数理解析研究所講究録
第 1983 巻 2016 年 27-40



# Pollution Thresholds under Uncertainty in Asymmetric Duopoly

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## 1 Introduction

In this paper, we study emission controls and market equilibrium. The global warming is an urgent issue to be addressed throughout the entire world. Industrialized nations have emitted far more greenhouse gas emissions, consequently, the significance of emission controls are discussed broadly in recent years. The argument of emission controls is developed at not only the political level, but also the academic level. In fact, there are many existing literature studying emission controls.

Existing literature consider pollution taxes, tradable emission permits and imperfect competition, however, they do not consider any uncertainty of pollution. von der Fehr (1993) studies tradable emission rights and strategic interaction. He shows strategic manipulation may have negative welfare effects in the market. Simpson (1995) studies optimal pollution tax in Cournot duopoly. He shows the optimal tax rate may exceed the marginal damage. Sartzetakis (1997) studies tradable emission permits regulations under imperfect competition. He shows social welfare is higher under the TEP relative to the CC regulation. Tanaka and Chen (2012) study tradable emission permits in electricity markets. They show diverting emission permits reduces the power and the permit prices. Mansur (2013) studies tradable permits are preferable to taxes under imperfect competition. He shows strategic behavior reduced local emissions by approximately 9%.

We consider uncertainty of pollution, firm's entry-exit decision and pollution tax in Cournot duopoly by combining Wirl (2006) and Simpson (1995). Outputs of the model are dynamics of pollution and equilibrium market supply. It is shown that environmental regulation decreases duration and amount of pollution, pollution uncertainty increases duration and amount of pollution, and market size increases increment of pollution.

The rest of the paper is organized as follows. Section 2 describes our entry-exit pollution model in monopoly and provides firm values of polluting/stopping/exit with options. In Section 3, we introduce Cournot competition and extend the entry-exit pollution model to the duopolistic framework. Section 4 shows some numerical results, and Section 5 concludes this paper.

## 2 Monopoly

First, we consider the firm (labeled 1) produces $q_1$ units using technology at cost per unit $c_1$ in monopoly market. Market demand is given by

$$P(q_1) = \theta - \eta q_1, \tag{1}$$

so the optimal output of firm 1 is

$$\frac{\partial}{\partial q_1}(Pq_1 - c_1 q_1) = 0 \quad \Rightarrow \quad q_1^* = \frac{\theta - c_1}{2\eta}, \tag{2}$$

and equilibrium market price is

$$P^* = P(q_1^*) = \frac{\theta + c_1}{2}. \tag{3}$$

Then, the firm's profit is given by

$$\pi_1^* = P^* q_1^* - c_1 q_1^* = \frac{(\theta - c_1)^2}{4\eta}. \tag{4}$$

We assume the firm has tax implications for pollution and the pollution tax is the form of

$$\tau(q, x) = \lambda x q, \tag{5}$$

where $\lambda$ is the constant tax rate and $x$ is the pollution rate which has the dynamics of

$$\mathrm{d}X_t = \alpha X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \qquad X_0 = x. \tag{6}$$

Then, firm's decision is as follows:

1. Pollute
   The firm earns the profit $\pi_1^*$ and pollutes $q_1^* X_t$.
2. Suspend pollution
   The firm maintains the technology at the cost $m$ and $q_1 = 0$.
3. Restart pollution
   Restart requires the fixed cost $K$.
4. Shut down
   The firm incurs no cost.

Next, we derive value functions of firm 1 in monopoly. Suppose firm 1 produces $q_1^*$ and pollutes forever, we have

$$V_0(x) = \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(\pi_1^* - \tau(q_1^*, X_t))\mathrm{d}t\right] = \frac{(\theta - c_1)^2}{4\eta r} - \frac{\theta - c_1}{2\eta}\frac{\lambda x}{r - \alpha}. \tag{7}$$

Value of polluting with the option to suspend is given by

$$V_1(x) = V_0(x) + A_1 x^{\beta_1}, \tag{8}$$

where $A_1$ is an unknown coefficient and

$$\beta_1 = \frac{-(\alpha - \sigma^2/2) + \sqrt{(\alpha - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} > 1. \tag{9}$$

