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Categorification of Coxeter groups and braid groups

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1. Introduction

This paper is mainly a summary of [M, AM], where we discuss preprojective algebras of Dynkin type. Preprojective algebras first appeared in the work of Gelfand-Ponomarev [GP]. Since then, they have been one of the important objects in the representation theory of algebras and they also appear in many branches of mathematics such as quantum groups.

Recently, the notion of support τ-tilting modules was introduced in [AIR], as a generalization of tilting modules. Support τ-tilting modules have several nice properties. For example, it is shown that there are deep connections between τ-tiltihg theory, torsion theory, silting theory and cluster tilting theory. Moreover, support τ-tilting modules over selfinjective algebras are useful to provide tilting complexes. It is therefore fruitful to investigate these remarkable modules for preprojective algebras. To explain our results, we give the following set-up.

Let Δ be a finite connected graph (without loop) with the set $\Delta_0 = \{1, \ldots, n\}$ of vertices, Λ the preprojective algebra of Δ and I_i the two-sided ideal of Λ generated by $1 - e_i$, where e_i is an idempotent of Λ corresponding to $i \in \Delta_0$. We denote by $\langle I_1, \ldots, I_n\rangle$ the set of ideals of Λ of the form $I_{i_1}I_{i_2}\cdots I_{i_k}$ for some $k \geq 0$ and $i_1, \ldots, i_k \in \Delta_0$.

These ideals are quite useful to study structure of categories [IR, BIRS, AIRT, ORT]. They also play important roles in Geiss-Leclerc-Schröer's construction of cluster monomials of certain types of cluster algebras [GLS1, GLS2] and Baumann-Kamnitzer-Tingley's works of MV polytopes [BK, BKT].

One of the results in this paper is to show that elements of $\langle I_1, \ldots, I_n\rangle$ are support τ-tilting modules over preprojective algebras of Dynkin type, and they are bijective to the elements of the Coxeter group.

Another aim is to study tilting complexes. It is known that derived equivalences are controlled by tilting complexes [Ric] and therefore these objects have been extensively studied. By applying the above result, we give a classification of tilting complexes.

Notation. Let K be an algebraically closed field and $D := \mathrm{Hom}_K(-, K)$. All modules are right modules. For a finite dimensional algebra Λ, we denote by $\mathrm{mod}\,\Lambda$ the category of finitely generated Λ-modules.

2. Preliminaries

2.1. **Support τ-tilting modules.** We recall the definition of support τ-tilting modules. We refer to [AIR] for the details about support τ-tilting modules. Let Λ be a finite dimensional algebra and τ denote the AR translation [ARS].

Definition 2.1. (a) We call X in $\operatorname{mod}\Lambda$ *τ-rigid* if $\operatorname{Hom}_\Lambda(X, \tau X) = 0$.

(b) We call X in $\operatorname{mod}\Lambda$ *τ-tilting* (respectively, *almost complete τ-tilting*) if X is τ-rigid and $|X| = |\Lambda|$ (respectively, $|X| = |\Lambda| - 1$), where $|X|$ denotes the number of non-isomorphic indecomposable direct summands of X.

(c) We call X in $\operatorname{mod}\Lambda$ *support τ-tilting* if there exists an idempotent e of Λ such that X is a τ-tilting $(\Lambda/\langle e\rangle)$-module.

(d) We call a pair (X, P) of $X \in \operatorname{mod}\Lambda$ and $P \in \operatorname{proj}\Lambda$ *τ-rigid* if X is τ-rigid and $\operatorname{Hom}_\Lambda(P, X) = 0$.

(e) We call a τ-rigid pair (X, P) a *support τ-tilting* (respectively, *almost complete support τ-tilting*) pair if $|X| + |P| = |\Lambda|$ (respectively, $|X| + |P| = |\Lambda| - 1$).

