

Highlights

- Influence of surrounding environment on the crack velocity in marble was studied.
- Crack velocity in air increased at higher temperature and/or humidity.
- Subcritical crack growth in marble occurs in grain boundary.
- Region II behavior of subcritical crack growth was observed in marble with low porosity in air.

1 Influence of surrounding environment on subcritical crack growth in marble

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16 **Abstract**

17

18 Understanding subcritical crack growth in rock is essential for determining
19 appropriate measures to ensure the long-term integrity of rock masses surrounding
20 structures and for construction from rock material. In this study, subcritical crack
21 growth in marble was investigated experimentally, focusing on the influence of the
22 surrounding environment on the relationship between the crack velocity and stress
23 intensity factor.

24 The crack velocity increased with increasing temperature and/or relative humidity. In
25 all cases, the crack velocity increased with increasing stress intensity factor. However,
26 for Carrara marble (CM) in air, we observed a region in which the crack velocity still
27 increased with temperature, but the increase in the crack velocity with increasing stress
28 intensity factor was not significant. This is similar to Region II of subcritical crack
29 growth observed in glass in air. Region II in glass is controlled by mass transport to the
30 crack tip. In the case of rock, the transport of water to the crack tip is important. In
31 general, Region II is not observed for subcritical crack growth in rock materials,
32 because rocks contain water. Because the porosity of CM is very low, the amount of
33 water contained in the marble is also very small. Therefore, our results imply that we
34 observed Region II in CM.

35 Because the crack velocity increased in both water and air with increasing
36 temperature and humidity, we concluded that dry conditions at low temperature are
37 desirable for the long-term integrity of a carbonate rock mass. Additionally, mass
38 transport to the crack tip is an important process for subcritical crack growth in rock
39 with low porosity.

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41 **Keywords:** subcritical crack growth, marble, relative humidity, temperature, water,
42 porosity

43 **1. Introduction**

44

45 The long-term stability of rock masses surrounding structures, such as underground
46 repositories of radioactive waste, caverns to store liquid natural gas and liquid
47 petroleum gas, and underground power plants, is crucial. In addition, it is important to
48 ensure the stability of rock slopes in open-pit mines for safety. Various studies have
49 examined time-dependent fracturing in rock to determine the time-dependency of rock
50 stability (Atkinson, 1984; Swanson, 1984; Meredith and Atkinson, 1985; Sano, 1988;
51 Nara and Kaneko, 2005, 2006). In particular, studies of time-dependent fracturing in
52 rock have been conducted to examine the natural hazards related to failure in rock, such
53 as the increase in seismicity seen prior to earthquake rupture, fault formation, growth,
54 sliding, and volcanic eruption (Kilburn and Voight, 1998; Ciccotti et al., 2000a, 2001;
55 Heap et al., 2011; Brantut et al., 2013, 2014a; Violay et al., 2013). Additionally, several
56 studies have evaluated the long-term strength and time-to-failure based on the
57 measurement of time-dependent fracturing (Schmidtke and Lajtai, 1986; Jeong et al.,
58 2007; Nara et al., 2013; Nara, 2015).

59 Although classical fracture mechanics postulates that crack propagation occurs when
60 the value of the stress intensity factor reaches that of the fracture toughness, the crack
61 can propagate even at a stress intensity factor lower than the fracture toughness. This is
62 known as subcritical crack growth, which is considered to be one of the main
63 mechanisms responsible for the time-dependent behavior of rock in the brittle regime
64 (Atkinson, 1984). Most studies of subcritical crack growth in rock have been conducted
65 on silicate rocks, such as igneous rocks (Sano and Kudo, 1992; Nara et al., 2009, 2010),
66 sandstones (Holder et al., 2001; Ponson, 2009; Nara et al., 2011, 2014), and novaculite
67 (Atkinson, 1980).

68 Only a few studies have examined subcritical crack growth in carbonate minerals and
69 rocks. Henry et al. (1977) reported that for micrite the crack velocity in water is higher
70 than that in air. Røyne et al. (2011) suggested that some plastic processes might affect
71 subcritical crack growth in calcite. Rostom et al. (2012) reported that the fluid salinity
72 influences the crack velocity in calcite in a NaCl solution; specifically, they showed that
73 the stress intensity factor decreased when the concentration of NaCl is <0.8 mol/L.
74 Bergsaker et al. (2016) examined the impact of the fluid composition on subcritical
75 crack growth in calcite single crystal, and concluded that a pH in the range of 5 – 7.5
76 has a negligible influences. However, subcritical crack growth in carbonate rocks under
77 different temperature and humidity conditions is poorly understood.

78 In this study, we investigated subcritical crack growth in marble experimentally in
79 both air and water. We focussed on examining the influence of the surrounding
80 environment on the relationship between the crack velocity and stress intensity factor by
81 conducting all measurements under controlled temperature and relative humidity.

82 2. Rock samples

83

84 We used examined two types of marble: Carrara marble (CM) quarried in Italy, and a
85 marble quarried in Skopje in Macedonia (MM).

86 Figure 1 shows photomicrographs of CM and MM observed with thin sections of
87 0.03 mm thickness. As shown in the photomicrograph, the grain size is around 0.2 mm
88 and 0.3 mm for CM, and MM, respectively. Figure 2 shows the results of X-ray
89 diffraction analysis of the marbles. Remarkable peaks can be seen for calcite (CaCO_3) in
90 CM and dolomite ($\text{CaMg}(\text{CO}_3)_2$) in MM. Small peaks of illite are also seen in MM.

91 For CM, the porosity determined by water saturation was 0.19%. The P-wave
92 velocities in three orthogonal directions were 6.04, 5.98, and 5.90 km/s under dry
93 conditions. CM is considered to be isotropic. The Brazilian tensile strength was 6.9 MPa.
94 The uniaxial compressive strength, Young's modulus, and Poisson's ratio were 77.8
95 MPa, 51.0 GPa, and 0.32, respectively, which were determined from uniaxial
96 compression tests with the loading rate at 10^{-5} strain/s.

97 For MM, the porosity measured by water saturation was 0.6%. The P-wave velocities
98 in three orthogonal directions were 4.15, 4.06, and 3.74 km/s. We named these three
99 orthogonal directions Axes 1, 2, and 3 in order of decreasing P-wave velocity.
100 Furthermore, we named the planes normal to these axes Planes 1, 2, and 3, respectively.
101 Slight anisotropy was observed in the P-wave velocity. Since investigation of
102 anisotropic properties was beyond the scope of this study, we treated the marble sample
103 as an isotropic material. The Brazilian tensile strength was 6.2 MPa when the fracturing
104 was parallel to Plane 3. The uniaxial compressive strength, Young's modulus, and
105 Poisson's ratio were 190 MPa, 80.2 GPa, and 0.46, respectively, which were determined
106 from uniaxial compression tests with the loading rate at 10^{-5} strain/s.

107 3. Methodology

108

109 3.1 Outline of double torsion method

110

111 In this study, the double torsion (DT) method was used. The DT method is a fracture
112 mechanics testing method used commonly to study subcritical crack growth. The
113 loading configuration of the DT method is shown in Figure 3. Three different types of
114 test can be performed using the DT arrangement, each using different loading
115 conditions: the constant load method (Kies and Clark, 1969), the constant displacement
116 rate method (Evans, 1972), and the load relaxation (RLX) method (Evans, 1972;
117 Williams and Evans, 1973). Using the RLX method, we can obtain a large amount of
118 data on the relationship between the stress intensity factor, K_I , and the crack velocity,
119 da/dt (the K_I - da/dt relation), which, in general, ranges from 10^{-2} to 10^{-9} m/s, using only
120 a single experimental run. Therefore, we used the RLX method to determine the
121 K_I - da/dt relation in this study.

122 In the RLX method, the displacement of the loading points must be kept constant
123 during the experiment while the temporally decreasing load (load relaxation) due to the
124 crack growth is measured. The stress intensity factor and the crack velocity are
125 expressed as follows (Williams and Evans, 1973):

$$126 \quad K_I = P w_m \sqrt{\frac{3(1+\nu)}{W d^3 d_n}} \quad (1)$$

$$127 \quad \frac{da}{dt} = -\varphi_c \times \frac{W d^3 G S_i P_i}{3 w_m^2 P^2} \frac{dP}{dt} \quad (2)$$

128 where P is the applied load, w_m is the moment arm (18 mm in this study), ν is Poisson's
129 ratio, W is the width of the specimen, d is the thickness of the specimen, d_n is the

130 reduced thickness of the specimen, P_i is the initial value of the applied load, S_i is the
131 compliance of the specimen at the initial crack length, dP/dt is the load relaxation rate,
132 and G is the shear modulus. φ_c is a constant that is dependent on the shape of the crack
133 front. We set the value of φ_c as 0.4 from observation of the crack front following Sano
134 (1988). Evans (1972, 1973), Williams and Evans (1973), and Meredith (1983)
135 concluded that the Mode-I stress intensity factor is evaluated from the DT test based on
136 their experimental results using polycrystalline materials including rocks. As shown in
137 Eq. (1), the stress intensity factor is independent of the crack length in the DT test.
138 Because it is impossible to detect the crack tip in rock specimens, various researchers
139 have employed DT tests to investigate subcritical crack growth (Atkinson, 1984;
140 Atkinson and Meredith, 1987). For similar reasons, we employed the DT test in this
141 study.

142 The size of the DT specimen must satisfy the following condition (Atkinson, 1979;
143 Pletka et al., 1979):

$$144 \quad 12d \leq W \leq L/2 \quad (3)$$

145 where L is the length of the specimen. Ciccotti et al. (2000b) performed a finite element
146 analysis to demonstrate the option of using specimens thicker than those recommended
147 by Atkinson (1979). Previous studies (Shyam and Lara-Curzio, 2006; Madjoubi et al.,
148 2007) recommended that the length of the specimen should be greater than twice the
149 width.

150 Taking the above studies into consideration, we set the width W , length L , and
151 thickness d to be 45, 140, and 3 mm, respectively. The width and depth of the guide
152 groove were both 1 mm.

153

154 3.2 Experimental apparatus and conditions

155

156 The experimental apparatus used in this study was the same as that used by Nara and
157 Kaneko (2005, 2006) in air, and Nara et al. (2009) in water. The apparatus was set in a
158 room where the temperature and relative humidity could be controlled over ranges of
159 278–353 K, and 40–90%, respectively.

160 The subcritical crack growth in air was measured under different temperatures at
161 fixed relative humidity and vice-versa to investigate the influences of temperature and
162 relative humidity separately. We performed experiments under low temperature (293 K,
163 47–50%), intermediate temperature (323–324 K, 50%), and high temperature (351 K,
164 50%) conditions to investigate the influence of temperature; and under low humidity
165 (323 K, 5–7%), intermediate humidity (323–324 K, 50%), and high humidity (323–324
166 K, 89–92 %) conditions to investigate the influence of relative humidity. We could only
167 control the temperature of the air in the apparatus. Therefore, the measurements in water
168 were made at slightly different temperatures than those in air. The subcritical crack
169 growth was measured at low temperature (290–291 K), and at intermediate temperature
170 (313–319 K) in water.

171

172 3.3 Experimental procedure

173

174 Initially, precracking was performed. We applied a displacement of 4 μm at the
175 loading point, and then maintained the displacement at this distance to observe the
176 surface of the specimen, with a digital microscope set under the specimen to determine
177 the crack length. We continued these operations until the crack length reached 25 mm,
178 which is the minimum length required so that the stress intensity factor is independent
179 of the crack length for a DT specimen (Trantina, 1977).

180 After precracking, the specimen was exposed to the experimental environment at a
181 constant temperature and relative humidity for more than 20 hours. Then, measurements
182 following the RLX method were made.

183 For rock, hysteresis in the K_I - da/dt relation has been reported during measurements
184 using the RLX method several times with a single specimen. Sano (1988) suggested that
185 the source of this hysteresis was friction and locking on the crack surfaces, because no
186 hysteresis was observed on soda-lime glass. When measurements using the RLX
187 method are performed, it is essential to minimise the influence of friction and locking.
188 Hence, it is important to open the crack as much as possible, not to repeat opening and
189 closing of the crack, and to perform only a single experimental run with one specimen
190 to avoid the significant influence of friction and locking.

191 Based on the above considerations, after applying a preload of 14–15 N (27–30% of
192 the maximum load) to avoid hitting the specimen, we applied displacements of 0.300
193 mm and 0.332 mm at the loading points of CM and MM specimens, respectively,
194 because the DT specimens completely broke if we increased the displacement.

195 For the measurements in water, we used the distilled water in which the marble
196 samples had been stored for more than 1 month. The pH of the water was measured just
197 after measuring the subcritical crack growth. Figure 4 shows a photo of the
198 experimental apparatus and a pH measurement.

199 **4. Results**

200

201 We performed several experiments under the same conditions to verify
202 reproducibility. Figure 5 shows the temporal changes of the applied load, which are
203 known as load-relaxation curves. Using the relations shown in Figure 5, we estimated
204 the crack velocity and the stress intensity factor (Williams and Evans, 1973).

205 The crack velocity da/dt is empirically related to the stress intensity factor K_I , as
206 follows (Charles, 1958; Wiederhorn and Bolz, 1970):

207
$$\frac{da}{dt} = AK_I^n \quad (4)$$

208
$$\frac{da}{dt} = v_0 \exp\left(\frac{-E^\ddagger + bK_I}{RT}\right) \quad (5)$$

209 where E^\ddagger is the stress-free activation energy, R is the gas constant, T is the absolute
210 temperature, and the other variables are experimentally-determined constants. n is the
211 subcritical crack growth index (Atkinson, 1984). We used these equations to summarize
212 the experimental results. In particular, we used the following equation of the
213 exponential law:

214
$$\left. \begin{aligned} \ln\left(\frac{da}{dt}\right) &= \alpha + \frac{b}{RT} K_I \\ \because \alpha &= \ln v_0 - \frac{E^\ddagger}{RT} \end{aligned} \right\} \quad (5')$$

215 The K_I - da/dt relation for CM and MM at constant temperature in air with different
216 relative humidities, and in water, are shown in Figure 6. The crack velocity increased
217 with increasing relative humidity in air. In addition, the crack velocity in water was
218 significantly higher than that in air. For CM in air, the slope was smaller when the crack
219 velocity was $> 10^{-4}$ m/s.

