other words, the contact manifold $(M, \tilde{\xi})$ does not have any weak symplectic filling if M is closed.*

If $n = 1$, this modification is the 3-dimensional Lutz twist along a pre-Lagrangian torus $H = S^1 \times S^1$. It makes not a bordered Legendrian open book, that is, an overtwisted disk, but the Giroux π-torsion domain.

We call, in this note, this generalization simply the *generalized Lutz twist along a ξ-round hypersurface modeled on the standard contact sphere*. The difference between the two cases, $n > 1$ and $n = 1$, comes from the shape of the (generalized) Lutz tube.

Remark. In dimension 3, the Lutz twist along a pre-Lagrangian torus, or the Giroux torsion, makes a difference between the weak, strong fillability and the tightness (see [Ga]). However, this generalization of the Lutz twist does not make such a difference. From this point of view, another modification of contact structure is introduced in [Ad4].

The generalized Lutz twist along an embedded transverse circle and that along a ξ-round hypersurface modeled on the standard contact sphere are described by contact round surgeries. Although the descriptions are not explicitly used in the definitions or statements of the theorems, these are the fundamental ideas of this note.

Theorem E. *We deal with a* $(2n+1)$*-dimensional contact manifold* (M, ξ).

(*1*) *The generalized Lutz twist along a transverse curve* $\Gamma \subset (M, \xi)$ *is realized by a certain ordered pair of contact round surgeries of index* 1 *and* $2n$ *with the model Lutz tube.*

(2) *The generalized Lutz twist along a* ξ*-round hypersurface* $H = S^{2n-1} \times S^1$ *modeled on the standard contact sphere is realized by contact round surgeries of index* $2n$ *and* 1 *with the model Lutz tube.*

As a matter of fact, the observations in Theorem E is a motivation of this work. In fact, it is proved in [Ad3] that the 3-dimensional Lutz twists are realized by contact round surgeries. Then it was expected that a similar operations by contact round surgeries corresponds to a higher-dimensional generalization of the Lutz twists.

From the view point of round surgery, it is interesting to regard the double $\left(\mathrm{DU}(\sqrt{\pi}), \tilde{\zeta}\right)$ a certain unit. The generalized Lutz twists dealt in Theorem E are considered as operations taking in this unit by contact round surgeries (see Figures 2).

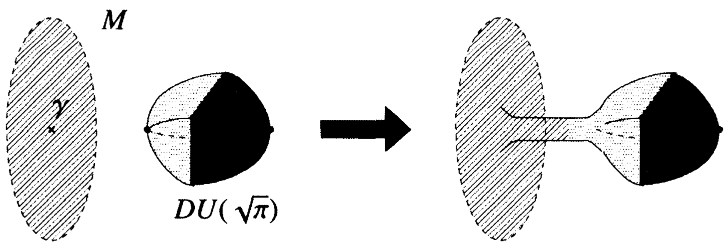

Figure 2: Generalized Lutz twist along a circle (mod $\times S^1$).

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