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On a Multi-valued Two-person Game.

Kenji Kimura
Learning Support Center,
Niigata Insutitute of Technology,

Masakazu Higuchi
Graduate School of Science and Engineering,
Saitama University,

Tamaki Tanaka
Department of Mathematical Science Graduate School of Science and Technology,
Niigata University

Abstract

In this paper, we introduce a multi-criteria and multi-valued game, which is formulated by utilizing two kinds of concepts of direct set-comparison between two sets. For the multi-criteria and multi-valued game, we define an optimal response strategy based on the two set comparisons. Also we define minimax and maximin value and strategies for the multi-criteria and multi-valued game.

1 Introduction

Multi-criteria two-person game is considered when we want to treat several criteria simultaneously. If we should deal with more than one vector-valued outcome at the same time as payoffs in a two-person game, we possibly convert the problem into an ordinary multicriteria game by merging the outcomes into one vector.

In contrast, we can treat its original problem as a game whose payoff outputs multiple vector-values in its objective space simultaneously by utilizing some methodology on comparison of each set, that is, we can formulate it as a game with a set-valued payoff function. Especially, when each players hope to improve preferentially the worst or best vector-values of several outcomes for a strategy, multi-valued game is more suitable than the multi-criteria game converted by merging outcomes. For instance, let $(1,1)$ and $(0,0)$ be expected payoff vector-values for a strategies pair of players 1 and 2. If the two values are merged into $(1,1,0,0)$, then $(1,1)$ and $(0,0)$ are handled evenly by both players. However if we consider $(1,1)$ and $(0,0)$ as $\{(1,1),(0,0)\}$, then a player and the other player focus $(1,1)$ and $(0,0)$ and improve it preferentially, respectively.

Moreover, when we have incomplete and/or uncertainty information on the payoff function of a game, some of such games can be managed well by considering it as a set-valued game, in which we simultaneously handle all prospective payoff values for each strategy. For another instance on validity of set-valued game, we consider a two-group game with group strategies and group payoffs, which can be divided into payoffs of each player. If every player of the both groups always join the game for any strategy, it can be formulated as an ordinary multi-objective game. However, depending on each selected strategy by their group, some players may fix their decision whether they participate in the game or not. That is, each payoff value of the function of the game may achieve a different cardinal number depending upon each strategy. In this case, it seems hard to formulate the game as a game with vector-valued payoffs, which will be referred to as a vector-valued game in this paper. By contrast, considering the game as a set-valued game, we can formulate it without any such difficulty.

For the foregoing reasons, in this paper, we consider a multicriteria two-person set-valued game. We construct a fixed form of two-person set-valued game by utilizing two direct set-comparisons, which are introduced as (iii)-type and (v)-type set-relations in [9]. Also we define an optimal response strategy for each player and minimax and maximin strategies based on the two set-comparisons. After that, we reformulate two-person non-zero-sum multicriteria matrix game as two-person multicriteria set-valued matrix game, and consider an optimal response value for each player and minimax and maximin values and strategies of the set-valued game.

2 Preliminaries

By the same approach in [10], we introduce two direct set comparisons as follows. Let U and V be two subsets of a topological vector space Z with an ordering cone C. In this paper, we use the following two types of set-relations between two sets:

$$\text{(i) } U \leq^{(l)} V \overset{\text{def}}{\rightleftharpoons} V \subset U + C;$$
$$\text{(ii) } U \leq^{(u)} V \overset{\text{def}}{\rightleftharpoons} U \subset V - C.$$

Let $\mathcal{U}$ be a family of non-empty sets of Z. We say that $\bar{U} \in \mathcal{U}$ is a lower efficient set of $\mathcal{U}$ w.r.t. (l) [resp. w.r.t. (u)], if for any $U \in \mathcal{U}$,

$$U \leq^{(l)} \bar{U} \Rightarrow \bar{U} \leq^{(l)} U, \qquad [\textit{resp. } U \leq^{(u)} \bar{U} \Rightarrow \bar{U} \leq^{(u)} U].$$