220 Tables 1 and 2 summarize the results from the subcritical crack growth measurements

221 for CM and MM in air and water under different relative humidities, respectively. The
222 stress intensity factor for $da/dt = 10^{-5}$ [m/s], $K_I(10^{-5})$, is listed to quantitatively compare
223 the values because the range in the crack velocity is 10^{-2} – 10^{-8} m/s. Table 1 lists the
224 values of the crack velocity at $K_I = 1.0$ [MN/m^{3/2}], $da/dt(1.0)$. In the case of CM in air,
225 the crack velocity values $< 10^{-4}$ m/s were analyzed. The values of the average and
226 standard deviation were estimated from several experimental results performed under
227 the same conditions. The crack velocity in air increases as the relative humidity
228 increases. Additionally, the crack velocity in water is significantly higher than that in air.

229 Figure 7 shows the K_I - da/dt relation at different temperatures in air at a constant
230 relative humidity, and in water for CM and MM. The crack velocity increases with
231 increasing temperature. In Figure 7a, the slope is small when the crack velocity is $> 10^{-4}$
232 m/s for CM in air. This decrease in the slope is remarkable, which implies Region II
233 subcritical crack growth (Lawn, 1974, 1993).

234 Tables 3 and 4 show the subcritical crack growth measurements for CM and MM,
235 respectively, at different temperatures, in air under a constant relative humidity and in
236 water. In the case of CM in air, the values of crack velocity $< 10^{-4}$ m/s were analyzed.
237 As the temperature increases, the crack velocity increases, while the stress intensity
238 factor decreases in both air and water. The crack velocity in water is higher than that in
239 air.

240 **5. Discussion**

241

242 Water significantly affects subcritical crack growth in marble. Specifically, the
243 increase in the crack velocity in water is remarkable.

244 If the fracture toughness of marble is obtained, we can verify the range of the stress
245 intensity factor where subcritical crack growth occurs. Therefore, determining fracture
246 toughness is important. Several researchers have measured the fracture toughness using
247 DT tests (Atkinson, 1979; Meredith, 1983; Meredith and Atkinson, 1985; Nara, 2015;
248 Nara et al., 2012). Since fracture toughness depends on the environmental conditions
249 such as temperature (Meredith and Atkinson, 1985; Funatsu et al., 2004, 2014), relative
250 humidity (Nara et al., 2012), water vapor pressure (Kataoka et al., 2015), and water
251 (Vavro and Souček, 2013), it is necessary to conduct fracture toughness measurements
252 under controlled environmental conditions. Consequently, we measured the fracture
253 toughness of marble under controlled temperatures and relative humidities~~y~~ using the
254 constant displacement rate method of DT test (Evans, 1972) with the same methodology
255 as Nara et al. (2012) (see Appendix).

256 Table 5 summarizes the results of the fracture toughness measurements in the marble
257 samples in air while controlling the temperature and relative humidity. For MM, the
258 fracture toughness was measured in air at different temperatures with the same relative
259 humidity. The fracture toughness of MM decreases as the temperature increases. This
260 tendency agrees well with that for the Mode-I fracture toughness of Kimachi sandstone
261 reported by Funatsu et al. (2004, 2014).

262 Table 5 also summarizes the results of the fracture toughness measurements for CM
263 in air (293K, 47%). The results of CM and MM were used to check the range of the
264 stress intensity factor in which subcritical crack growth occurs. Figure 8 shows the

265 relationships between the crack velocity and the stress intensity factor normalized by the
266 fracture toughness determined under the same environmental conditions. The subcritical
267 crack growth of marble occurs in the same range as the normalized stress intensity
268 factor, which is higher than 74% of the fracture toughness in this study. If the values of
269 the subcritical crack growth index n (in Eq. (4)) and constant b (in Eq. (5)) increase, the
270 range of the stress intensity factor causing subcritical crack growth decreases. This can
271 contribute to the long-term stability of rock (Nara et al., 2013).

272 It is important to compare the results in this study to previous studies. Henry et al.
273 (1977) showed that the crack velocity in micrite in water is higher than that in air, and
274 the slope (the value of subcritical crack growth index) in water is lower than that in air.
275 These tendencies agree well with the results in this study. On the other hand, according
276 to the result of calcite crystal by Bergsaker et al. (2016), the difference between the
277 crack velocity in distilled water and that in air is unclear. It is considered that the extent
278 of the influence of water on subcritical crack growth in rock completely differs from a
279 single crystal. Røyne et al. (2011) showed the results of subcritical crack growth
280 measurement on calcite single crystals. They reported that the variation in the crack
281 velocity at a given energy release rate might be caused by the plastic process or other
282 unclarified process.

283 Figure 9 depicts the relationships between the crack velocity and the stress intensity
284 factor for CM, MM, and calcite crystal (Røyne et al., 2011; Rostom et al., 2012). Since
285 the measured values of the stress intensity factor differ significantly between the
286 marbles in this study and the calcite crystals (Røyne et al., 2011; Rostom et al., 2012),
287 the stress intensity factors in Figure 9 are normalized by the fracture toughness. For CM
288 and MM, the values obtained in air at the same temperature are used (see Table 5). The
289 fracture toughness of the calcite crystal is $0.22 \text{ MN/m}^{3/2}$ (Chen et al., 2001; Shushakova

290 et al., 2013). The shape of the relationship for the calcite crystal is nonlinear, although
291 the shape for marbles is linear. The nonlinearity for the calcite crystal can be recognized
292 in different environmental conditions (Røyne et al., 2011; Rostom et al., 2012;
293 Bergsaker et al., 2016). It is considered that the mechanism controlling subcritical crack
294 growth in marble differs from that in calcite crystal. To understand why the mechanism
295 controlling subcritical crack growth differs, the crack path must be observed.

296 Figure 10 shows images of the crack paths in CM (Figure 10a) and MM (Figure 10b)
297 from thin sections (0.03mm thickness) observed using an optical microscope under an
298 open nicol, because this allowed the most clear crack path to be observed. On the other
299 hand, the crack path could not be observed clearly under crossed nicols. To prepare a
300 thin section, first an epoxy resin was permeated into the cracks. Then thin sections were
301 prepared to observe the tension plane (the lower plane in Figure 3). In Figure 10, green,
302 which corresponds to the epoxy resin, indicates the crack path. The crack path is mainly
303 inter-granular, and trans-granular crack paths are quite rare. Sano (1981) suggested that
304 microcracking occurs ahead of the crack tip in DT specimens of granite using the source
305 location of the acoustic emission. Swanson (1984) suggested that the fracture process
306 zone is generated ahead of the crack tip in DT specimen of rock. Nasser et al. (2007)
307 suggested that the fracture process zone was generated ahead of the crack tip and the
308 crack grows by connecting smaller microcracks in the fracture process zone in granite.

309 It is considered that the fracture process zone is generated in marble. Considering that
310 the inter-granular crack path is dominant for marble, microcracking in the fracture
311 process zone occurs at the boundary of mineral grains mainly. Thus, the crack growth in
312 the grain boundary is more important in marble than the crack growth within a mineral
313 grain. In particular, it is suggested that fracturing in cement materials at a grain
314 boundary is important for subcritical crack growth in marble.

315 The crack path in marble is not perfectly planer (Figure 10). Previous studies did not
316 observe planer crack paths for subcritical crack growth in rocks (Meredith and Atkinson,
317 1983; Swanson, 1985, 1987; Kudo et al., 1992; Nara and Kaneko, 2005; Nara et al.,
318 2006, 2011). In this study, a rectangular guide groove is put on the specimen (Figure 3),
319 generating a zigzag path. Nara and Kaneko (2005) tried to constrain the crack path to a
320 straight line by preparing a semi-circular guide groove and a tri-angular guide groove on
321 the DT specimen; they demonstrated that straight crack paths are not generated.
322 Consequently, the larger scattering is obtained due to the crack propagation away from
323 the bottom of the semi-circular and triangular grooves, where the crack thickness is not
324 constant. Considering this result, the rectangular guide groove has been used to measure
325 subcritical crack growth (e.g., Nara and Kaneko, 2006; Nara et al., 2011, 2013) and
326 fracture toughness (e.g., Nara et al., 2012).

327 The results in this study are also compared to the results in a previous study of
328 subcritical crack growth in igneous rocks shown in Nara et al. (2013). In igneous rocks,
329 the crack velocity in distilled water was higher than that in air at 50% relative humidity
330 by two to four orders of magnitude (Nara et al., 2013). The increase in the crack
331 velocity in marble was greater than that recorded in igneous rock. As shown in Figure 2,
332 MM contained illite. Francisca et al. (2005) reported that the strength of sediment
333 containing illite decreased with increasing water content. Nara et al. (2011) showed that
334 the crack velocity increased in rock containing illite with increasing water content. The
335 results of these two studies agreed with those of this study. Thus, the presence of clay in
336 rock increases the crack velocity in water.

337 Temperature affected the increase in the crack velocity in marble. Weakening
338 processes, i.e. stress corrosion, can occur at the crack tip under tension (Anderson and
339 Grew, 1977). In the case of silicate materials, it has been suggested that weakening of

340 siloxane bonds via chemical reaction with water occurs at the crack tip under tension
341 (Michalske and Freiman, 1982). However, alternative mechanisms should also be
342 considered for carbonate rocks.

343 Additionally, other chemical reactions between a carbonate mineral and water should
344 be considered. Various researchers have studied the dissolution kinetics of calcite (e.g.,
345 Garrels and Christ, 1965; Sjöberg and Rickard, 1984; MacInnis and Brantley, 1992;
346 Liang et al., 1996; Shiraki et al., 2000). The solubility of many carbonate minerals in
347 water decreases as the temperature increases (Garrels and Christ, 1965). This property
348 may suppress subcritical crack growth in carbonate materials. However, Nara et al.
349 (2010) suggested that the electric double layer formed around the crack tip affects the
350 acceleration of subcritical crack growth. Since the aperture of the crack close to the
351 crack tip is very small, it is possible that the water vapor turns to liquid water by
352 capillary condensation in this zone. Consequently, the crack path close to the crack tip is
353 immersed in liquid water (Thomson, 1871; Nara et al., 2010; Nakao et al., 2016). In this
354 case, an electric double layer is formed in the condensed water around the crack tip and
355 a repulsive force acts between the crack planes. The electric double layer increases as
356 the temperature increases (Shaw, 1980), causing the repulsive force in the crack tip to
357 increase, which can decrease the activation energy of subcritical crack growth. This
358 effect can reduce the stress intensity factor applied at the crack tip at a given crack
359 velocity.

360 Variations in the slope of the K_I - da/dt relation for CM in air were observed. A similar
361 trend has been observed in soda-lime glass (Wiederhorn, 1967), and in alumina (Evans,
362 1972) in air. Subcritical crack growth is divided into three key regions based on the
363 mechanisms that control the crack velocity. In Region I, the crack velocity is controlled
364 by the rate of stress corrosion. In Region II, the crack velocity is controlled by the rate

365 of transport of the reactive agent to the crack tip (Lawn, 1974, 1993). In Region III, the
366 crack propagation is insensitive to the chemical environment and occurs mechanically
367 (Wiederhorn et al., 1974). This is the classic tri-modal behaviour of the K_I-da/dt relation
368 for subcritical crack growth. Figure 11 shows schematic illustrations of the tri-modal
369 behaviour of the K_I-da/dt relation (Figure 11a) and that obtained for silica glass (Figure
370 11b). The change in the slope can be seen clearly for glass in air.

371 For CM in air, the region with a low slope can be seen clearly. This is likely to
372 represent Region II of subcritical crack growth, in which the mass-transport rate of the
373 reactive agent (in this case, water) controls the crack velocity. Since water is included
374 within the rock materials, Region II is not usually observed in rock in air. According to
375 Nakao et al. (2016), rock materials contain water in cracks and pores of tiny aperture
376 because of the capillary condensation of water vapour. In the case of CM, the porosity is
377 very low (0.19 %), and the water content is also very low, so that the transport rate of
378 the water to the crack tip largely depends on the surrounding environmental conditions,
379 similar to glass and alumina. For this reason, we were able to observe the region in
380 which the transport of the reactive agent (water) controlled the crack velocity for
381 subcritical crack growth in CM in air. In contrast, the porosity of MM is 0.6%, which is
382 higher than that of CM, and Region II crack growth was not observed, even in air.

383 The subcritical crack growth index n in water was lower than that in air for both types
384 of marble. This tendency was similar to that previously observed in igneous rocks (Nara
385 et al., 2013). Lower subcritical crack growth index results in lower long-term strength
386 and shorter time-to-failure (Nara et al., 2013). This implies that long-term integrity will
387 be realized under dry condition if structures are constructed in marble rock masses.

388 The results of this study provide insight on time-dependent fracturing under tension.
389 Therefore, data under confining pressures is necessary to consider the long-term

390 stability in a rock mass in the underground. According to Brantut et al. (2014b),
391 pressure solution can occur in calcite under compression. In addition, data under higher
392 temperature is required. De Bresser et al. (2005) reported that the compressive strength
393 of Carrara marble at a confining pressure of 300 MPa decreases as the temperature
394 increases up to 1000 °C. Even in a compressive stress field such as underground, tensile
395 stress can occur locally. Thus, the information and knowledge under tension as well as
396 those under compression are important to ensure the long-term stability of rock masses.

397 **6. Conclusion**

398

399 Subcritical crack growth in marble was investigated experimentally in both air and
400 water. Specifically, the influence of the surrounding environment on the relationship
401 between the crack velocity and the stress intensity factor was investigated by conducting
402 all measurements under controlled temperature and relative humidity.

403 The crack velocity in water was significantly higher than that in air. The crack
404 velocity increased with increasing relative humidity in air, and with increasing
405 temperature in both air and water. For CM in air, Region II crack growth was observed,
406 which was due to the low porosity of CM.

407 The increasing crack velocity with increasing temperature and humidity implies that
408 dry conditions at low temperature are desirable for the long-term integrity of carbonate
409 rock masses. Additionally, mass transport to the crack tip represents a key process for
410 subcritical crack growth in rock with low porosity.

411

412 **Appendix – Fracture toughness measurement using double torsion test**

413

414 The constant displacement rate method of the DT test (DT-CDR test) was used for the
415 measurement of the fracture toughness in this study. In the DT-CDR test, the
416 displacement rate of the loading points has to be kept constant during the experiment.