Suppose the firm suspends pollution ($q_1 = 0$) and maintain ($-m$) the technology forever, we have

$$\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(-m)\mathrm{d}t\right] = -\frac{m}{r}. \tag{10}$$

Value of stopping with the options to restart and exit is given by

$$V_2(x) = -\frac{m}{r} + B_1 x^{\beta_1} + B_2 x^{\beta_2}, \tag{11}$$

where $B_1$ and $B_2$ are unknown coefficients and

$$\beta_2 = \frac{-(\alpha - \sigma^2/2) - \sqrt{(\alpha - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2} < 0. \tag{12}$$

Suppose the firm shuts down the technology, we have

$$V_3(x) = 0. \tag{13}$$

Finally, we have value-matching and smooth-pasting conditions at three types of pollution thresholds. At restart threshold $\underline{X}$, we have

$$V_1(\underline{X}) - K = V_2(\underline{X}), \tag{14}$$
$$V_1'(\underline{X}) = V_2'(\underline{X}). \tag{15}$$

At suspend threshold $\bar{X}$, we have

$$V_1(\bar{X}) = V_2(\bar{X}), \tag{16}$$
$$V_1'(\bar{X}) = V_2'(\bar{X}). \tag{17}$$

At exit threshold $\hat{X}$, we have

$$V_2(\hat{X}) = 0, \tag{18}$$
$$V_2'(\hat{X}) = 0. \tag{19}$$

We find three unknown coefficients and three thresholds numerically using six boundary conditions (14)–(19).

Figure 1 depicts value functions of firm 1 in monopoly for the base case parameter set: $K = 1$, $m = 0.1$, $\lambda = 0.25$, $r = 0.1$, $\alpha = 0.05$, $\sigma = 0.3$, $\theta = 1.3$, $\eta = 0.05$ and $c_1 = 0.5$. Then, we have $\pi_1^* = 3.2$, $q_1^* = 8$, $\underline{X} = 1.206$, $\bar{X} = 2.120$ and $\hat{X} = 7.626$.

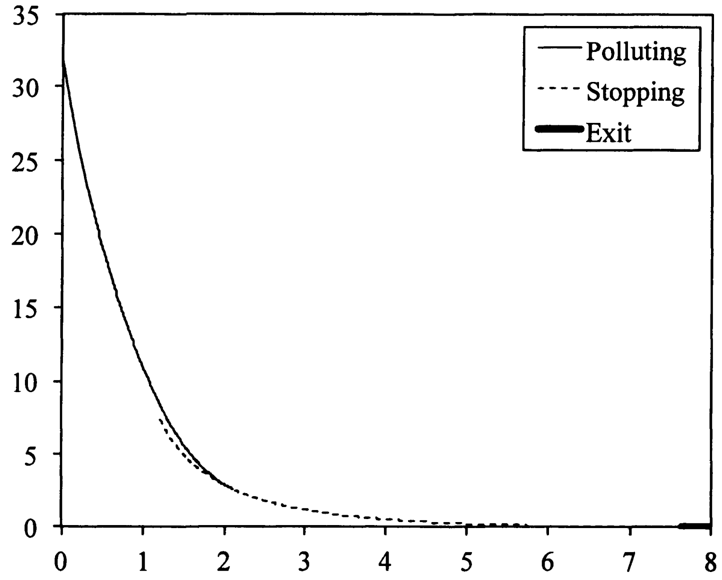


Figure 1: Value functions in monopoly.

## 3 Duopoly

In this section, we consider a duopoly market where two firms $i \neq j \in \{1, 2\}$ face Cournot competition. Market demand is given by

$$P(Q) = \theta - \eta Q, \qquad Q = q_1 + q_2, \tag{20}$$

so the optimal output is

$$\frac{\partial}{\partial q_i}(Pq_i - c_i q_i) = 0 \quad \Rightarrow \quad q_i^{**} = \frac{\theta + c_j - 2c_i}{3\eta}, \quad Q^{**} = \frac{2\theta - c_1 - c_2}{3\eta}, \tag{21}$$

and equilibrium market price is

$$P^{**} = P(Q^{**}) = \frac{\theta + c_1 + c_2}{3}. \tag{22}$$

Then, the firm's profit is given by

$$\pi_i^{**} = P^{**} q_i^{**} - c_i q_i^{**} = \frac{(\theta + c_j - 2c_i)^2}{9\eta}. \tag{23}$$