We call (X, P) *basic* if X and P are basic, and we say that (X, P) is a *direct summand* of (X', P') if X is a direct summand of X' and P is a direct summand of P'. Note that (X, P) is a τ-rigid (respectively, support τ-tilting) pair for Λ if and only if X is a τ-rigid (respectively, τ-tilting) $(\Lambda/\langle e\rangle)$-module, where e is an idempotent of Λ such that $\operatorname{add} P = \operatorname{add} e\Lambda$ [AIR, Proposition 2.3]. Moreover, if (X, P) and (X, P') are support τ-tilting pairs for Λ, then we get $\operatorname{add} P = \operatorname{add} P'$. Thus, a basic support τ-tilting module X determines a basic support τ-tilting pair (X, P) uniquely and we can identify basic support τ-tilting modules with basic support τ-tilting pairs.

We denote by $s\tau\text{-tilt}\,\Lambda$ the set of isomorphism classes of basic support τ-tilting Λ-modules.

Next we recall some properties of support τ-tilting modules. The set of support τ-tilting modules has a natural partial order as follows.

Definition 2.2. [AIR, Theorem 2.18] Let Λ be a finite dimensional algebra. For $T, T' \in s\tau\text{-tilt}\,\Lambda$, we write

$$T' \geq T$$

if $\operatorname{Fac} T' \supset \operatorname{Fac} T$. Then $\geq$ gives a partial order on $s\tau\text{-tilt}\,\Lambda$.

Then we give the following results, which play important roles in this paper.

Definition-Theorem 2.3. [AIR, Theorem 2.28] Let Λ be a finite dimensional algebra. Then

(i) any basic almost support τ-tilting pair (U, Q) is a direct summand of exactly two basic support τ-tilting pairs (T, P) and (T', P'). Moreover, we have $T > T'$ or $T < T'$.

Under the above setting, let X be an indecomposable Λ-module satisfying either $T = U \oplus X$ or $P = Q \oplus X$. We write $(T', P') = \mu_{(X,0)}(T, P)$ if X is a direct summand of T and $(T', P') = \mu_{(0,X)}(T, P)$ if X is a direct summand of P, and we say that (T', P') is a *mutation* of (T, P). In particular, we say that (T', P') is a *left mutation* (respectively, *right mutation*) of (T, P) if $T > T'$ (respectively, if $T < T'$) and write $\mu^-(T, P) = (T', P')$ (respectively, $\mu^+(T, P) = (T', P')$). By (i), exactly one of the left mutation or right mutation occurs.

Now, assume that X is a direct summand of T and $T = U \oplus X$. In this case, for simplicity, we write a left mutation $T' = \mu_X^-(T)$ and a right mutation $T' = \mu_X^+(T)$.

Finally, we define the *support τ-tilting quiver* $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\,\Lambda)$ as follows.

- The set of vertices is $\mathrm{s}\tau\text{-tilt}\,\Lambda$.
- Draw an arrow from T to T' if T' is a left mutation of T (i.e. $T' = \mu_X^-(T)$).

The following theorem relates $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\,\Lambda)$ with partially orders of $\mathrm{s}\tau\text{-tilt}\,\Lambda$.

Theorem 2.4. [AIR, Corollary 2.34] *The support τ-tilting quiver $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\,\Lambda)$ is the Hasse quiver of the partially ordered set* $\mathrm{s}\tau\text{-tilt}\,\Lambda$.

2.2. **Preprojective algebras.** In this subsection, we recall definitions and some properties of preprojective algebras. We refer to [BBK, BGL, Ri] for basic properties and background information.

Definition 2.5. Let Q be a finite connected acyclic quiver with vertices $Q_0 = \{1, \dots, n\}$. The preprojective algebra associated to Q is the algebra

$$\Lambda = K\overline{Q}/\langle \sum_{a \in Q_1} (aa^* - a^*a) \rangle$$

where $\overline{Q}$ is the double quiver of Q, which is obtained from Q by adding for each arrow $a : i \to j$ in Q_1 an arrow $a^* : i \leftarrow j$ pointing in the opposite direction.

We remark that Λ does not depend on the orientation of Q. Hence, for a graph Δ, we define the preprojective algebra by $\Lambda_\Delta = \Lambda_Q$, where Q is a quiver whose undrlying graph is Δ. We denote by Δ_0 vertices of Δ.

Let Δ be a Dynkin (ADE) graph. The preprojective algebra of Δ is finite dimensional and selfinjective. We denote the Nakayama permutation of Λ by $\iota : \Delta_0 \to \Delta_0$ (i.e. $D(\Lambda e_{\iota(i)}) \cong e_i\Lambda$).