417 When this method is used for the fracture toughness measurement, the displacement rate
418 of the loading points should be large (Shyam and Lara-Curzio, 2006); this is especially
419 important for applying this particular method to the calculation of fracture toughness.

420 Selçuk and Atkinson (2000) reported an influence of measured stress intensity factor
421 with loading rate when employing the DT-CDR method, with the use of a high
422 displacement rates (above approximately 0.07 mm/s) mitigating this effect.

423 In the DT-CDR test as a fracture toughness measurement, the maximum value of
424 applied load at failure is used to calculate fracture toughness via the following equation:

$$425 \quad K_{Ic} = P_{\max} w_m \sqrt{\frac{3(1+\nu)}{Wd^3d_n}} \quad (A1)$$

426 where K_{Ic} is Mode-I fracture toughness, and P_{\max} is the maximum value of the applied
427 load.

428 Atkinson (1979), Meredith (1983) and Meredith and Atkinson (1985) applied the
429 DT-CDR method using displacement rates of approximately 20 mm/min (around 0.33
430 mm/s) (Atkinson, 1979; Meredith, 1983) and 10 mm/min (around 0.17 mm/s) (Meredith,
431 1983; Meredith and Atkinson, 1985), respectively. These displacement rates are higher
432 than that mentioned by Selçuk and Atkinson (2000). Meredith (1983) reported that the
433 fracture toughness values obtained by DT-CDR method agreed well with those obtained
434 by the short-rod method which is the standard method for Mode-I fracture toughness
435 measurement of rock (International Society for Rock Mechanics, 1988). Based on these

436 considerations, DT-CDR test in this study was performed using by applying a load at a
437 displacement rate of 0.23 mm/s, which is the maximum rate of our apparatus and higher
438 than the rate suggested by Selçuk and Atkinson (2000).

439 Experimental procedure is same as Nara et al. (2012). Firstly, the rock specimens are
440 pre-cracked in order to introduce a small starting crack as described in Section 3.3. Then
441 the apparatus and specimen were exposed to the environmental condition of interest
442 with the same temperature and relative humidity for approximately 20 hours. Following
443 this period, a fracture toughness measurement with DT-CDR method was performed
444 using by applying a load at a displacement rate of 0.23 mm/s, after applying a small
445 amount of pre-load around 10 N.

446 A photo of the apparatus used in this study is shown in Figure A-1. This apparatus
447 consists of a speed-control (stepping) motor that drives the loading axis moving
448 perpendicular to the DT specimen. This apparatus is housed in a temperature and
449 humidity controlled room.

450 In Figure A-2, a photo of the DT specimen of MM used in the fracture toughness
451 measurement is shown. All DT specimens were completely broken in the fracture
452 toughness measurement. In Figure A-3, examples of the raw data from CM and MM are
453 presented in the form of load vs. time plots. Using the maximum value of the applied
454 load, the fracture toughness can be estimated.

455

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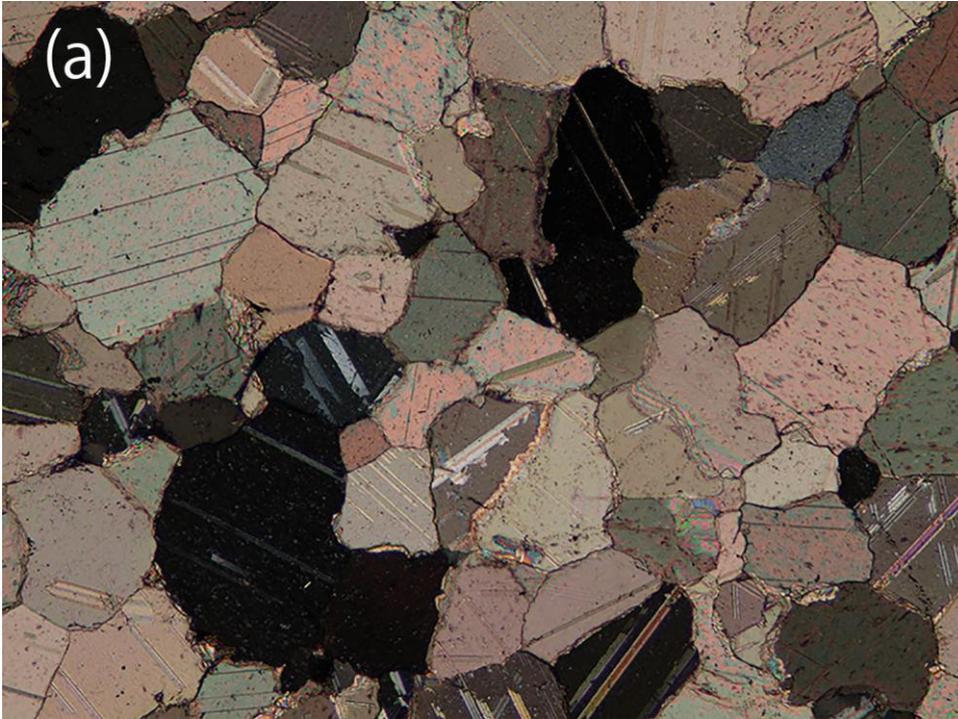
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641 **Figures**

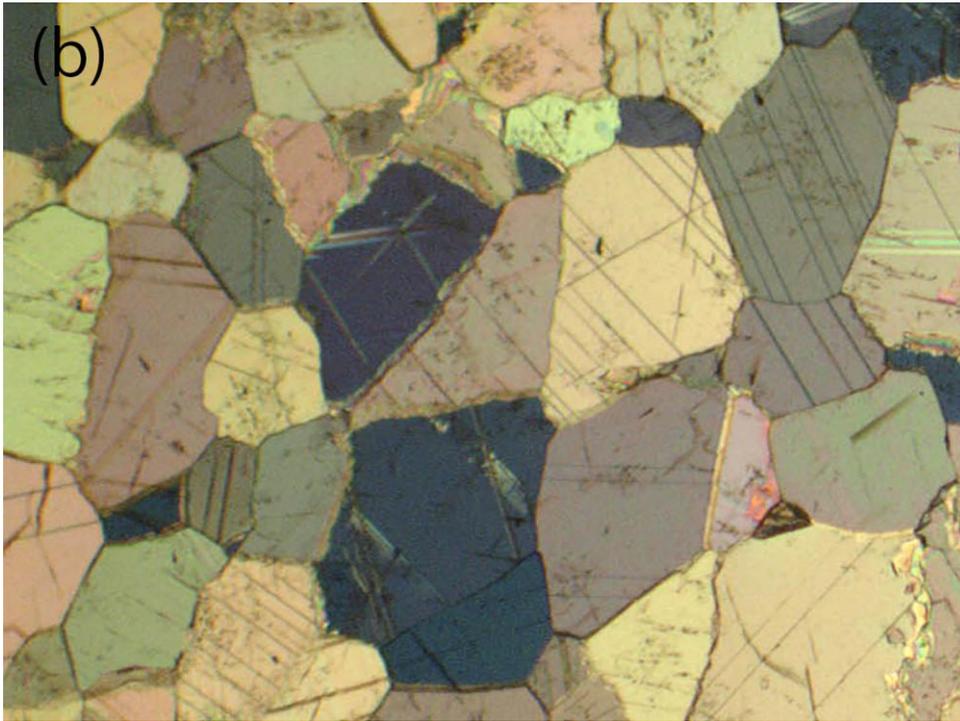
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644 Figure 1. Photomicrographs of (a) Carrara marble (CM) and (b) Macedonian marble
645 (MM) observed with a thin section under crossed nicols. The width and height of the
646 sections are 1.5 mm and 1.1 mm, respectively.

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649 Figure 1. Photomicrographs of (a) Carrara marble (CM) and (b) Macedonian marble

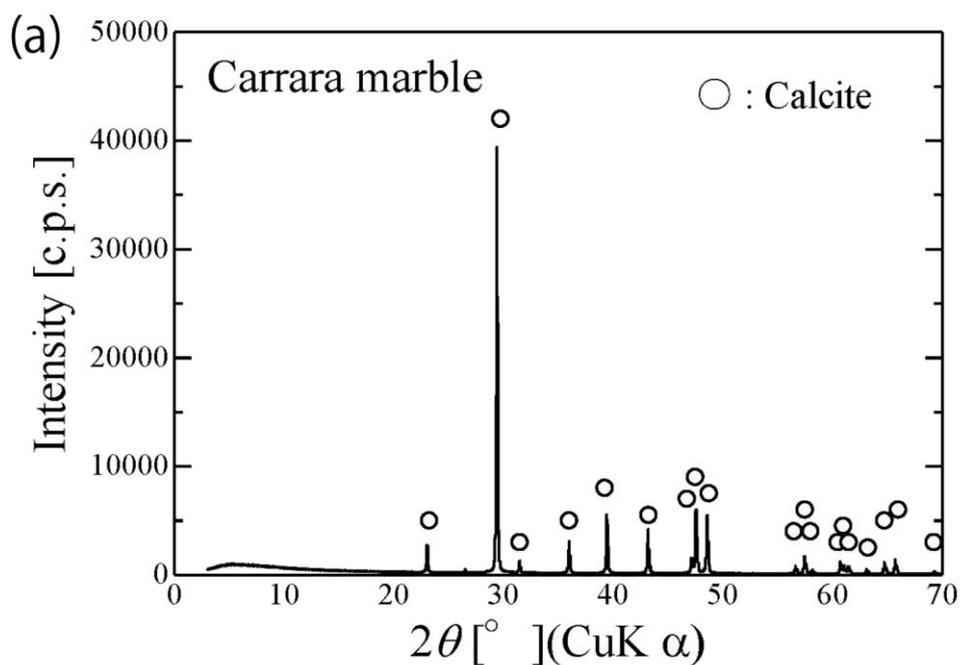
650 (MM) observed with a thin section under crossed nicols. The width and height of the

651 sections are 1.5 mm and 1.1 mm, respectively.

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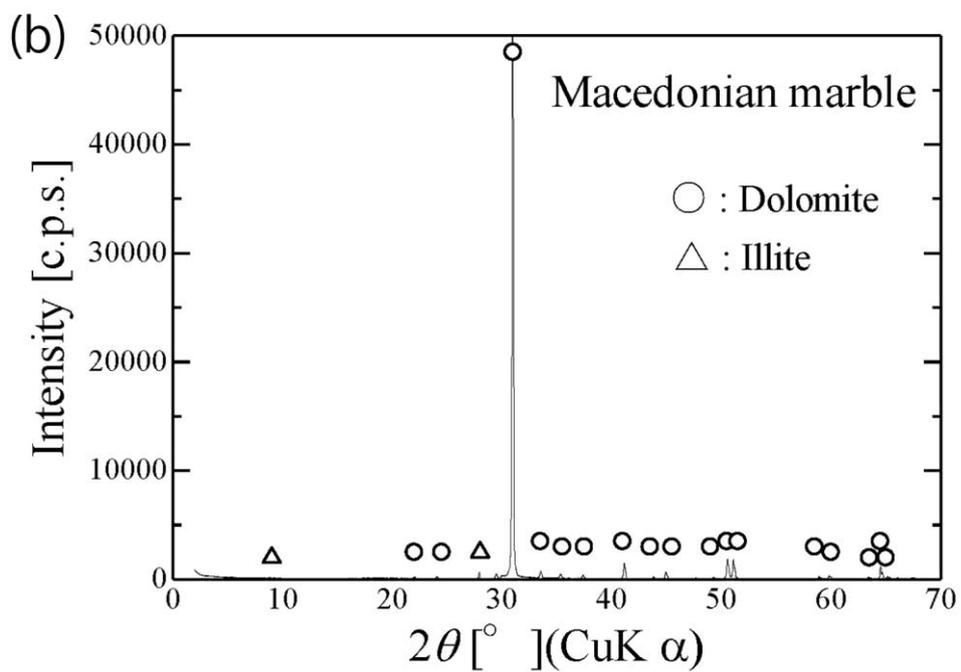
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656 Figure 2. X-ray diffraction patterns for (a) CM and (b) MM.

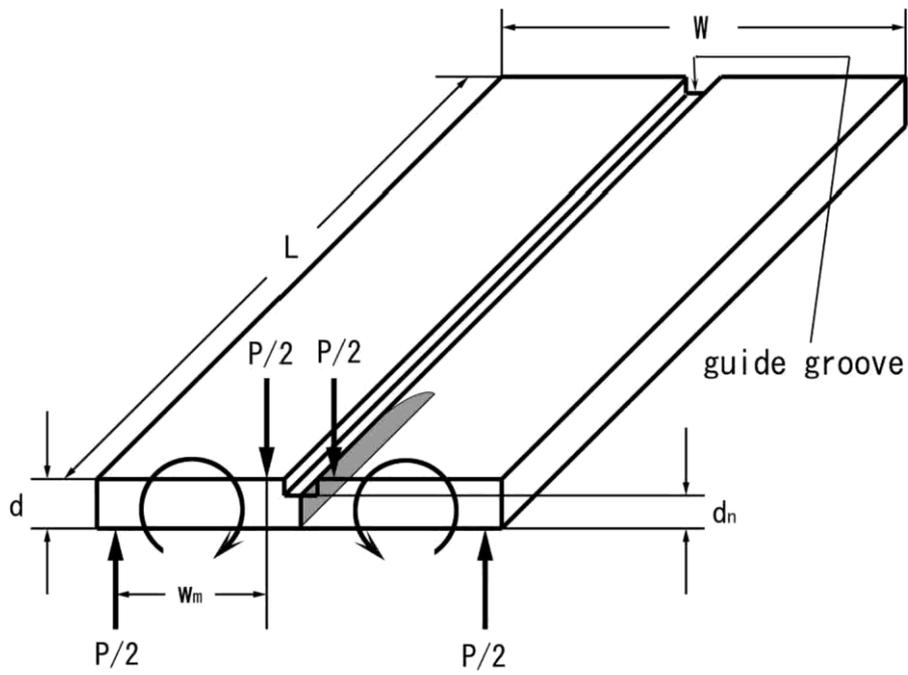
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659 Figure 2. X-ray diffraction patterns for (a) CM and (b) MM.