W.l.o.g., we assume $c_1 < c_2$ so that firm 2 suspends and exits earlier than firm 1. Then market supply is as follows:

1. Both firms produce and pollute:

$$q_i^{**} = \frac{\theta + c_j - 2c_i}{3\eta}, \qquad Q^{**} = \frac{2\theta - c_1 - c_2}{3\eta}, \tag{24}$$

Table 1: Possible market status for the base case.

| | | Firm 2 | | |
|---|---|---|---|---|
| | | Polluting | Stopping | Exit |
| Firm 1 | Polluting | 1** | 2* | 3* |
| | Stopping | — | — | 4 |
| | Exit | — | — | 5 |

$$1 \underset{\underline{X}_2}{\overset{\bar{X}_2}{\rightleftarrows}} 2 \overset{\hat{X}_2}{\longrightarrow} 3 \underset{\bar{X}_1}{\overset{\bar{X}_1}{\rightleftarrows}} 4 \overset{\hat{X}_1}{\longrightarrow} 5$$

Figure 2: Status transition for the base case.

2. Firm 1 produces and firm 2 suspends/shuts down:

$$Q^{**} = q_1^* = \frac{\theta - c_1}{2\eta}, \qquad q_2 = 0, \tag{25}$$

3. Both firms suspend/shut down:

$$q_i = 0. \tag{26}$$

Suppose $\underline{X}_2 < \underline{X}_1 < \bar{X}_2 < \hat{X}_2 < \bar{X}_1 < \hat{X}_1$, possible market status is shown in Table 1. Statuses 1 and 2 are in duopoly, while statuses 3–5 are in monopoly. Then status transition is shown in Figure 2.

Next, we derive value functions of firm 2 which is always in duopoly. The value of polluting is given by

$$V_{02}(x) = \mathbb{E}\left[\int_0^\infty e^{-rt}(\pi_2^{**} - \tau(q_2^{**}, X_t))\mathrm{d}t\right] \tag{27}$$

$$= \frac{(\theta + c_1 - 2c_2)^2}{9\eta r} - \frac{\theta + c_1 - 2c_2}{3\eta}\frac{\lambda x}{r - \alpha}, \tag{28}$$

$$V_{12}(x) = V_{02}(x) + A_{12}x^{\beta_1}, \tag{29}$$

the value of stopping is given by

$$V_{22}(x) = -\frac{m}{r} + B_{12}x^{\beta_1} + B_{22}x^{\beta_2}, \tag{30}$$

and value of exit is given by

$$V_{32}(x) = 0. \tag{31}$$

We have value-matching and smooth-pasting conditions at restart threshold $\underline{X}_2$:

$$V_{12}(\underline{X}_2) - K = V_{22}(\underline{X}_2), \tag{32}$$

$$V'_{12}(\underline{X}_2) = V'_{22}(\underline{X}_2), \tag{33}$$

suspend threshold $\bar{X}_2$:

$$V_{12}(\bar{X}_2) = V_{22}(\bar{X}_2), \tag{34}$$

$$V'_{12}(\bar{X}_2) = V'_{22}(\bar{X}_2), \tag{35}$$

and exit threshold $\hat{X}_2$:

$$V_{22}(\hat{X}_2) = 0, \tag{36}$$

$$V'_{22}(\hat{X}_2) = 0. \tag{37}$$

We find three unknown coefficients and three thresholds numerically using six boundary conditions.

Finally, we derive value functions of firm 1. Firm 1 is in duopoly for statuses 1 and 2, while in monopoly for statuses 3–5. For status 1 where both firms pollute, the value function of firm 1 is given by

$$V_{01}(x) = \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(\pi_1^{**} - \tau(q_1^{**}, X_t))\mathrm{d}t\right] \tag{38}$$

$$= \frac{(\theta + c_2 - 2c_1)^2}{9\eta r} - \frac{\theta + c_2 - 2c_1}{3\eta}\frac{\lambda x}{r - \alpha}, \tag{39}$$