2.3. **Coxeter group.** Let Δ be a Dynkin graph of type A to F. The *Coxeter group* W_Δ associated to Δ is defined by the generators s_i ($i \in \Delta_0$) and relations $(s_is_j)^{m(i,j)} = 1$, where

$$m(i,j) := \begin{cases} 1 & \text{if } i = j; \\ 2 & \text{if no edge between } i \text{ and } j; \\ 3 & \text{if there is an edge } i - j, \\ 4 & \text{if there is an edge } i \overset{4}{-} j. \end{cases}$$

Each element $w \in W_\Delta$ can be written in the form $w = s_{i_1} \cdots s_{i_k}$. If k is minimal among all such expressions for w, then k is called the *length* of w and we denote by $l(w) = k$. In this case, we call $s_{i_1} \cdots s_{i_k}$ a *reduced expression* of w.

Let ι be a permutation of Δ_0. Then ι acts on an element of the Coxeter group W_Δ by $\iota(w) := s_{\iota(i_1)} s_{\iota(i_2)} \cdots s_{\iota(i_\ell)}$ for $w = s_{i_1} s_{i_2} \cdots s_{i_\ell} \in W_Q$. We define the subgroup W_Δ^ι of W_Δ by

$$W_\Delta^\iota := \{w \in W \mid \iota(w) = w\}.$$

Then the following result is well-known.

Theorem 2.6. *Let Δ be a Dynkin (A,D,E) quiver and W_Δ the Coxeter group of Δ. Let $\Delta' = \Delta$ if Δ is type D_{2n}, E_7 and E_8. Otherwise, let Δ' be a quiver, respectively, given by the following type.*

Δ	A_{2n-1}, A_{2n}	D_{2n+1}	E_6
Δ'	B_n	B_{2n}	F_4

Then, for the Nakayama permutation ι of the preprojective algebra of Δ, W_Δ^ι is isomorphic to $W_{\Delta'}$.

3. Preprojective algebras and the Coxeter groups

Let Δ be a Dynkin graph with $\Delta_0 = \{1, \ldots, n\}$ and Λ the preprojective algebra of Δ. We denote by $I_i := \Lambda(1 - e_i)\Lambda$ for $i \in \Delta_0$. We denote by $\langle I_1, \ldots, I_n \rangle$ the set of ideals of Λ which can be written as

$$I_{i_1} I_{i_2} \cdots I_{i_k}$$

for some $k \geq 0$ and $i_1, \ldots, i_k \in \Delta_0$.

The following lemma plays a key role.

Lemma 3.1. *Let $T \in \langle I_1, \ldots, I_n \rangle$. If $I_i T \neq T$, then there is a left mutation of T :*

$$\mu_{e_i T}^{-}(T) \cong I_i T.$$

Moreover we recall the following important result.

Theorem 3.2. [IR, BIRS] *There exists a bijection $W_\Delta \to \langle I_1, \ldots, I_n \rangle$. It is given by $w \mapsto I_w = I_{i_1} I_{i_2} \cdots I_{i_k}$ for any reduced expression $w = s_{i_1} \cdots s_{i_k}$.*

Then using Lemma 3.1 and Theorem 3.2, we obtain the following result.

Theorem 3.3. *The map in Theorem 3.2 gives a bijection between the elements of the Coxeter group W_Δ and* sτ-tiltΛ.