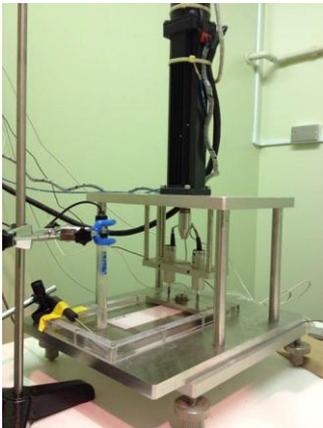
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662 Figure 3. Schematic illustration of double torsion specimen and loading configuration.

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665 Figure 4. Photo of experimental apparatus of DT-RLX test with pH measurement.

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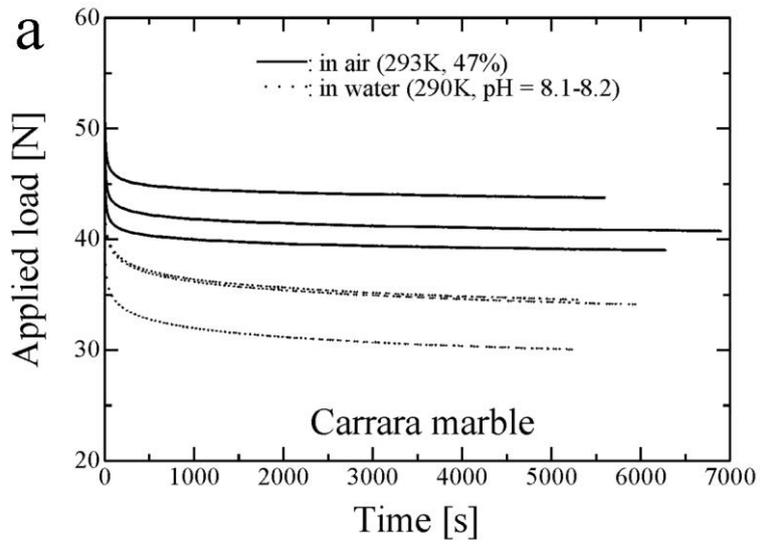
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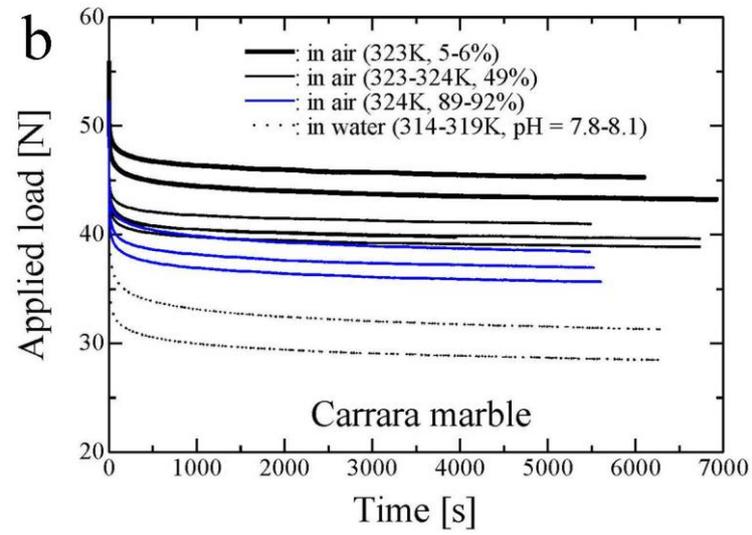
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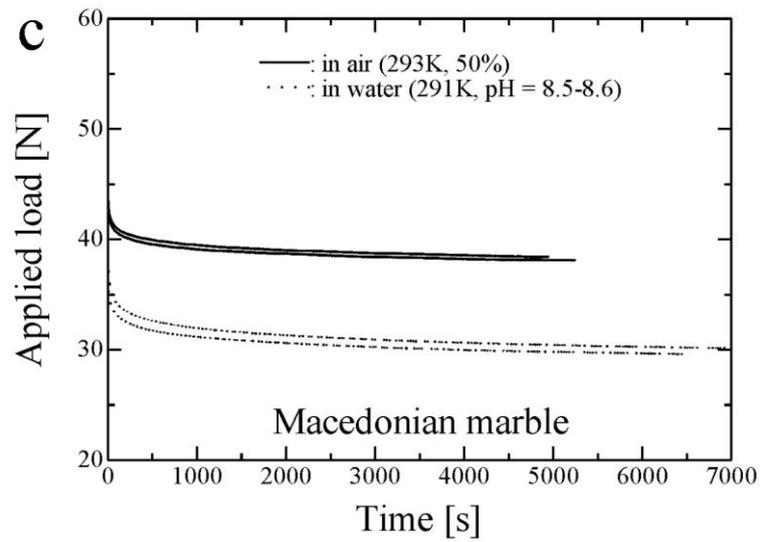
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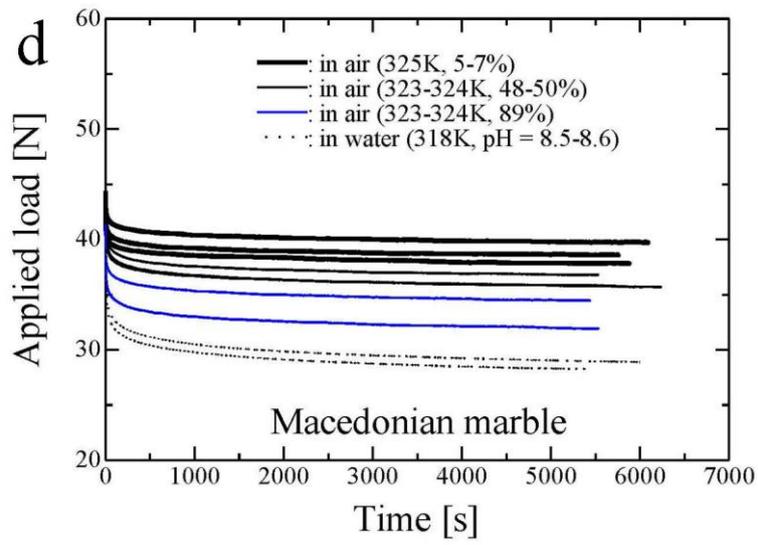
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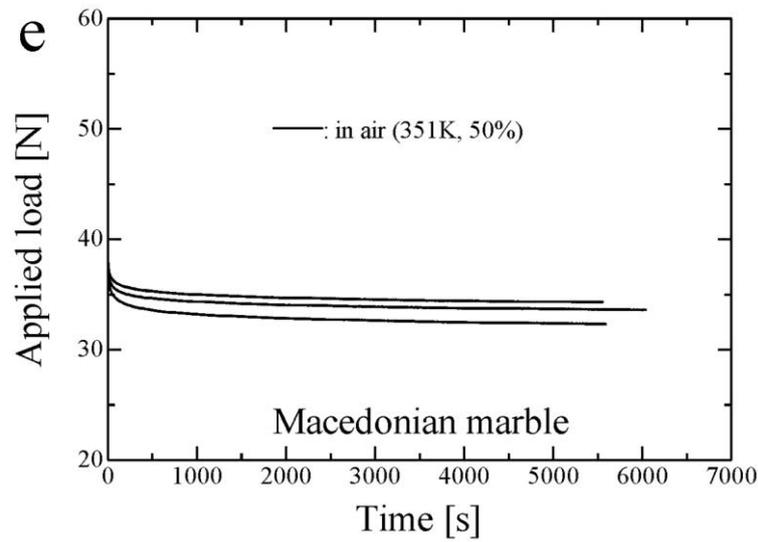
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677 Figure 5. Load-relaxation curves for marbles. (a): CM at 290-293K, and (b): CM at
 678 313-324K, (c): MM at 290-293K, (d): MM at 318-324K, (e): MM at 351K.

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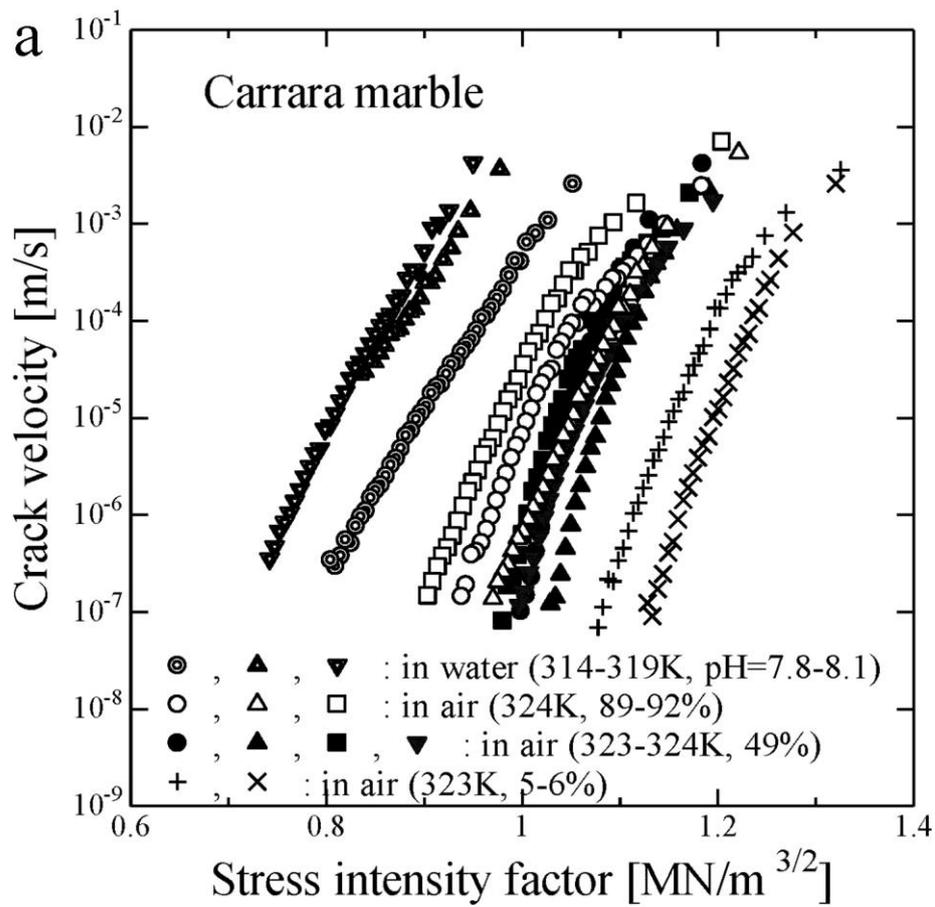
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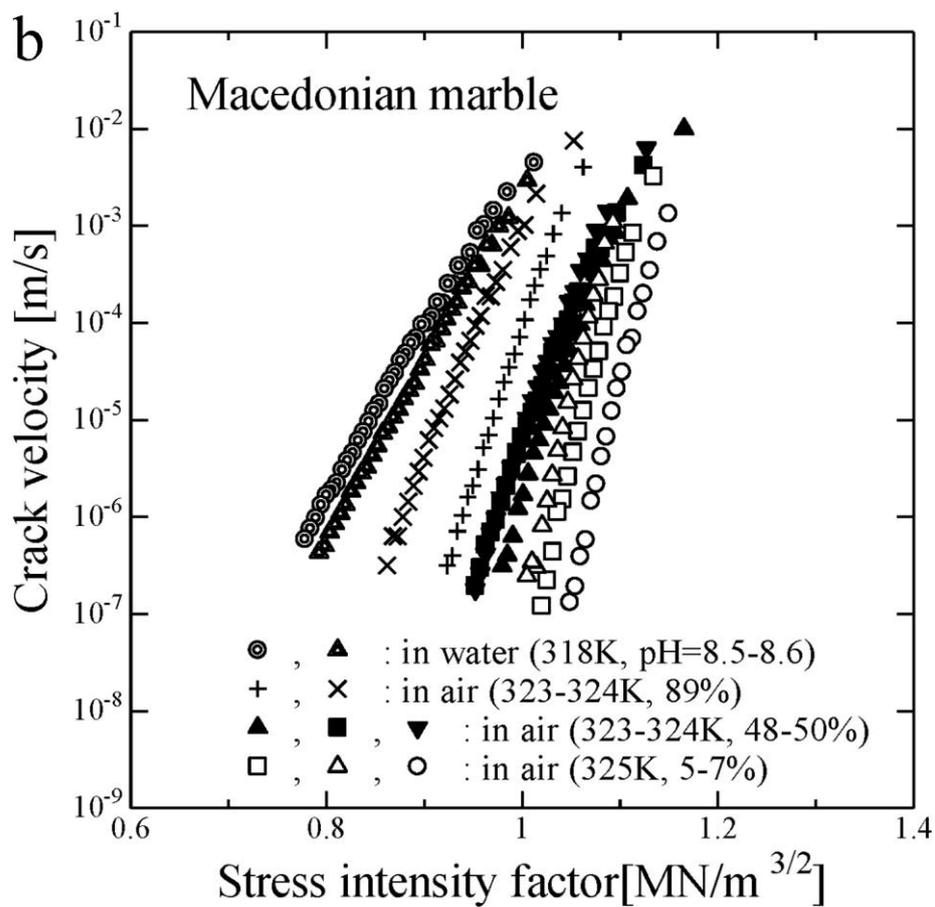


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687 Figure 6. Relationship between crack velocity and stress intensity factor for (a) CM

688 and (b) MM in air with different relative humidities, and in water.

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691 Figure 6. Relationship between crack velocity and stress intensity factor for (a) CM

692 and (b) MM in air with different relative humidities, and in water.