We also say that $\bar{U} \in \mathcal{U}$ is an upper efficient set of $\mathcal{U}$, if for any $U \in \mathcal{U}$,

$$\bar{U} \leq^{(u)} U \Rightarrow U \leq^{(u)} \bar{U}, \qquad [\textit{resp. } \bar{U} \leq^{(l)} U \Rightarrow U \leq^{(l)} \bar{U}].$$

Let F be a non-empty set-valued function from $X \times Y$ to Z, where X and Y are strategy sets. Using the above methods of set optimization, we formulate two types of two-person multicriteria game with set-valued payoff function as the following tables.

(1)

Player	Strategies	Payoff function	Preferences
1	X	$F(x,y)$	lower efficient w.r.t. (l),
2	Y	$-F(x,y)$	upper efficient w.r.t. (u)

Table 1: Two-person multicriteria and multivalued game type (I).

(2)

Player	Strategies	Payoff function	Preferences
1	X	$F(x,y)$	lower efficient w.r.t. (u),
2	Y	$-F(x,y)$	upper efficient w.r.t. (l)

Table 2: Two-person multicriteria and multivalued game type (II).

Two-person multicriteria and multivalued game type (I) is suitable for the case where each players hope to improve preferentially the best values of each payoff set. On contrast, two-person multicriteria and multivalued game type (II) is suitable for the case where each players hope to improve preferentially the worst values of each payoff set.
We have considered concepts of optimal response strategy, minimax value, maximin value, minimax strategies, and maximin strategies for two-person multicriteria and multivalued game type (I), in [20, 21]. So, in this paper, we consider these concepts for for two-person multicriteria and multivalued game type (II).

Definition 1. Each set of optimal response strategies of player 1 for y and player 2 for x is defined by the following R_1 and R_2, respectively: For each fixed $y \in Y$,

$$R_1(y) := \{x \in X : F(x,y) \text{ is a lower efficient set of } \{F(u,y) : u \in X\} \text{ w.r.t. } (u)\}$$

and for each fixed $x \in X$,

$$R_2(x) := \{y \in Y : F(x,y) \text{ is an upper efficient set of } \{F(x,v) : v \in Y\} \text{ w.r.t. } (l)\}.$$

Definition 2. Each set of optimal responses of player 1 at y and player 2 at x is defined by the following statements, respectively: For each fixed $y \in Y$,

$$\text{MIN}\, F(X,y) := \{F(x,y) \in 2^Z : \ x \in R_1(y)\}$$

and for each fixed $x \in X$,

$$\mathrm{MAX}\, F(x, Y) := \{F(x, y) \in 2^Z : y \in R_2(x)\}$$

.

Remark 1. Remark that $F(x,y) \in \mathrm{MIN}\, F(X,y)$ does not mean $x \in R_1(y)$. Also $F(x,y) \in \mathrm{MAX}\, F(x,Y)$ does not mean $y \in R_2(y)$.

Definition 3. Minimax value of F and maximin value of F are defined in the following way:

$$\mathrm{MAX}\,\mathrm{MIN}\, F := \{F(x,y) : F(x,y) \text{ is an upper efficient set of } \bigcup_{y \in Y} \mathrm{MIN}\, F(X,y) \text{ w.r.t. } (l)\},$$

$$\mathrm{MIN}\,\mathrm{MAX}\, F := \{F(x,y) : F(x,y) \text{ is a lower efficient set of } \bigcup_{x \in X} \mathrm{MAX}\, F(x,Y) \text{ w.r.t. } (u)\}.$$

Remark 2. Remark that $F(x,y) \in \mathrm{MAX}\,\mathrm{MIN}\, F$ does not mean $F(x,y) \in \mathrm{MIN}\, F(X,y)$ nor $x \in R_1(y)$. Similarly, $F(x,y) \in \mathrm{MIN}\,\mathrm{MAX}\, F$ does not mean $F(x,y) \in \mathrm{MAX}\, F(x,Y)$ nor $y \in R_2(x)$.