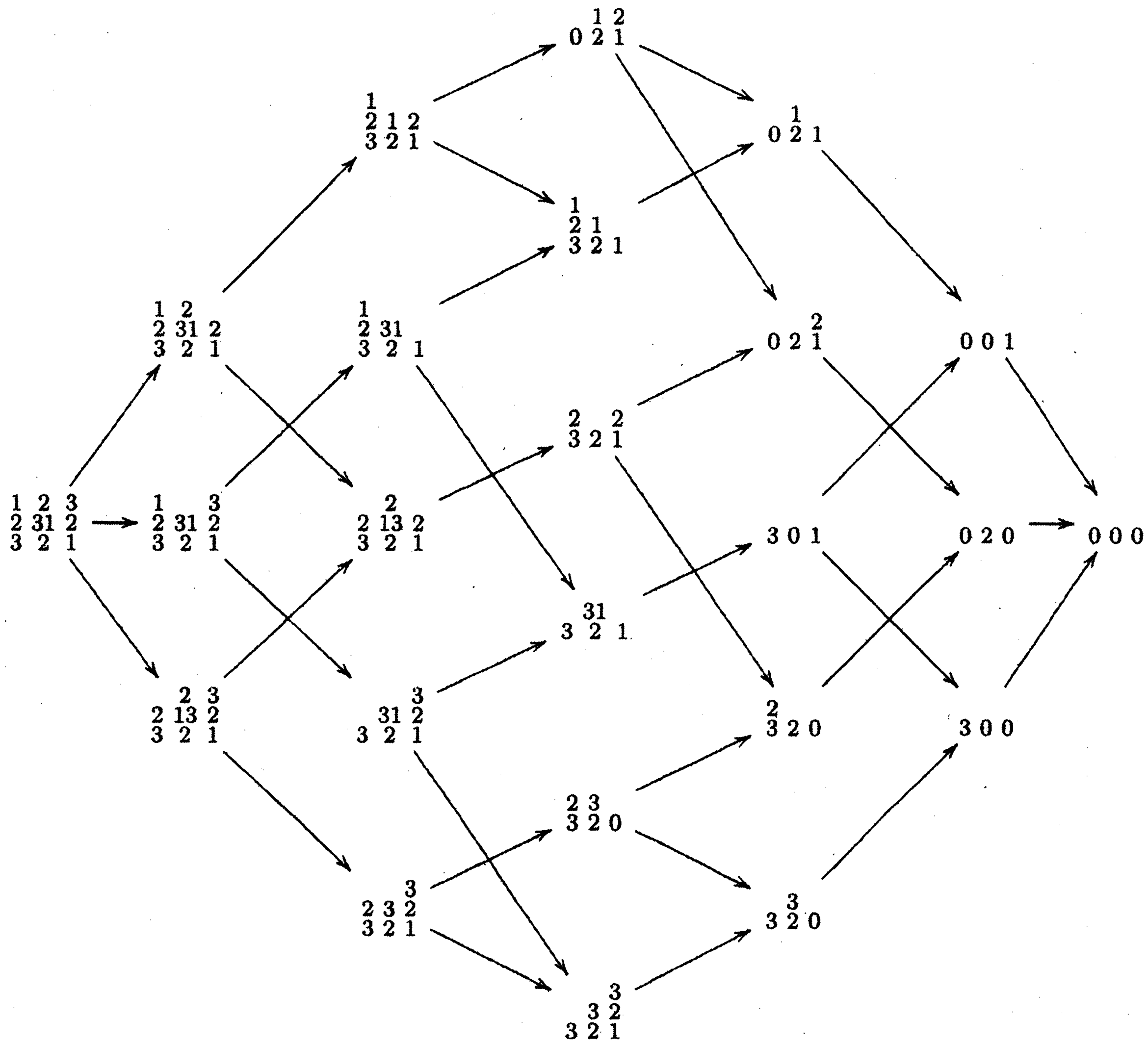
We remark that the above ideals I_w are tilting modules in the case of non-Dynkin type [IR, BIRS].

Example 3.4. (a) Let Λ be the preprojective algebra of type A_2. In this case, $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\,\Lambda)$ is given as follows.

$$\begin{tikzcd}
 & {\begin{smallmatrix}1\\2\end{smallmatrix}\,1} \arrow[r, "I_1"] & 0\,1 \arrow[dr, "I_2"] & \\
{\begin{smallmatrix}1&2\\2&1\end{smallmatrix}} \arrow[ur, "I_2"] \arrow[dr, "I_1"'] & & & 0\,0 \\
 & 2\,{\begin{smallmatrix}2\\1\end{smallmatrix}} \arrow[r, "I_2"'] & 2\,0 \arrow[ur, "I_1"'] &
\end{tikzcd}$$

Here we represent modules by their radical filtrations and we write a direct sum $X \oplus Y$ by XY. For example, ${\begin{smallmatrix}1\\2\end{smallmatrix}}\,1$ denotes the support τ-tilting module $e_1\Lambda \oplus S_1$, where S_1 is the simple module associated with the vertex 1.