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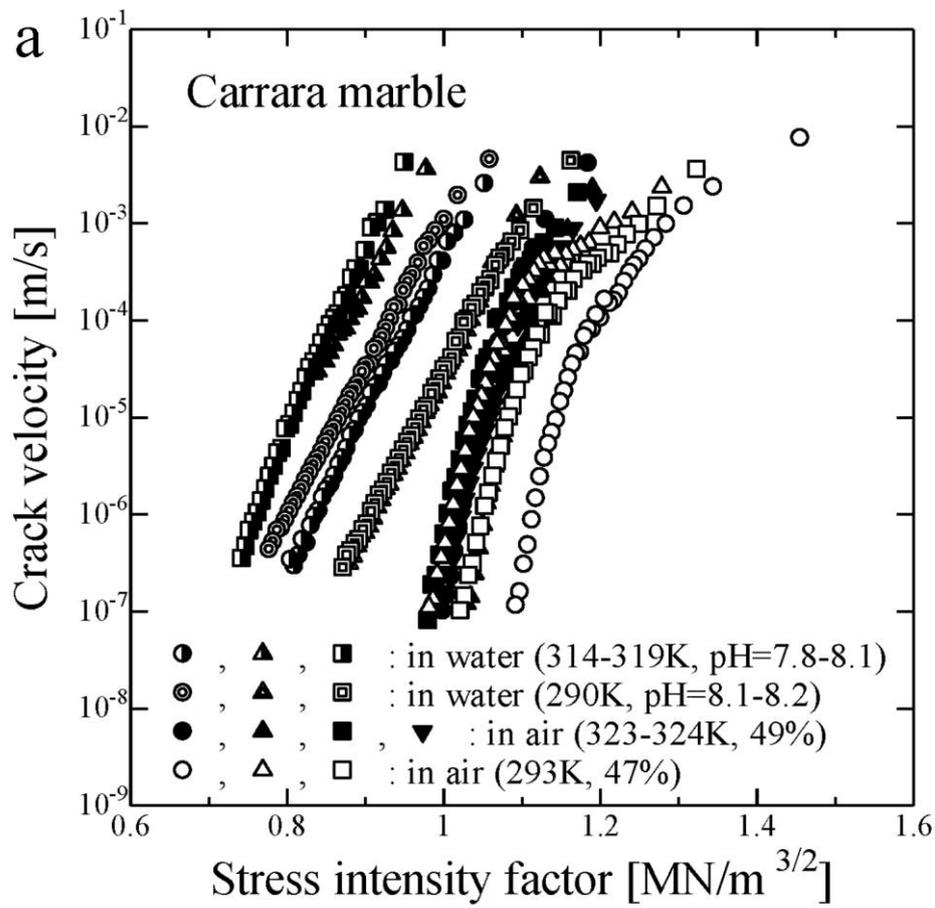
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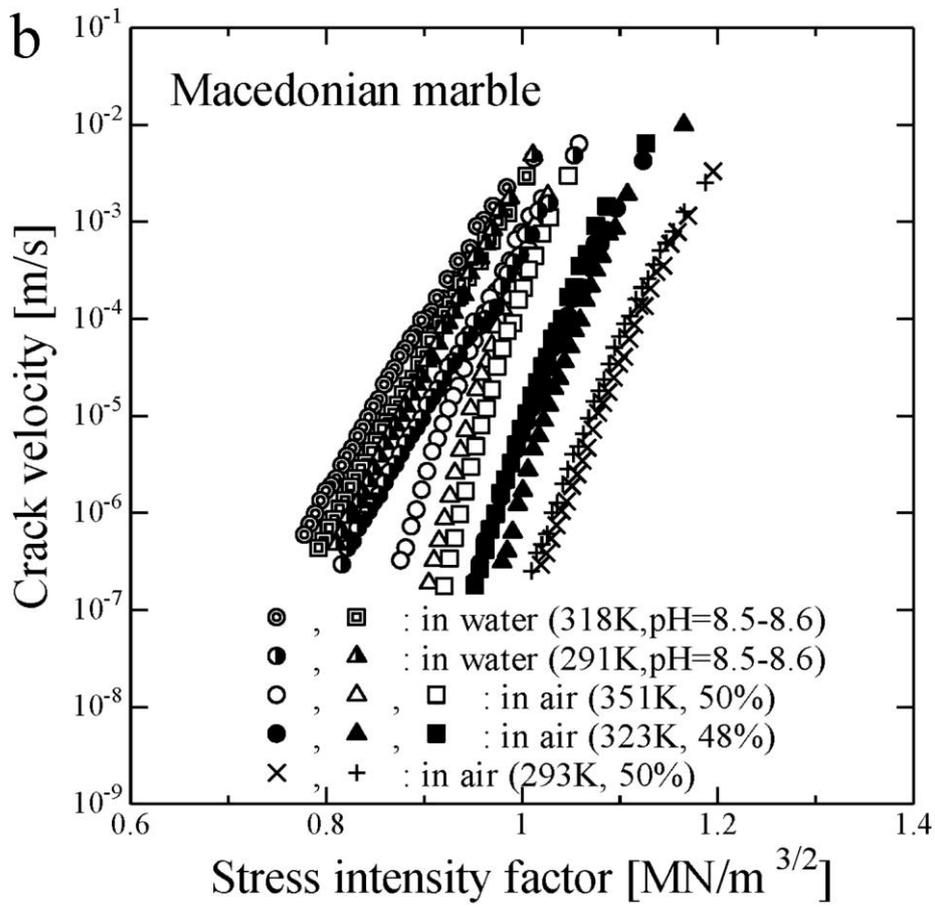


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703 Figure 7. Relationship between crack velocity and stress intensity factor for (a) CM

704 and (b) MM in air and water at different temperatures.

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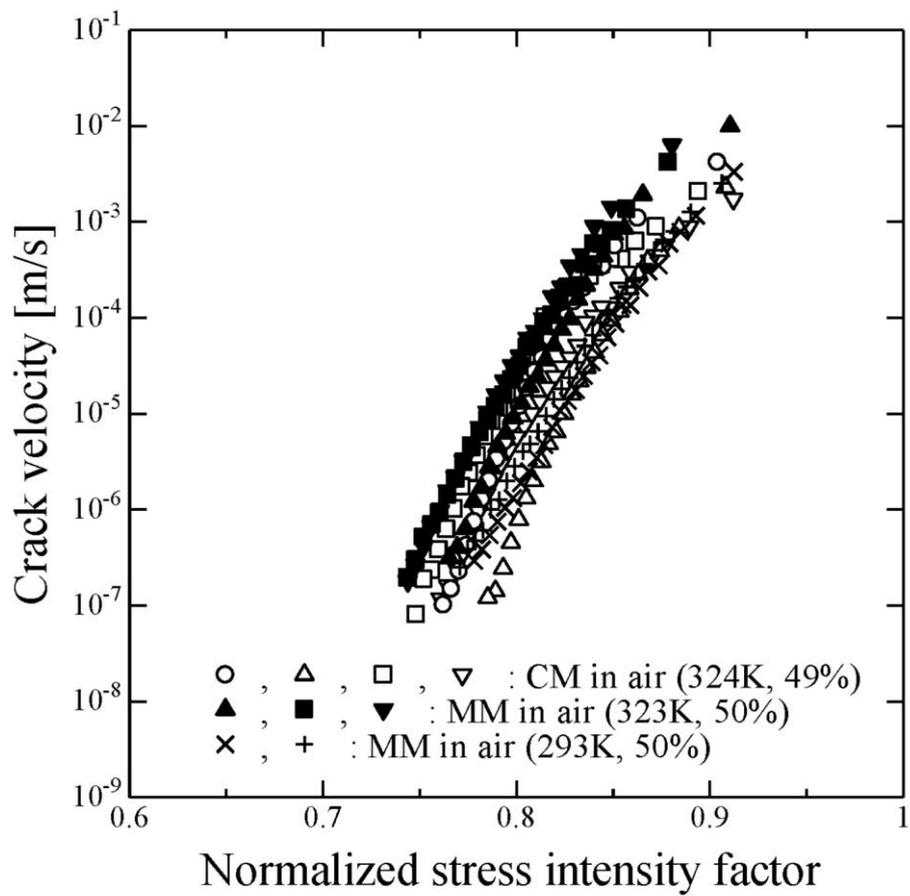
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707 Figure 7. Relationship between crack velocity and stress intensity factor for (a) CM

708 and (b) MM in air and water at different temperatures.

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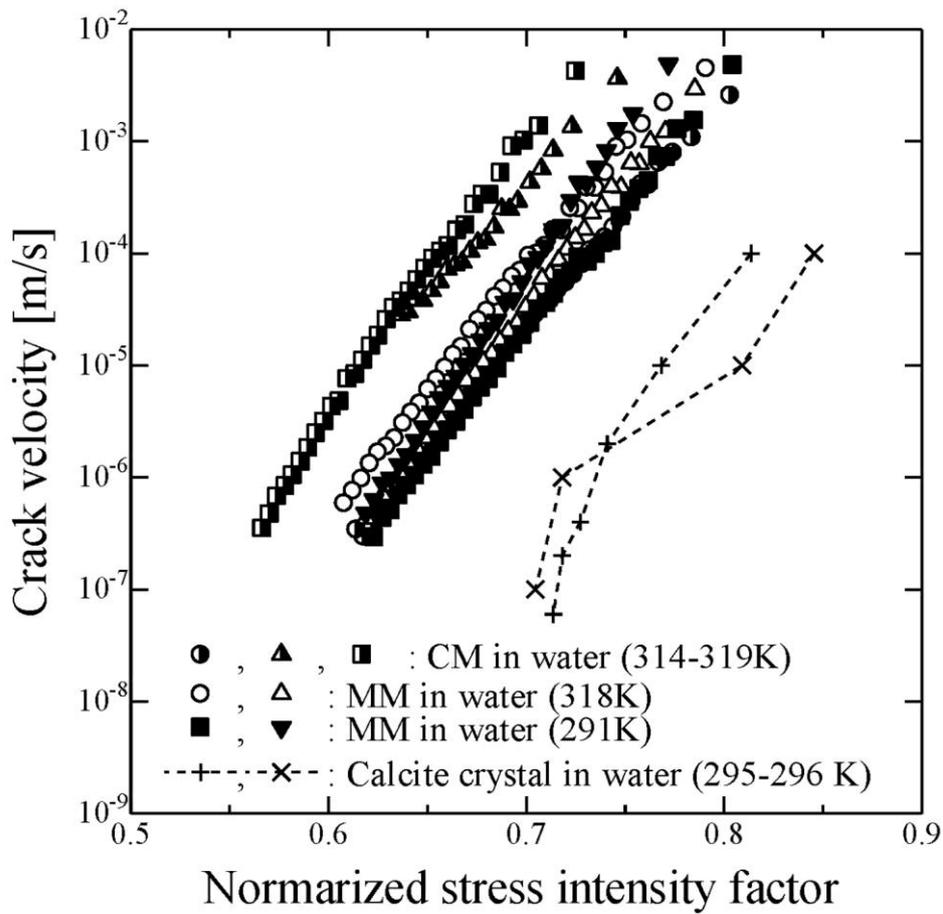


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712 Figure 8. Relationship between crack velocity and stress intensity factor normalized

713 by fracture toughness for marble in air.

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716 Figure 9. Relationship between crack velocity and stress intensity factor normalized

717 by fracture toughness for marble and calcite crystal. “x” and “+” are the

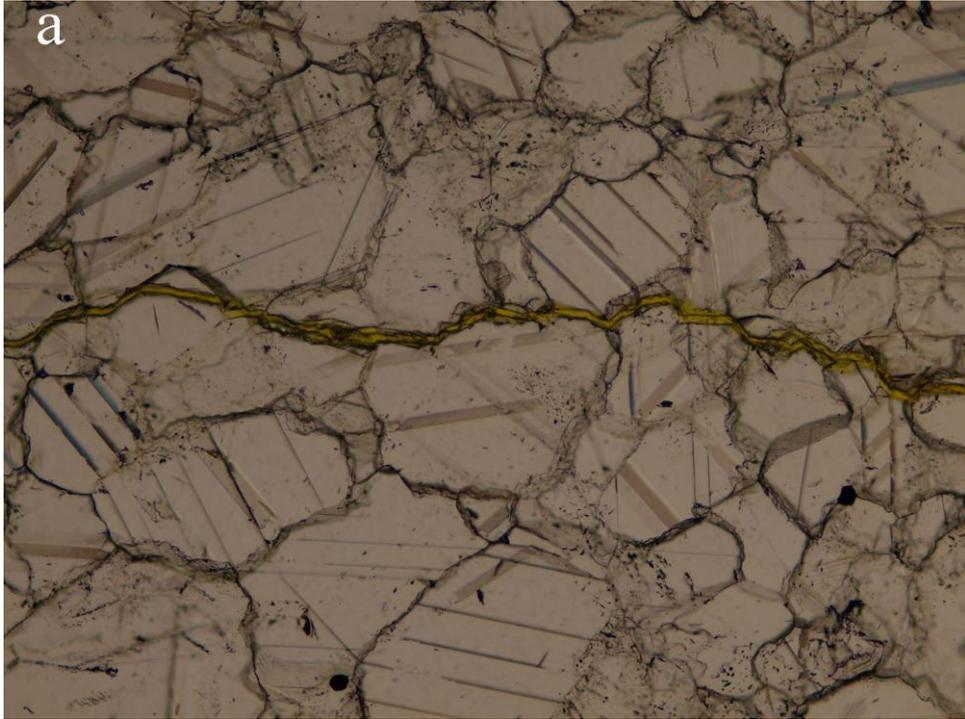
718 relationships obtained by Røyne et al. (2011) and Rostom et al. (2012),

719 respectively.