$$V_{11}(x) = V_{01}(x) + A_{11}x^{\beta_1}. \tag{40}$$

For status 2 where firm1 pollutes and firm 2 suspends, we have

$$V_{21}(x) = V_0(x) + B_{11}x^{\beta_1} + B_{21}x^{\beta_2}. \tag{41}$$

Firm 1 in monopoly is already solved, that is, the value function for status 3 is $V_1(x)$, $V_2(x)$ status 4 and $V_3(x) = 0$ for status 5. From Figure 2, we have only value-matching conditions in duopoly at firm 2's restart threshold $\underline{X}_2$:

$$V_{11}(\underline{X}_2) = V_{21}(\underline{X}_2), \tag{42}$$

at firm 2's suspend threshold $\bar{X}_2$:

$$V_{11}(\bar{X}_2) = V_{21}(\bar{X}_2), \tag{43}$$

and at firm 2's exit threshold:

$$V_{21}(\hat{X}_2) = V_1(\hat{X}_2). \tag{44}$$

We find three unknown coefficients numerically using three boundary conditions.

## 4 Numerical Analyses

In this section, we investigate dynamics of pollution and equilibrium market supply. We use base case parameters: $K = 1$, $m = 0.1$, $\lambda = 0.25$, $r = 0.1$, $\alpha = 0.05$, $\sigma = 0.3$, $\theta = 1.3$,

Table 2: Pollution thresholds for the base case.

| | $\pi$ | $q$ | $\underline{X}$ | $\bar{X}$ | $\hat{X}$ |
|---|---|---|---|---|---|
| monopoly | 3.2 | 8 | 1.206 | 2.120 | 7.626 |
| firm 1 | 1.8 | 6 | | | |
| firm 2 | 0.8 | 4 | 0.515 | 1.287 | 1.433 |

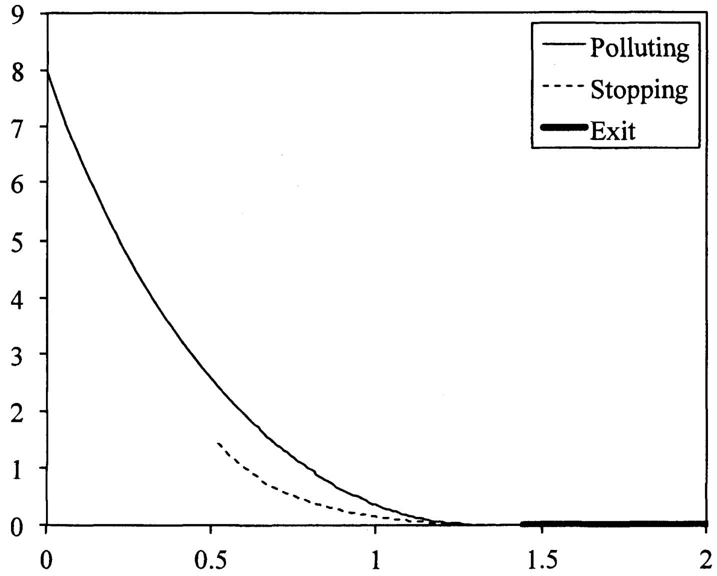


Figure 3: Value functions of firm 2 in duopoly.

$\eta = 0.05$, $c_1 = 0.5$, $c_2 = 0.6$. Additionally, we compute comparative statistics with respect to environmental regulation $\lambda$, market size $\theta$ and pollution uncertainty $\sigma$.

Table 2 shows pollution thresholds for the base case. Figures 3 and 4 show value functions of firm 2 and 1 in duopoly. Figure 5 shows dynamics of pollution and equilibrium market supply when pollution increases (+) and decreases (−). Figure 6 shows pollution dynamics when environmental regulation is strict ($\lambda = 0.35$). In this case, market supply does not change, however, duration and amount of pollution decreases.

Next, we investigate the impact of market size $\theta$. When $\theta = 1.5$, pollution thresholds are in the order of $\underline{X}_2 < \underline{X}_1 < \bar{X}_2 < \bar{X}_1 < \hat{X}_2 < \hat{X}_1$. In this case, possible market status and status transition are shown in Table 3 and Figure 7, respectively. Statuses 1, 2 and 6 are in duopoly, while statuses 3–5 are in monopoly. Note that status 6 affects only firm 1's value function. For status 6 where both firms suspend, the value function of firm 1 is given by