(b) Let Λ be the preprojective algebra of type A_3. In this case, $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\,\Lambda)$ is given as follows.



Morever we study a close relationship between partial orders of W_Δ and sτ-tiltΛ. This lemma is crucial.

Lemma 3.5. *Let $w \in W_\Delta$ and $i \in \Delta_0$.*

(i) *If $l(w) < l(s_i w)$, then $I_i I_w = I_{s_i w} \subsetneq I_w$ and we have a left mutation $\mu_i^-(I_w, P_w)$.*

(ii) *If $l(w) > l(s_i w)$, then $I_i I_w = I_w \subsetneq I_{s_i w}$ and we have a right mutation $\mu_i^+(I_w, P_w)$.*

We denote by $\leq$ the (*left*) *weak order* of W_Δ and by $\mathcal{H}(W_\Delta, \leq)$ the Hasse quiver induced by weak order on W_Δ.

Then, by Lemma 3.5, we have the following result [M].

Theorem 3.6. *The bijection $W_\Delta \to$ sτ-tiltΛ in Theorem 3.2 gives an isomorphism of partially ordered sets $(W_\Delta, \leq)$ and (sτ-tilt$\Lambda, \leq)^{\mathrm{op}}$.*

4. Silting-discreteness

In this section, we discuss some properties of silting complexes. First we recall the notion of silting complexes.

4.1. **Silting complexes.** Silting complexes are a generalization of tilting complexes, which were introduced by Keller-Vossieck [KV]. They were originally invented as a tool for studying tilting complexes. Nonetheless, silting complexes have turned out to have deep connections with several important complexes such as t-structures [KY, BY].

We recall the definition of silting complexes as follows.

Definition 4.1. Let A be a finite dimensional algebra and $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}A)$ the bounded homotopy category of the finitely generated projective A-modules. Let $\mathcal{T} := \mathsf{K}^{\mathrm{b}}(\mathsf{proj}A)$ for simplicity.

(a) We call a complex P in $\mathcal{T}$ (or in the derived category of $\operatorname{mod} A$) is *presilting* (respectively, *pretilting*) if it satisfies $\operatorname{Hom}_{\mathcal{T}}(P, P[i]) = 0$ for any $i > 0$ (respectively, $i \neq 0$).

(b) We call a complex P in $\mathcal{T}$ *silting* (respectively, *tilting*) if it is presilting (respectively, pretilting) and the smallest thick subcategory containing P is $\mathcal{T}$.

We denote by $\operatorname{silt} A$ (respectively, $\operatorname{tilt} A$) the set of non-isomorphic basic silting (respectively, tilting) complexes in $\mathcal{T}$.

For complexes P and U of $\mathcal{T}$, we write $P \geq U$ if $\operatorname{Hom}_{\mathcal{T}}(P, U[i]) = 0$ for any $i > 0$. Then the relation $\geq$ gives a partial order on $\operatorname{silt} A$ [AI, Theorem 2.11].

Moreover, a complex $P \in \mathcal{T}$ is called *2-term* provided it is concerned in the degree 0 and -1. We denote by $2\text{-}\operatorname{silt} A$ (respectively, $2\text{-}\operatorname{tilt} A$) the subset of $\operatorname{silt} A$ (respectively, $\operatorname{tilt} A$) consisting of 2-term complexes. Note that a complex P is 2-term if and only if $A \geq T \geq A[1]$.

Then we give the definition of silting-discrete triangulated categories as follows.

Definition 4.2. (a) We call $\mathcal{T}$ *silting-discrete* if the set $\{T \in \mathsf{silt}\,\mathcal{T} \mid A \geq T \geq A[\ell]\}$ is finite for any $\ell > 0$. Similarly, we call $\mathcal{T}$ *tilting-discrete* if the set $\{T \in \mathsf{tilt}\,\mathcal{T} \mid A \geq T \geq A[\ell]\}$ is finite for any $\ell > 0$.