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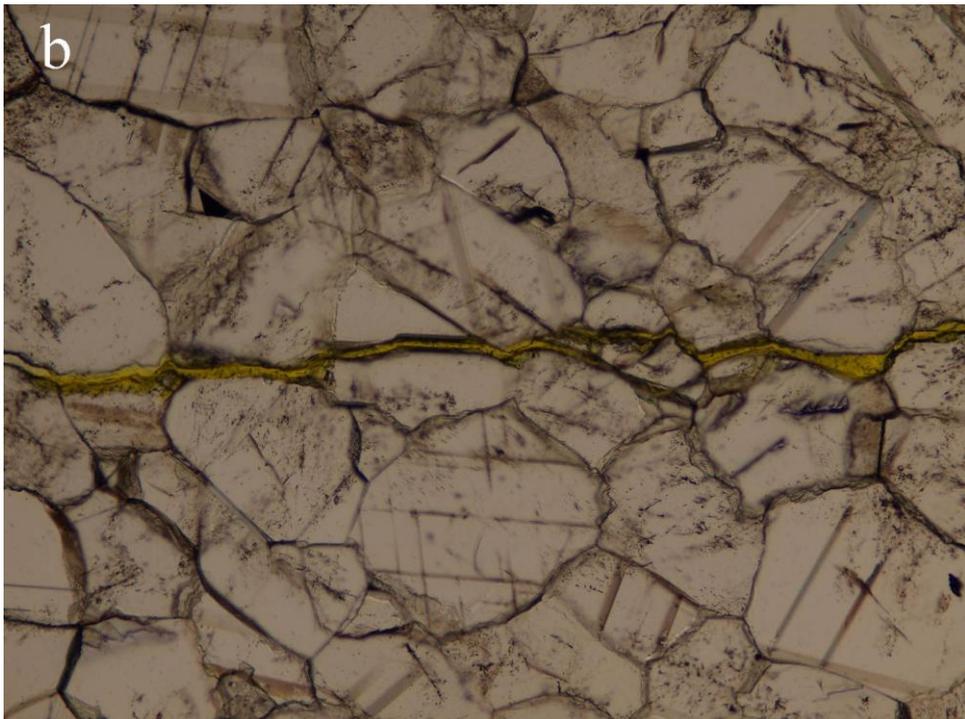
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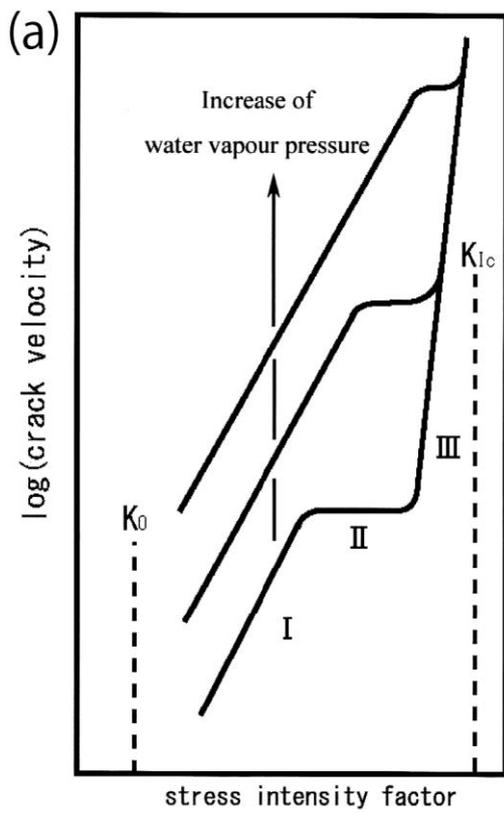
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727 Figure 10. Images of crack path observed for (a) CM and (b) MM. The width and

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height of the images are 1.5 mm and 1.1 mm, respectively.

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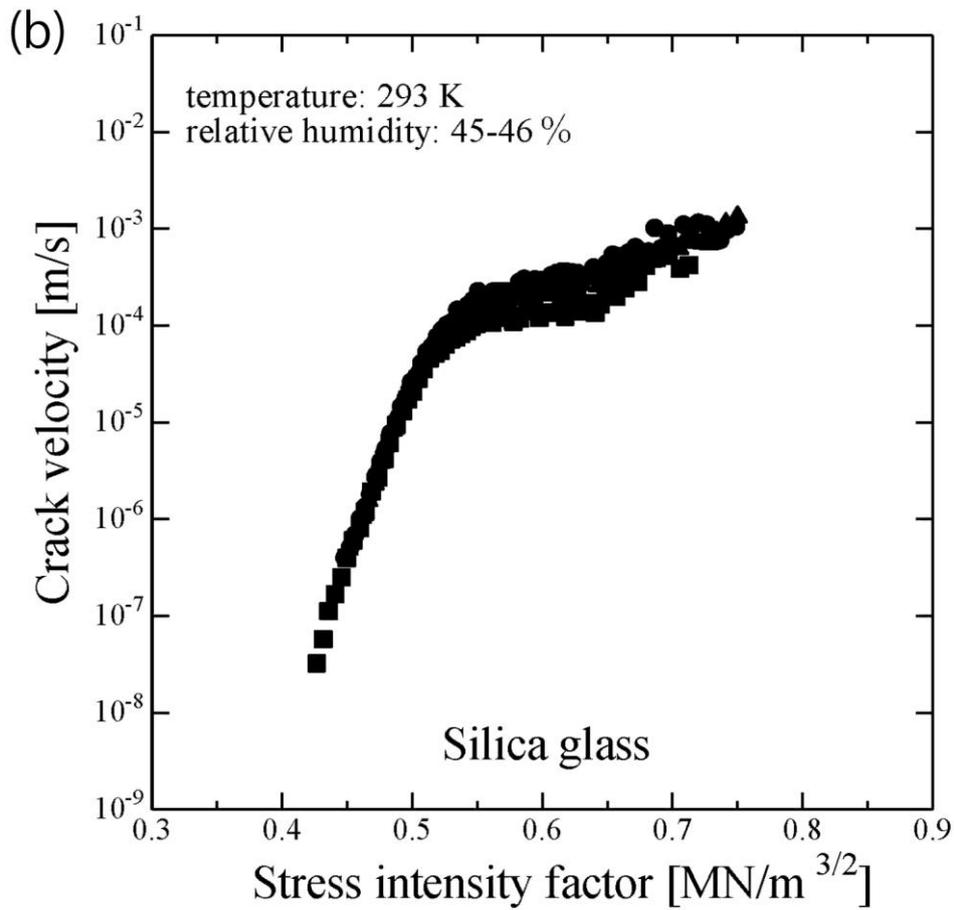
731 Figure 11. Tri-modal behaviour of relationship between crack velocity and stress

732 intensity factor for subcritical crack growth. (a): Schematic illustration, (b):

733 Relationship in silica glass in air.

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737 Figure 11. Tri-modal behaviour of relationship between crack velocity and stress

738 intensity factor for subcritical crack growth. (a): Schematic illustration, (b):

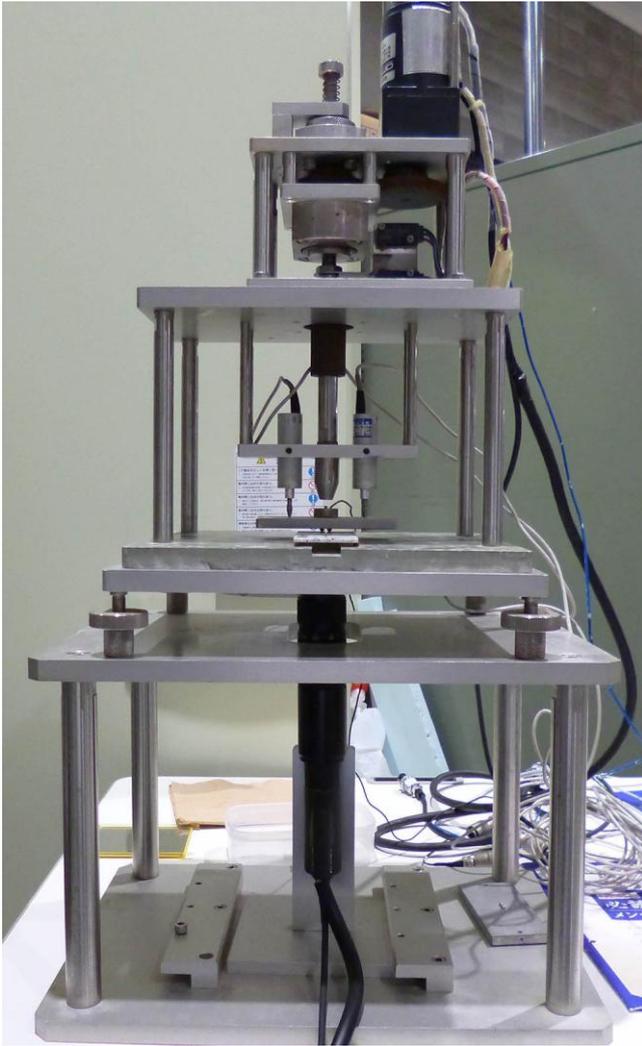
739 Relationship in silica glass in air.

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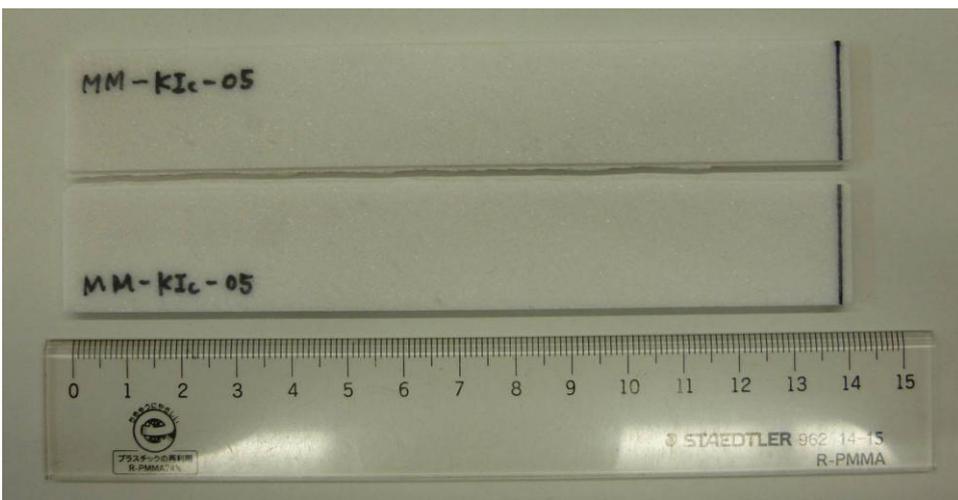
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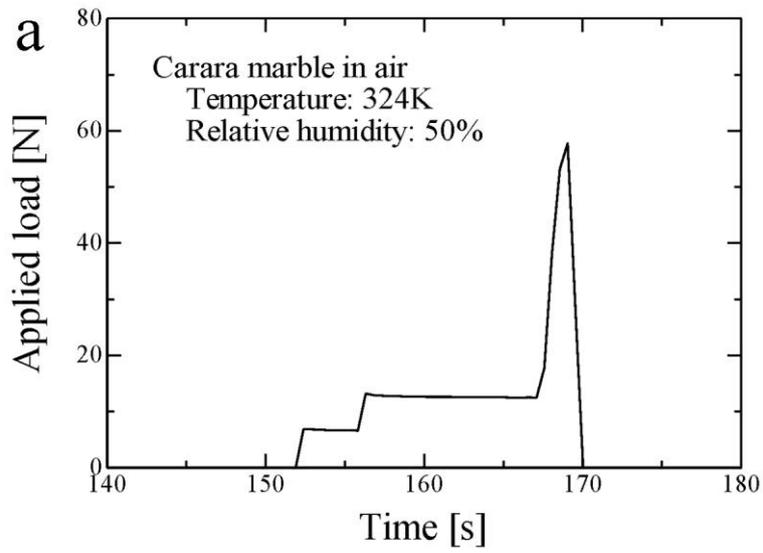
745 Figure A-1. Photo of apparatus of fracture toughness measurement by DT-CDR test.



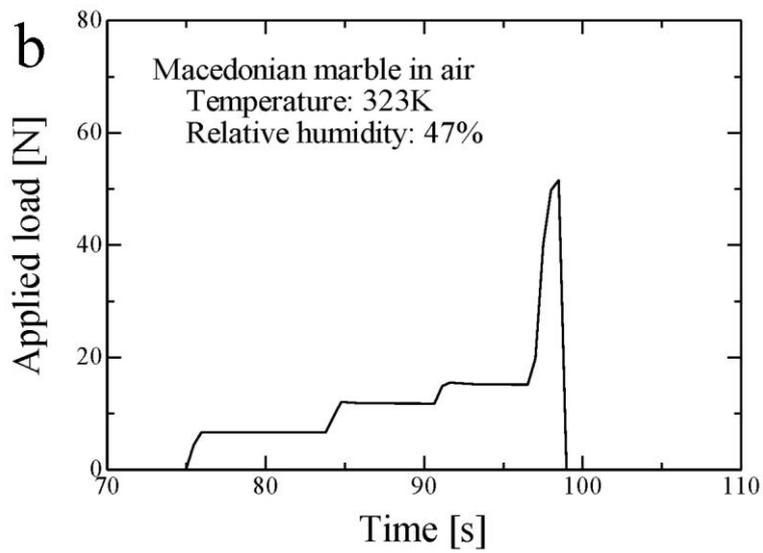
746

747 Figure A-2. Photo of DT specimen of Macedonian marble used in fracture toughness

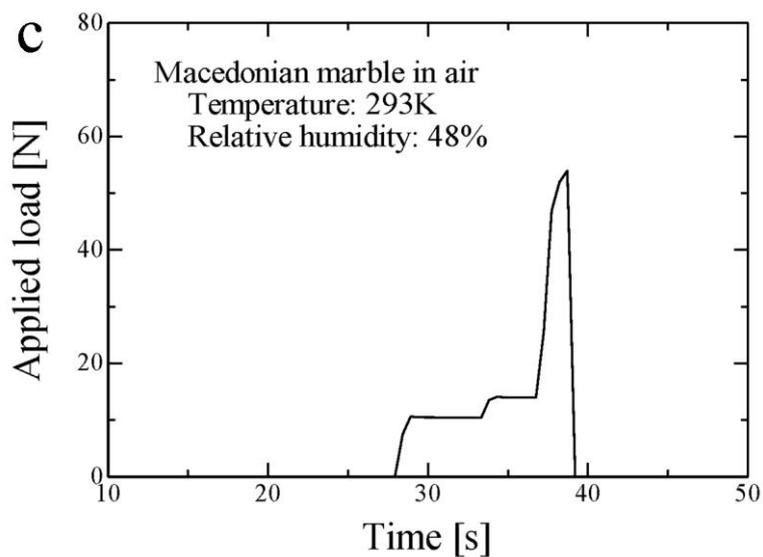
748 measurement



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752 Figure A-3. Temporal change of applied load on DT specimen for fracture toughness

753 measurement. (a): CM in air at 324 K with 50 % relative humidity, (b) MM in air
754 at 323 K with 47% relative humidity, (c): MM in air at 293 K with 48 % relative
755 humidity.

756

757

758 **Tables**

759 **Table 1.** Summary of subcritical crack growth measurements for Carrara marble in air

760 with different relative humidities, and in water.