$$V_{61}(x) = -\frac{m}{r} + C_{11}x^{\beta_1} + C_{21}x^{\beta_2}. \tag{45}$$

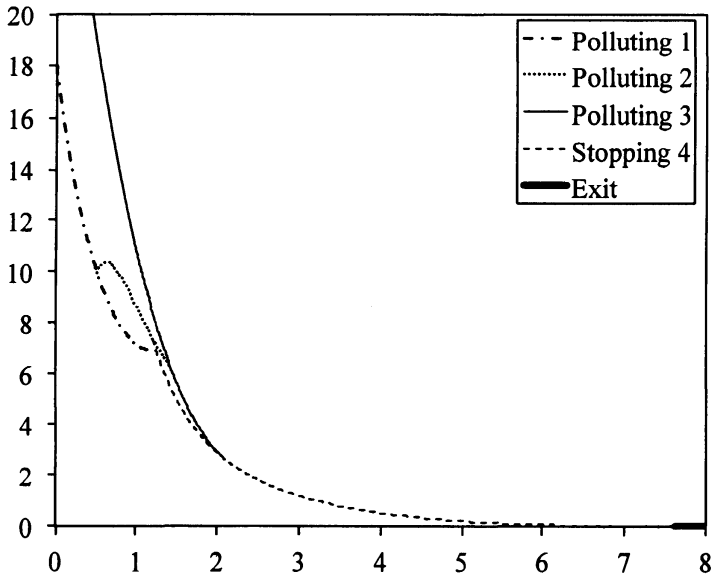


Figure 4: Value functions of firm 1 in duopoly.

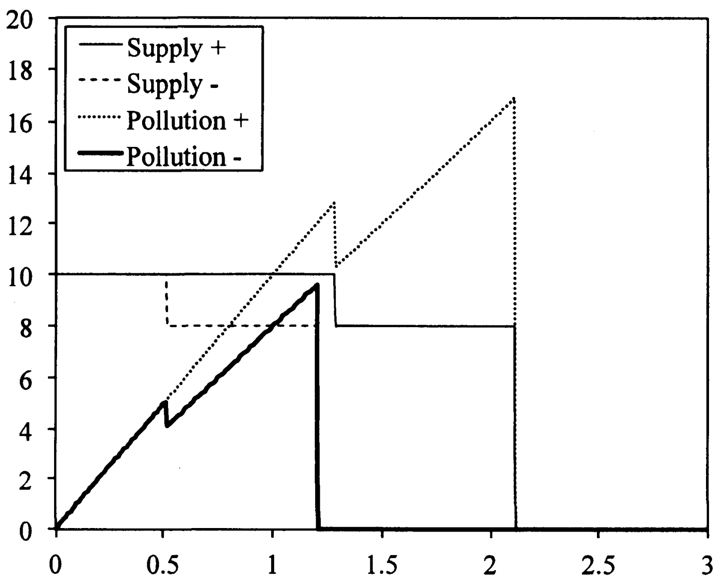


Figure 5: Dynamics of pollution for the base case.

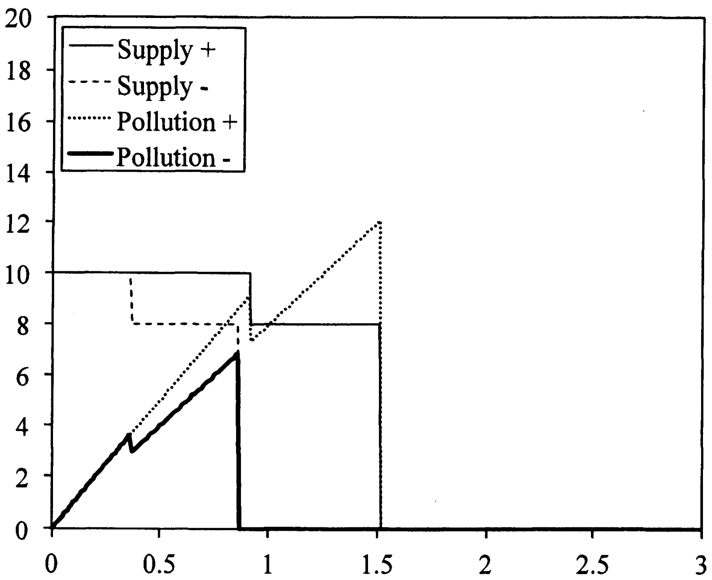


Figure 6: Dynamics of pollution when environmental regulation is strict ($\lambda = 0.35$).