Example 1. Let X and Y be $\{\lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 0 \\ 1 \end{pmatrix} : \lambda \in [0,1]\}$. Let F be a non-empty set-valued function from $X \times Y$ to $\mathbb{R}^2$ defined by

$$F(x,y) = \left\{ \begin{pmatrix} x^t \begin{pmatrix} -4 & -1 \\ -3 & -2 \end{pmatrix} y \\ x^t \begin{pmatrix} -2 & -1 \\ -7 & -5 \end{pmatrix} y \end{pmatrix}, \begin{pmatrix} x^t \begin{pmatrix} 3 & 6 \\ 1 & 5 \end{pmatrix} y \\ x^t \begin{pmatrix} 7 & 6 \\ 1 & 2 \end{pmatrix} y \end{pmatrix} \right\}.$$

Then, $R_1(y) = \{(0,1)^t\}$, $R_2(x) = \{(1,0)\}$,

$$\mathrm{MIN}\, F(X,y) = \left\{ \lambda \begin{pmatrix} -3 \\ -7 \end{pmatrix} + (1-\lambda) \begin{pmatrix} -2 \\ -5 \end{pmatrix}, \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 5 \\ 2 \end{pmatrix} : \lambda \in [0,1] \text{ s.t. } y = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\},$$

and

$$\mathrm{MAX}\, F(x,Y) = \left\{ \lambda \begin{pmatrix} -1 \\ -1 \end{pmatrix} + (1-\lambda) \begin{pmatrix} -2 \\ -5 \end{pmatrix}, \lambda \begin{pmatrix} 6 \\ 6 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 5 \\ 2 \end{pmatrix} : \lambda \in [0,1] \text{ s.t. } y = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}.$$

Moreover,

$$\mathrm{MAX}\,\mathrm{MIN}\, F = \mathrm{MIN}\,\mathrm{MAX}\, F = \left\{ \begin{pmatrix} -2 \\ -5 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right\},$$

$$\hat{x} \in R_1(\hat{y}) \text{ and } \hat{y} \in R_2(\hat{x}),$$

where $\hat{x} = (0,1)^t$ and $\hat{y} = (0,1)^t$.

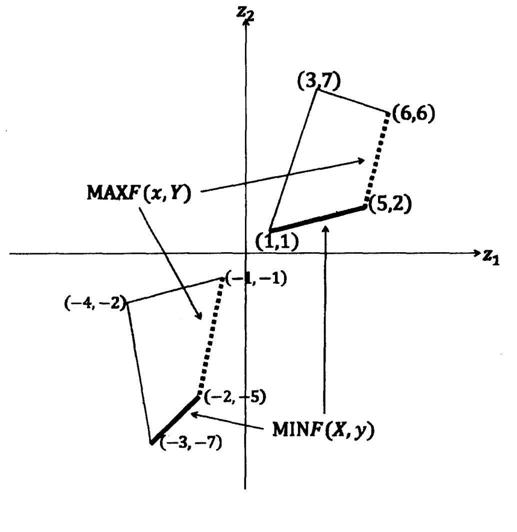

Figure 1: Example 1

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Kenji Kimura
Learning Support Center
Niigata Institute of Technology
Kashiwazaki 945-1195
Japan
E-mail:kkimura@adm.niit.ac.jp

Masakazu Higuchi
Graduate School of Science and Engineering
Saitama University

Saitama-shi 338-8570
Japan
E-mail:kkimura@adm.niit.ac.jp

Tamaki Tanaka
Department of Mathematical Science
Graduate School of Science and Technology
Niigata University
Niigata 950-2181
Japan
E-mail:tamaki@math.sc.niigata-u.ac.jp