(b) For a silting complex P of $\mathcal{T}$, we denote by $2\text{-}\mathsf{silt}_P\,\mathcal{T}$ the subset of $\mathsf{silt}\,\mathcal{T}$ such that U with $P \geq U \geq P[1]$. We call $\mathcal{T}$ *2-silting-finite* if $2\text{-}\mathsf{silt}_P\,\mathcal{T}$ is a finite set for any silting complex P of $\mathcal{T}$. Similarly, we denote by $2\text{-}\mathsf{tilt}_P\,\mathcal{T}$ the subset of $\mathsf{tilt}\,\mathcal{T}$ such that U with $P \geq U \geq P[1]$.

Moreover we recall mutation for silting complexes [AI, Theorem 2.31].

Definition 4.3. Let P be a basic silting complex of $\mathcal{T}$ and decompose it as $P = X \oplus M$. We take a triangle

$$X \xrightarrow{f} M' \longrightarrow Y \longrightarrow X[1]$$

with a minimal left $(\mathsf{add}\,M)$-approximation f of X. Then $\mu_X^-(P) := Y \oplus M$ is again a silting complex, and we call it the *left mutation* of P with respect to X. Dually, we define the right mutation $\mu_X^+(P)$. Mutation will mean either left or right mutation. If X is indecomposable, then we say that mutation is *irreducible.* In this case, we have $P > \mu_X^-(P)$ and there is no silting complex Q satisfying $P > Q > \mu_X^-(P)$ [AI, Theorem 2.35].

Moreover, if P and $\mu_X^-(P)$ are tilting complexes, then we call it the (left) *tilting mutation.* In this case, if there exists no non-trivial direct summand X' of X such that $\mu_{X'}^-(T)$ is tilting, then we say that tilting mutation is *irreducible.*

The following theorem play a key role.

Theorem 4.4. [AM] *Let A be a finite dimensional algebra and $\mathcal{T} := \mathsf{K}^{\mathsf{b}}(\mathsf{proj}A)$. The following are equivalent.*

(a) *$\mathcal{T}$ is silting-discrete.*
(b) *$\mathcal{T}$ is 2-silting-finite.*
(c) $\mathsf{2\text{-}silt}_P\,\mathcal{T}$ *is a finite set for any silting complex P which is given by iterated irreducible left mutation from A.*

Moreover if A is selfinjective, we have the following result.

Corollary 4.5. *Assume that A is selfinjective and let $\mathcal{T} := \mathsf{K}^{\mathsf{b}}(\mathsf{proj}A)$. The following are equivalent.*

(a) *$\mathcal{T}$ is tilting-discrete.*
(b) *$\mathcal{T}$ is 2-tilting-finite.*
(c) $\mathsf{2\text{-}silt}_P\,\mathcal{T}$ *is a finite set for any tilting complex P which is given by iterated irreducible tilting left mutation from A.*

5. Preprojective algebras and the braid groups

Using the previous results, we study tilting complexes over the preprojective algebra of Dynkin type.

First we recall the following nice correspondence.

Theorem 5.1. [AIR, Theorem 3.2] *Let A be a finite dimensional algebra. There exists a bijection*

$$\mathrm{s}\tau\text{-tilt}\,A \leftrightarrow \text{2-silt}\,A.$$

By the above correspondence, we can give a description of 2-term silting complexes by calculating support τ-tilting modules, which is much simpler than calculations of silting complexes.

From now on, let Δ be a Dynkin graph and Λ the preprojective algebra of Δ. Then, as a consequence of Theorem 3.3 and 5.1, we have the following corollary.