Table 3: Possible market status when market size is large ($\theta = 1.5$).

| | | Firm 2 | | |
|---|---|---|---|---|
| | | Polluting | Stopping | Exit |
| Firm 1 | Polluting | 1** | 2* | 3* |
| | Stopping | — | 6 | 4 |
| | Exit | — | — | 5 |

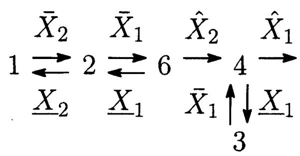


Figure 7: Status transition when market size is large ($\theta = 1.5$).

Table 4: Pollution thresholds when market size is large ($\theta = 1.5$).

| | $\pi$ | $q$ | $\underline{X}$ | $\bar{X}$ | $\hat{X}$ |
|---|---|---|---|---|---|
| monopoly | 5 | 10 | 1.567 | 2.542 | 12.935 |
| firm 1 | 2.69 | 7.33 | 1.567 | 2.542 | |
| firm 2 | 1.42 | 5.33 | 0.738 | 1.559 | 2.886 |

Table 5: Possible market status when pollution uncertainty is small ($\sigma = 0.2$).

| | | Firm 2 | | |
|---|---|---|---|---|
| | | Polluting | Stopping | Exit |
| Firm 1 | Polluting | 1** | — | 3* |
| | Stopping | — | — | 4 |
| | Exit | — | — | 5 |

We have value-matching and smooth-pasting conditions at firm 1's restart threshold $\underline{X}_1$:

$$V_{21}(\underline{X}_1) - K = V_{61}(\underline{X}_1), \tag{46}$$

$$V'_{21}(\underline{X}_1) = V'_{61}(\underline{X}_1), \tag{47}$$

and at firm 1's suspend threshold $\bar{X}_1$:

$$V_{21}(\bar{X}_1) = V_{61}(\bar{X}_1), \tag{48}$$

$$V'_{21}(\bar{X}_1) = V'_{61}(\bar{X}_1). \tag{49}$$

In addition to value-matching conditions (42) at firm 2's restart threshold $\underline{X}_2$ and (43) at firm 2's suspend threshold $\bar{X}_2$, we have

$$V_{61}(\hat{X}_2) = V_2(\hat{X}_2), \tag{50}$$

at firm 2's exit threshold $\hat{X}_2$.

Table 4 shows pollution thresholds when market size is large ($\theta = 1.5$). Figure 8 shows value functions of firm 1. Figure 9 shows dynamics of pollution when pollution increases (+) and decreases (−). In this case, duration and amount of pollution increase, additionally, increment of pollution increases compared to the base case.

Finally, we investigate the impact of pollution uncertainty $\sigma$. When $\sigma = 0.2$, pollution thresholds are in the order of $X_2 < \hat{X}_2 < \bar{X}_2 < \underline{X}_1 < \bar{X}_1 < \hat{X}_1$. In this case, possible market status and status transition are shown in Table 6 and Figure 10, respectively. Statuses 1 is in duopoly, while statuses 3–5 are in monopoly.

Note that firm 2 exits before suspends, therefore, status 2 does not exist in this case. We no longer consider the value of suspend for firm 2, and we have value-matching and smooth-pasting

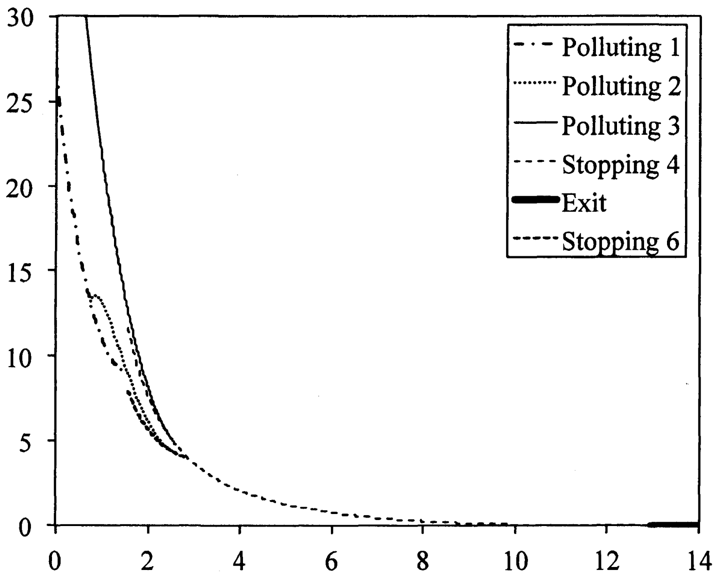


Figure 8: Value functions of firm 1 in duopoly when market size is large ($\theta = 1.5$).

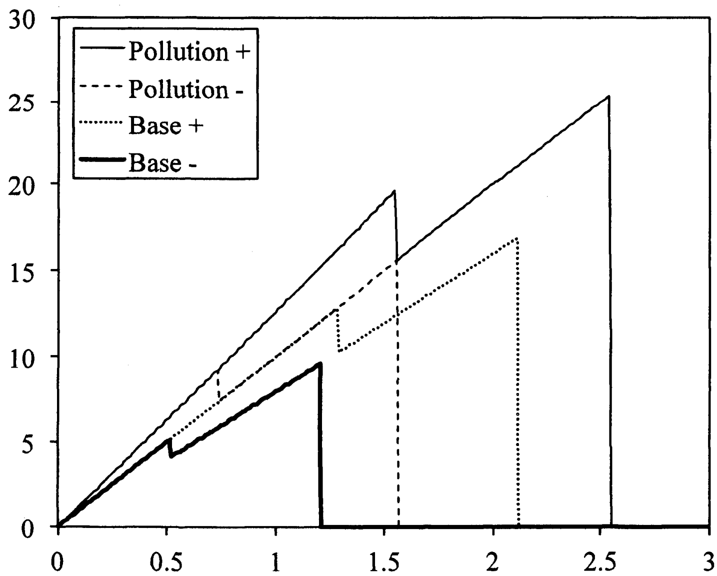


Figure 9: Dynamics of pollution when market size is large ($\theta = 1.5$).

$$1 \xrightarrow{\hat{X}_2} 3 \underset{\underline{X}_1}{\overset{\bar{X}_1}{\rightleftarrows}} 4 \xrightarrow{\hat{X}_1} 5$$

Figure 10: Status transition when pollution uncertainty is small ($\sigma = 0.2$).

Table 6: Pollution thresholds when pollution uncertainty is small ($\sigma = 0.2$).

| | $\pi$ | $q$ | $\underline{X}$ | $\bar{X}$ | $\hat{X}$ |
|---|---|---|---|---|---|
| monopoly | 3.2 | 8 | 1.269 | 1.963 | 2.881 |
| firm 1 | 1.8 | 6 | | | |
| firm 2 | 0.8 | 4 | | | 1.057 |

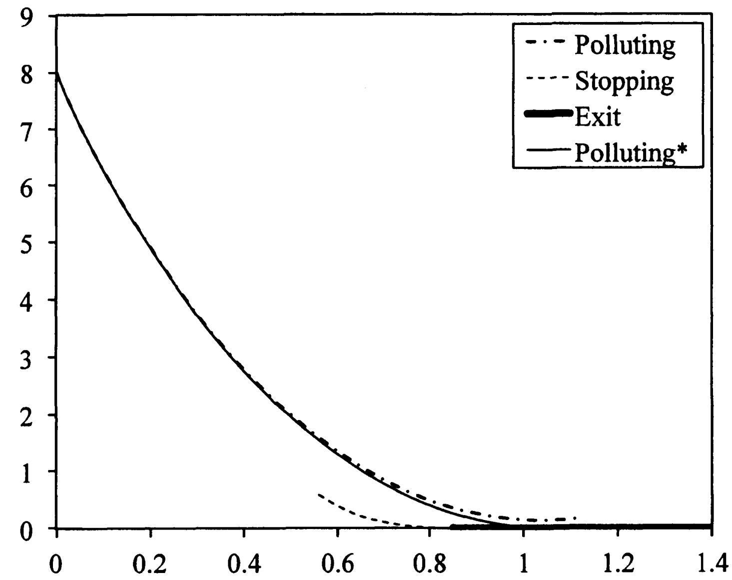


Figure 11: Value functions of firm 2 in duopoly when pollution uncertainty is small ($\sigma = 0.2$).

conditions in duopoly only at firm 2's exit threshold $\hat{X}_2$:

$$V_{12}(\hat{X}_2) = 0, \tag{51}$$

$$V'_{12}(\hat{X}_2) = 0, \tag{52}$$

$$V_{11}(\hat{X}_2) = V_1(\hat{X}_2). \tag{53}$$

Table 6 shows pollution thresholds when pollution uncertainty is small ($\sigma = 0.2$). Figures 11 and 12 show value functions of firm 2 and 1. Figure 13 shows dynamics of pollution when pollution increases (+) and decreases (−). In this case, duration of pollution is longer than that for the base case when pollution decreases due to early exit.