Corollary 5.2. *We have a bijection*

$$W_\Delta \leftrightarrow \text{2-silt}\,\Lambda.$$

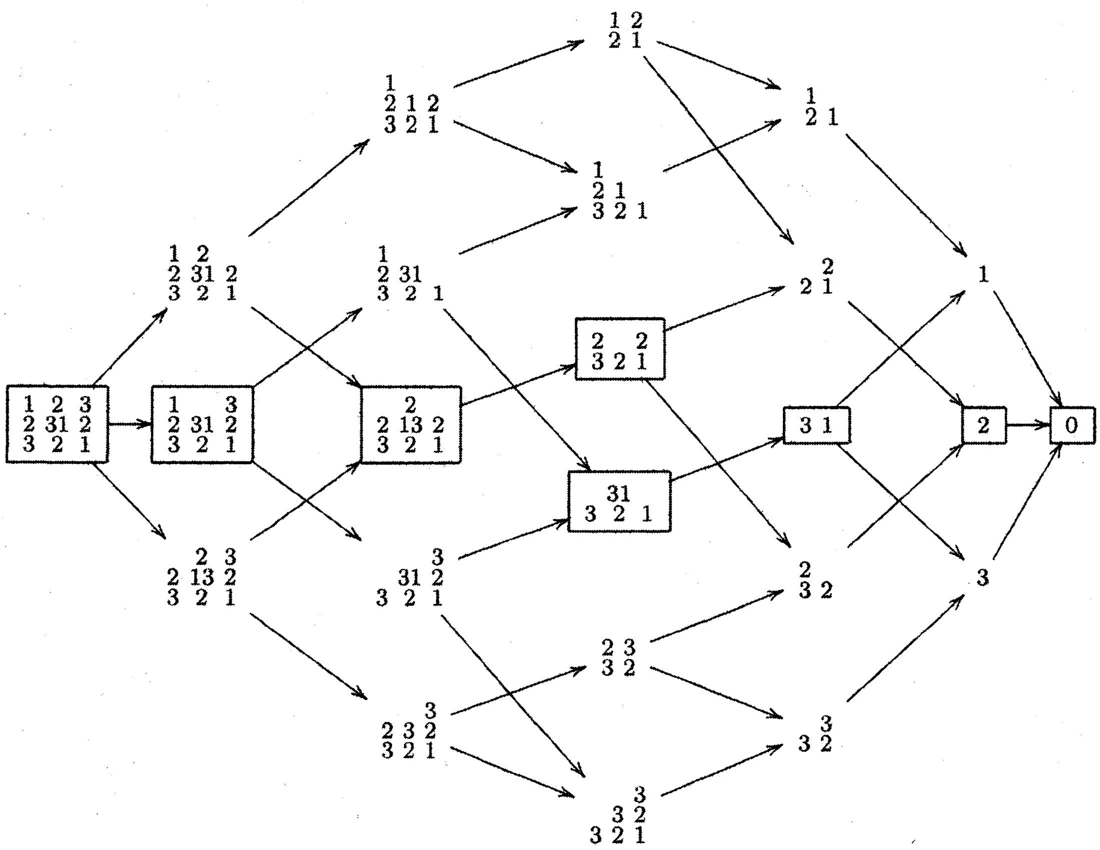
Thus we can parameterize 2-term silting complexes by the Coxeter group. Moreover, we can describe 2-term tilting complexes in terms of the Coxeter group as follows.

Proposition 5.3. *Let $\nu := D\operatorname{Hom}_\Lambda(-,\Lambda)$ the Nakayama functor of Λ and $\iota : \Delta_0 \to \Delta_0$ the Nakayama permutation of Λ. Then $\nu(I_w) \cong I_w$ if and only if $\iota(w) = w$. In particular, We have a bijection*

$$W_\Delta^\iota \leftrightarrow \text{2-tilt}\,\Lambda.$$

Then, by Theorem 2.6, we can understand W_Δ^ι as another type of the Coxeter group.

Example 5.4. Let Δ be a Dynkin graph of type A_3 and Λ the preprojective algebra of Δ. Then the support τ-tilting quiver of Λ is given as follows.



The framed modules indicate *ν-stable* modules (i.e. $I_w \cong \nu(I_w)$), which is equivalent to say that $\iota(w) = w$.

Let $\Lambda = X \oplus Y$. We denote by $\mu_X^-(\Lambda)$ the irreducible tilting left mutation of Λ with respect to X.

Proposition 5.5. *Assume that $\mu_X^-(\Lambda)$ is an irreducible tilting left mutation of Λ. Then we have an isomorphism*

$$\operatorname{End}_{K^b(\operatorname{proj}\Lambda)}(\mu_X^-(\Lambda)) \cong \Lambda.$$

In particular, by Corollary 4.5, Λ is tilting-discrete.

Consequently, we extend Proposition 5.3 and obtain the following result.

Theorem 5.6. *We denote the braid group by $B_{\Delta'}$. Then we have a bijection*

$$B_{\Delta'} \leftrightarrow \operatorname{tilt} \Lambda.$$

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