## 5 Conclusion

We have investigated pollution thresholds of entry-exit in Cournot competition where only a cost-advantaged firm can enjoy monopoly. Especially, we consider uncertainty of pollution by combining Wirl (2006) and Simpson (1995), while existing literature do not consider it. It is shown that transition of market status varies with power of a disadvantaged firm. Analyses

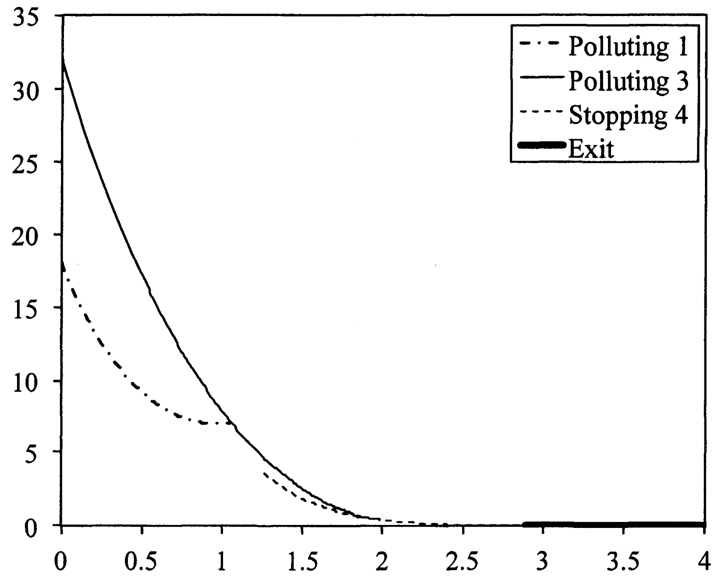


Figure 12: Value functions of firm 1 in duopoly when pollution uncertainty is small ($\sigma = 0.2$).

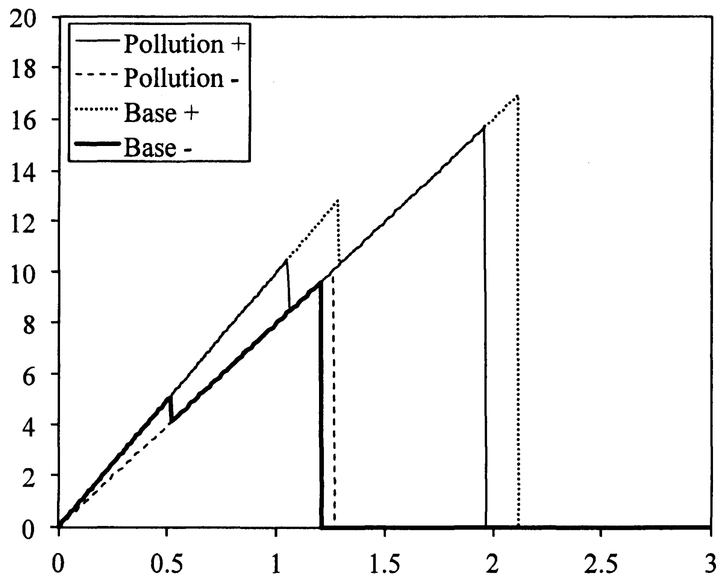


Figure 13: Dynamics of pollution when pollution uncertainty is small ($\sigma = 0.2$).

of dynamics of pollution find that environmental regulation decreases duration and amount of pollution, pollution uncertainty increases duration and amount of pollution, and market size increases increment of pollution.

Our remainders are welfare analysis and empirical studies. As a future study, it seems important to consider the game theoretic approach in the duopolistic setting. To this end, we need to develop a new equilibrium concept of entry-exit.

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