Condition		n	$\log A$	$K_I(10^{-5})$ [MN/m ^{3/2}] (power law)	$da/dt(1.0)$ [m/s] (power law)	b [m ^{5/2} /mol]	α	$K_I(10^{-5})$ [MN/m ^{3/2}] (exponential law)	$da/dt(1.0)$ [m/s] (exponential law)
air	323K, 5%	69	-9.26	1.15	5.50×10^{-10}	0.132	-68.3	1.15	9.08×10^{-10}
	323K, 6%	71	-11.15	1.20	6.31×10^{-12}	0.179	-91.3	1.20	1.95×10^{-11}
	average	70 ±1	-10.21 ±1.33	1.18 ±0.04	ave: 6.17×10^{-11} std: 2.14×10^1 (in log)	0.156 ±0.033	-79.8 ±16.3	1.18 ±0.04	ave: 1.33×10^{-10} std: 1.51×10^1 (in log)
	324K, 49%	92	-6.91	1.05	1.23×10^{-7}	0.239	-104.6	1.05	1.41×10^{-7}
	324K, 49%	88	-7.97	1.08	1.07×10^{-8}	0.221	-100.1	1.08	1.48×10^{-8}
	324K, 49%	80	-6.23	1.04	5.89×10^{-7}	0.211	-92.7	1.04	6.25×10^{-7}
	323K, 49%	69	-6.73	1.06	1.86×10^{-7}	0.177	-81.4	1.06	1.93×10^{-7}
	average	82 ±10	-6.96 ±0.73	1.06 ±0.02	ave: 1.10×10^{-7} std: 5.37×10^0 (in log)	0.212 ±0.026	-94.7 ±10.1	1.06 ±0.02	ave: 1.26×10^{-7} std: 4.79×10^0 (in log)
	324K, 89%	56	-5.19	1.01	6.46×10^{-6}	0.152	-68.6	1.01	6.21×10^{-6}
	324K, 92%	54	-6.15	1.05	7.08×10^{-7}	0.142	-66.9	1.05	6.95×10^{-7}
	324K, 91%	53	-4.50	0.98	3.16×10^{-5}	0.147	-65.1	0.98	3.20×10^{-5}
	average	54 ±2	-5.28 ±0.83	1.01 ±0.04	ave: 5.25×10^{-6} std: 6.76×10^0 (in log)	0.147 ±0.005	-66.9 ±1.8	1.01 ±0.04	ave: 5.13×10^{-6} std: 6.92×10^0 (in log)
water	319K, pH8.0	34	-3.37	0.89	4.27×10^{-4}	0.102	-46.0	0.90	5.64×10^{-4}
	314K, pH7.8	25	-2.63	0.80	2.34×10^{-3}	0.075	-34.5	0.80	3.20×10^{-3}
	314K, pH8.1	39	-1.34	0.81	4.57×10^{-2}	0.128	-51.1	0.81	1.19×10^{-1}
	average	33 ±7	-2.45 ±1.03	0.83 ±0.05	ave: 3.57×10^{-3} std: 1.05×10^1 (in log)	0.102 ±0.027	-43.9 ±8.5	0.84 ±0.06	ave: 5.99×10^{-3} std: 1.55×10^1 (in log)

761 ave: average, std: standard deviation

762

763

764 Table 2. Summary of subcritical crack growth measurements for Macedonian marble

765 in air with different relative humidities, and in water.

766

Condition		n	$\log A$	$K_I(10^{-5})$ [MN/m ^{3/2}] (power law)	$da/dt(1.0)$ [m/s] (power law)	b [m ^{5/2} /mol]	α	$K_I(10^{-5})$ [MN/m ^{3/2}] (exponential law)	$da/dt(1.0)$ [m/s] (exponential law)
air	325K, 7%	97	-7.54	1.06	2.88×10^{-8}	0.245	-107.9	1.06	3.44×10^{-8}
	325K, 5%	106	-6.96	1.04	1.10×10^{-7}	0.274	-117.3	1.04	1.31×10^{-7}
	325K, 5%	104	-8.96	1.09	1.10×10^{-9}	0.256	-115.0	1.09	1.67×10^{-9}
	average	103 ±5	-7.82 ±1.03	1.06 ±0.03	ave: 1.51×10^{-8} std: 1.07×10^1 (in log)	0.258 ±0.015	-113.4 ±4.9	1.06 ±0.03	ave: 1.96×10^{-8} std: 9.33×10^0 (in log)
	324K, 48%	61	-5.25	1.08	5.62×10^{-6}	0.150	-71.8	1.08	1.03×10^{-7}
	324K, 49%	66	-5.72	1.03	1.91×10^{-6}	0.166	-74.9	1.03	1.77×10^{-6}
	323K, 50%	65	-5.19	1.01	6.46×10^{-6}	0.170	-75.5	1.01	5.21×10^{-6}
	average	64 ±2	-5.39 ±0.29	1.03 ±0.04	ave: 4.11×10^{-6} std: 1.95×10^0 (in log)	0.162 ±0.011	-74.1 ±2.0	1.03 ±0.04	ave: 9.83×10^{-7} std: 7.56×10^0 (in log)
	324K, 89%	70	-4.07	0.97	8.51×10^{-5}	0.191	-80.6	0.97	6.37×10^{-5}
	323K, 89%	52	-3.00	0.92	1.00×10^{-3}	0.148	-62.0	0.92	1.05×10^{-3}
average	61 ±13	-3.54 ±0.76	0.94 ±0.04	ave: 2.92×10^{-4} std: 5.75×10^0 (in log)	0.170 ±0.030	-71.3 ±13.2	0.95 ±0.04	ave: 2.58×10^{-4} std: 7.24×10^0 (in log)	
water	318K, pH8.5	35	-2.43	0.84	3.72×10^{-3}	0.106	-44.8	0.84	8.92×10^{-3}
	318K, pH8.6	37	-2.66	0.87	2.19×10^{-3}	0.111	-47.8	0.87	2.94×10^{-3}
	average	36 ±1	-2.55 ±0.16	0.86 ±0.02	ave: 2.85×10^{-3} std: 1.45×10^0 (in log)	0.109 ±0.035	-46.3 ±2.1	0.86 ±0.02	ave: 5.12×10^{-3} std: 2.19×10^0 (in log)

767 ave: average, std: standard deviation

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770

771

772 Table 3. Summary of subcritical crack growth measurements for Carrara marble in air
 773 and water at different temperatures.

774

Condition		n	$\log A$	$K_I(10^{-5})$ [MN/m ^{3/2}] (power law)	$da/dt(1.0)$ [m/s] (power law)	b [m ^{5/2} /mol]	α	$K_I(10^{-5})$ [MN/m ^{3/2}] (exponential law)	$da/dt(1.0)$ [m/s] (exponential law)
air	293K, 47%	81	-9.85	1.15	1.41×10^{-10}	0.174	-93.5	1.15	2.70×10^{-10}
	293K, 47%	75	-6.31	1.04	4.90×10^{-7}	0.179	-88.1	1.04	4.47×10^{-7}
	293K, 47%	73	-7.59	1.09	2.57×10^{-8}	0.165	-85.2	1.09	2.65×10^{-8}
	average	77 ±4	-7.92 ±1.79	1.09 ±0.06	ave: 1.21×10^{-8} std: 6.17×10^1 (in log)	0.173 ±0.007	-88.9 ±4.2	1.09 ±0.06	ave: 1.47×10^{-8} std: 4.17×10^1 (in log)
	324K, 49%	92	-6.91	1.05	1.23×10^{-7}	0.239	-104.6	1.05	1.41×10^{-7}
	324K, 49%	88	-7.97	1.08	1.07×10^{-8}	0.221	-100.1	1.08	1.48×10^{-8}
	324K, 49%	80	-6.23	1.04	5.89×10^{-7}	0.211	-92.7	1.04	6.25×10^{-7}
	323K, 49%	69	-6.73	1.06	1.86×10^{-7}	0.177	-81.4	1.06	1.93×10^{-7}
	average	82 ±10	-6.96 ±0.73	1.06 ±0.02	ave: 1.10×10^{-7} std: 5.37×10^0 (in log)	0.212 ±0.026	-94.7 ±10.1	1.06 ±0.02	ave: 1.26×10^{-7} std: 4.79×10^0 (in log)
	water	290K, pH8.2	30	-3.14	0.87	7.24×10^{-4}	0.093	-46.0	0.90
290K, pH8.1		35	-4.58	0.97	2.63×10^{-5}	0.088	-47.0	0.97	2.91×10^{-5}
290K, pH8.2		35	-4.52	0.97	3.02×10^{-5}	0.088	-46.9	0.97	3.11×10^{-5}
average		33 ±3	-4.08 ±0.81	0.94 ±0.06	ave: 8.32×10^{-5} std: 6.46×10^0 (in log)	0.090 ±0.003	-46.6 ±0.6	0.95 ±0.04	ave: 8.27×10^{-5} std: 5.75×10^0 (in log)
319K, pH8.0		34	-3.37	0.89	4.27×10^{-4}	0.102	-46.0	0.90	5.64×10^{-4}
314K, pH7.8		25	-2.63	0.80	2.34×10^{-3}	0.075	-34.5	0.80	3.20×10^{-3}
314K, pH8.1		39	-1.34	0.81	4.57×10^{-2}	0.128	-51.1	0.81	1.19×10^{-1}
average		33 ±7	-2.45 ±1.03	0.83 ±0.05	ave: 3.57×10^{-3} std: 1.05×10^1 (in log)	0.102 ±0.027	-43.9 ±8.5	0.84 ±0.06	ave: 5.99×10^{-3} std: 1.55×10^1 (in log)

775 ave: average, std: standard deviation

776

777

778 Table 4. Summary of subcritical crack growth measurements for Macedonian marble

779 in air and water at different temperatures.

780

Condition		n	$\log A$	$K_I(10^{-5})$ [MN/m ^{3/2}] (power law)	$da/dt(1.0)$ [m/s] (power law)	b [m ^{5/2} /mol]	α	$K_I(10^{-5})$ [MN/m ^{3/2}] (exponential law)	$da/dt(1.0)$ [m/s] (exponential law)
air	293K, 50%	61	-7.02	1.08	9.55×10^{-8}	0.136	-71.8	1.08	1.19×10^{-7}
	293K, 50%	60	-6.75	1.07	1.78×10^{-7}	0.133	-70.0	1.07	2.10×10^{-7}
	average	61 ± 1	-6.89 ± 0.19	1.08 ± 0.01	ave: 1.30×10^{-7} std: 1.55×10^0 (in log)	0.135 ± 0.002	-70.9 ± 1.3	1.08 ± 0.01	ave: 1.58×10^{-7} std: 1.48×10^0 (in log)
	324K, 48%	61	-5.25	1.08	5.62×10^{-6}	0.150	-71.8	1.08	1.03×10^{-7}
	324K, 49%	66	-5.72	1.03	1.91×10^{-6}	0.166	-74.9	1.03	1.77×10^{-6}
	323K, 50%	65	-5.19	1.01	6.46×10^{-6}	0.170	-75.5	1.01	5.21×10^{-6}
	average	64 ± 2	-5.39 ± 0.29	1.03 ± 0.04	ave: 4.11×10^{-6} std: 1.95×10^0 (in log)	0.162 ± 0.011	-74.1 ± 2.0	1.03 ± 0.04	ave: 9.83×10^{-7} std: 7.56×10^0 (in log)
	351K, 50%	55	-3.14	0.93	7.24×10^{-4}	0.167	-64.7	0.93	5.84×10^{-4}
	351K, 50%	74	-3.34	0.95	4.57×10^{-4}	0.224	-84.4	0.95	4.99×10^{-4}
	351K, 50%	77	-3.79	0.96	1.62×10^{-4}	0.228	-87.0	0.96	1.46×10^{-4}
	average	69 ± 12	-3.42 ± 0.33	0.95 ± 0.02	ave: 4.55×10^{-4} std: 2.14×10^0 (in log)	0.206 ± 0.034	-78.7 ± 12.2	0.95 ± 0.02	ave: 3.49×10^{-5} std: 2.14×10^0 (in log)
water	291K, pH8.6	37	-3.29	0.90	5.13×10^{-4}	0.096	-47.3	0.90	5.39×10^{-4}
	291K, pH8.5	42	-2.56	0.87	2.75×10^{-3}	0.112	-52.3	0.88	2.54×10^{-3}
	average	40 ± 4	-2.93 ± 0.52	0.89 ± 0.02	ave: 1.19×10^{-3} std: 3.31×10^0 (in log)	0.104 ± 0.011	-49.8 ± 3.5	0.89 ± 0.01	ave: 1.17×10^{-3} std: 3.02×10^0 (in log)
	318K, pH8.5	35	-2.43	0.84	3.72×10^{-3}	0.106	-44.8	0.84	8.92×10^{-3}
	318K, pH8.6	37	-2.66	0.87	2.19×10^{-3}	0.111	-47.8	0.87	2.94×10^{-3}
	average	36 ± 1	-2.55 ± 0.16	0.86 ± 0.02	ave: 2.85×10^{-3} std: 1.45×10^0 (in log)	0.109 ± 0.035	-46.3 ± 2.1	0.86 ± 0.02	ave: 5.12×10^{-3} std: 2.19×10^0 (in log)

781 ave: average, std: standard deviation

782

783 Table 5. Summary of fracture toughness measurement for marble in air.

Rock	Condition	Fracture toughness [MN/m ^{3/2}]
CM	323K, 50%	1.36
	323K, 50%	1.32
	323K, 50%	1.26
	average	1.31±0.05
MM	293K, 47%	1.32
	293K, 48%	1.30
	average	1.31±0.01
	323K, 45%	1.33
	323K, 48%	1.27
	323K, 47%	1.25
	average	1.28±0.04

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788 The English in this document has been checked by at least two professional editors, both

789 native speakers of English, from an English editing service company:

790 <https://www.zenis.co.jp/eng/index.html>

791

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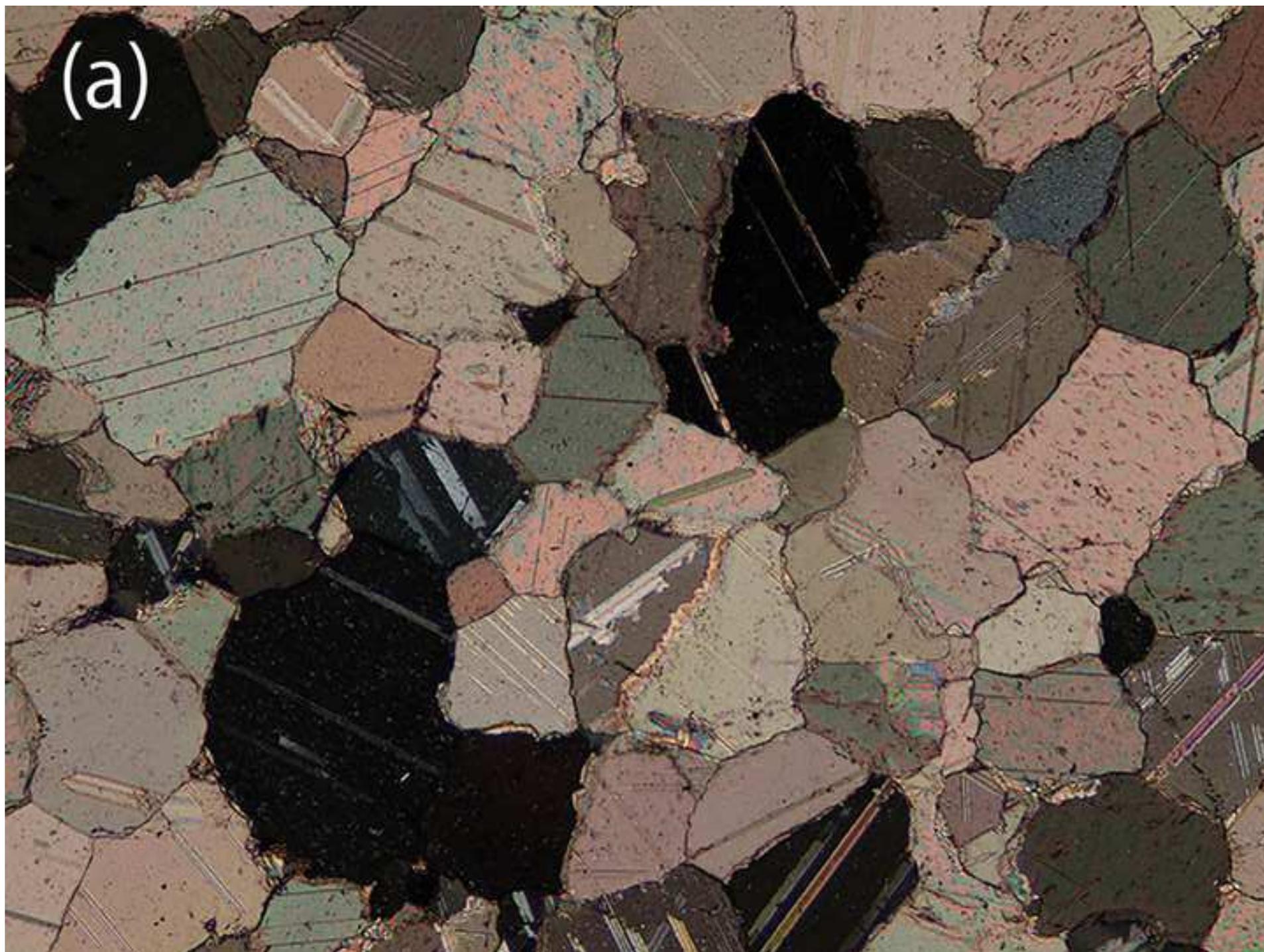
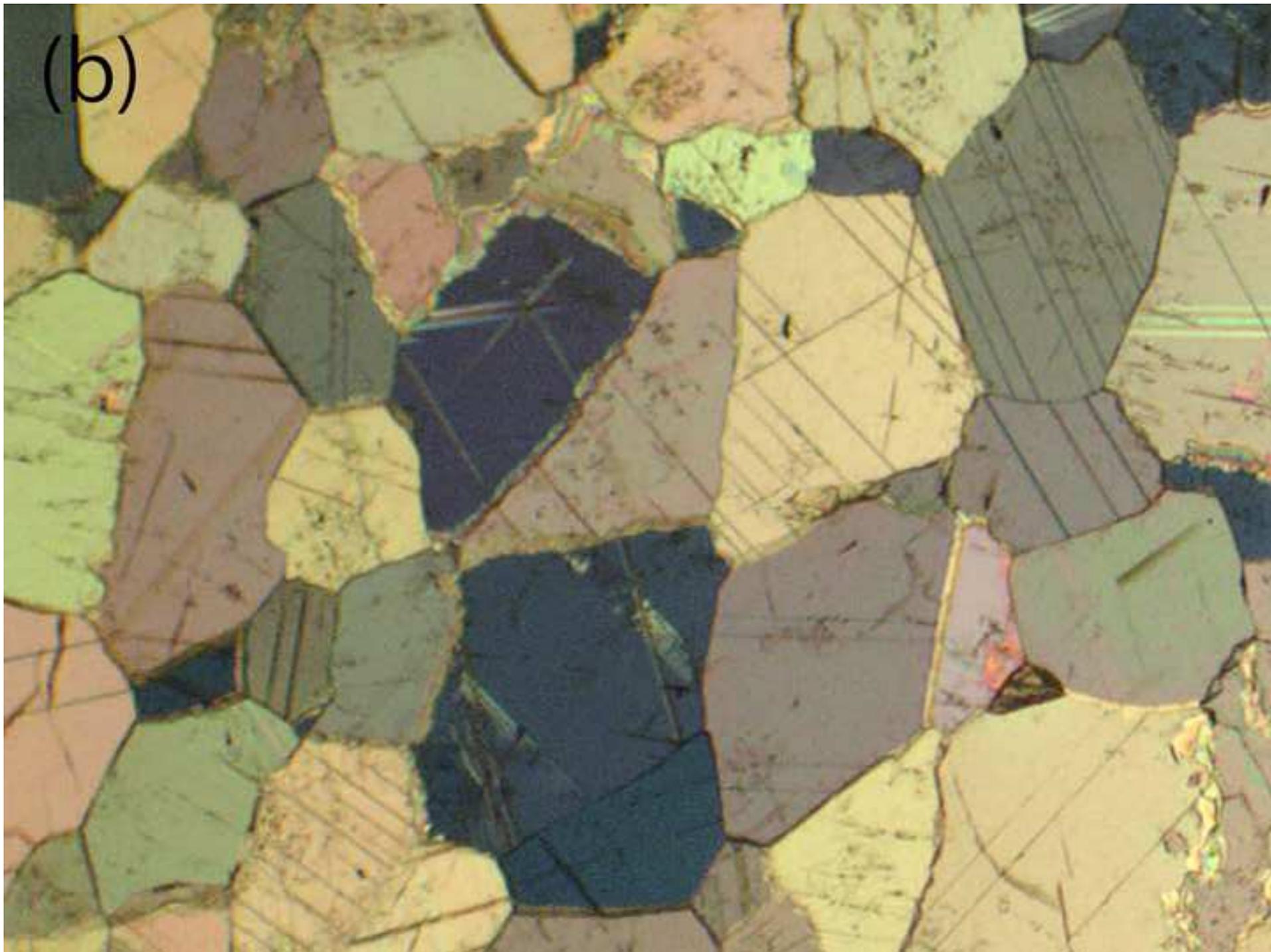
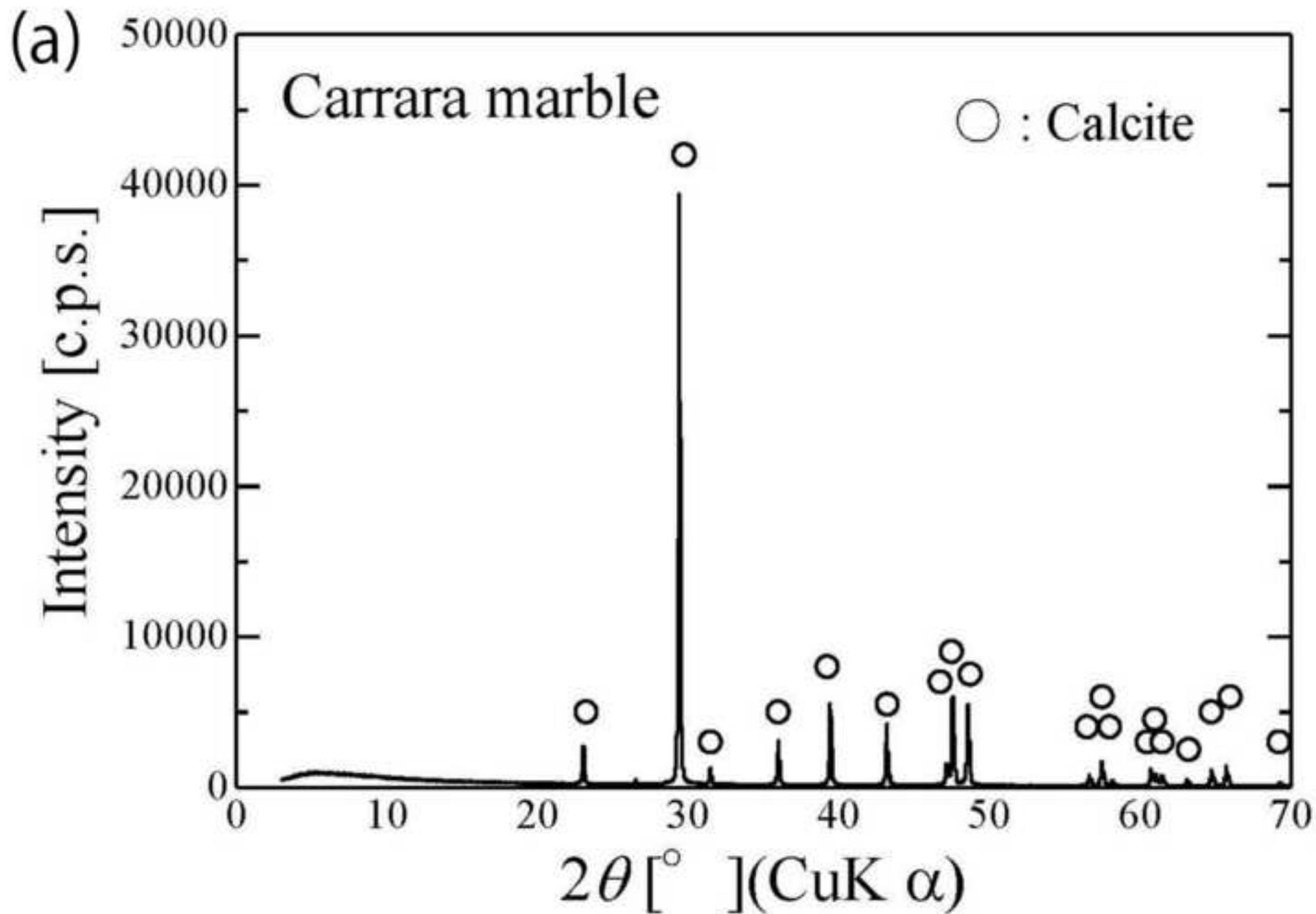


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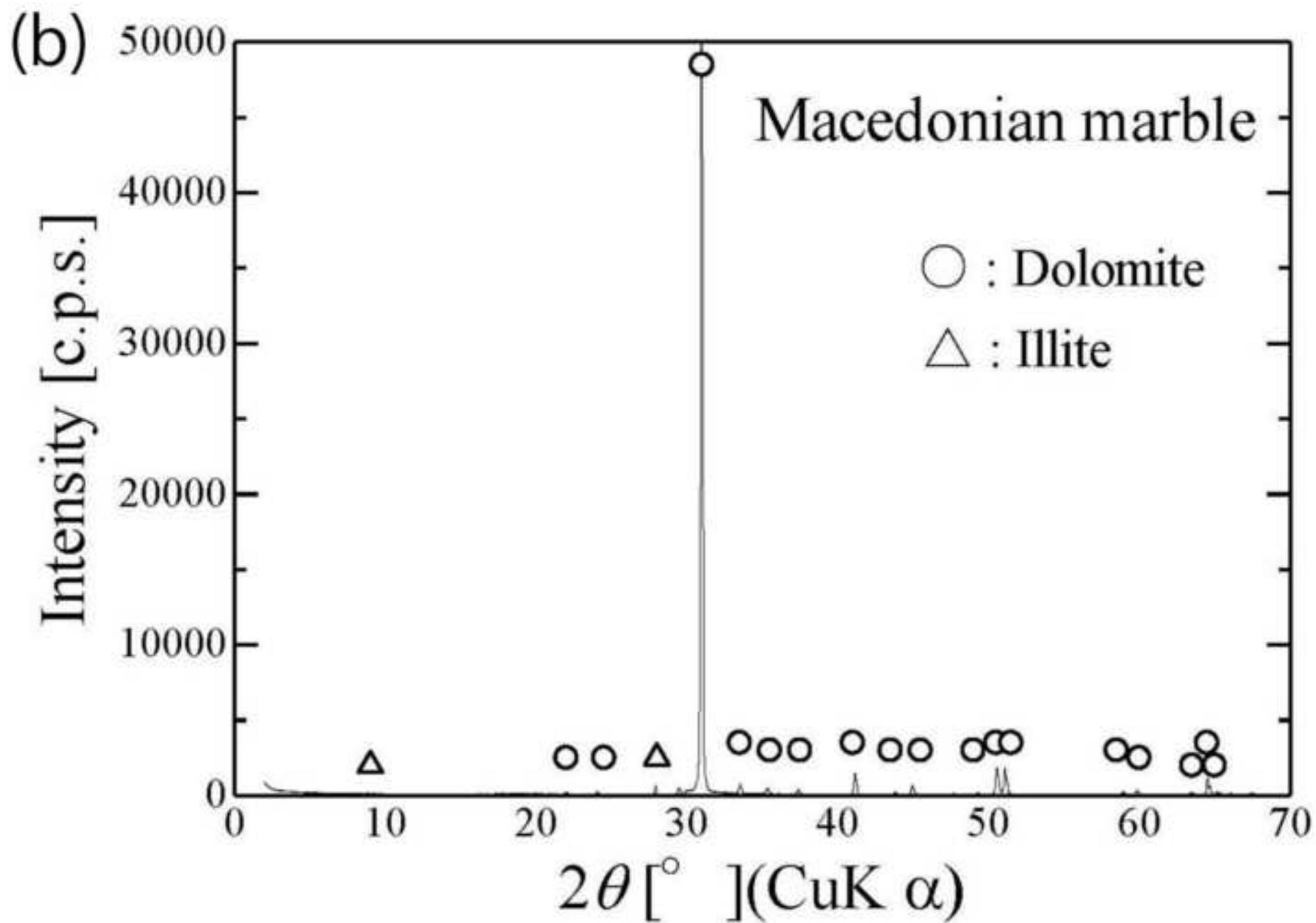


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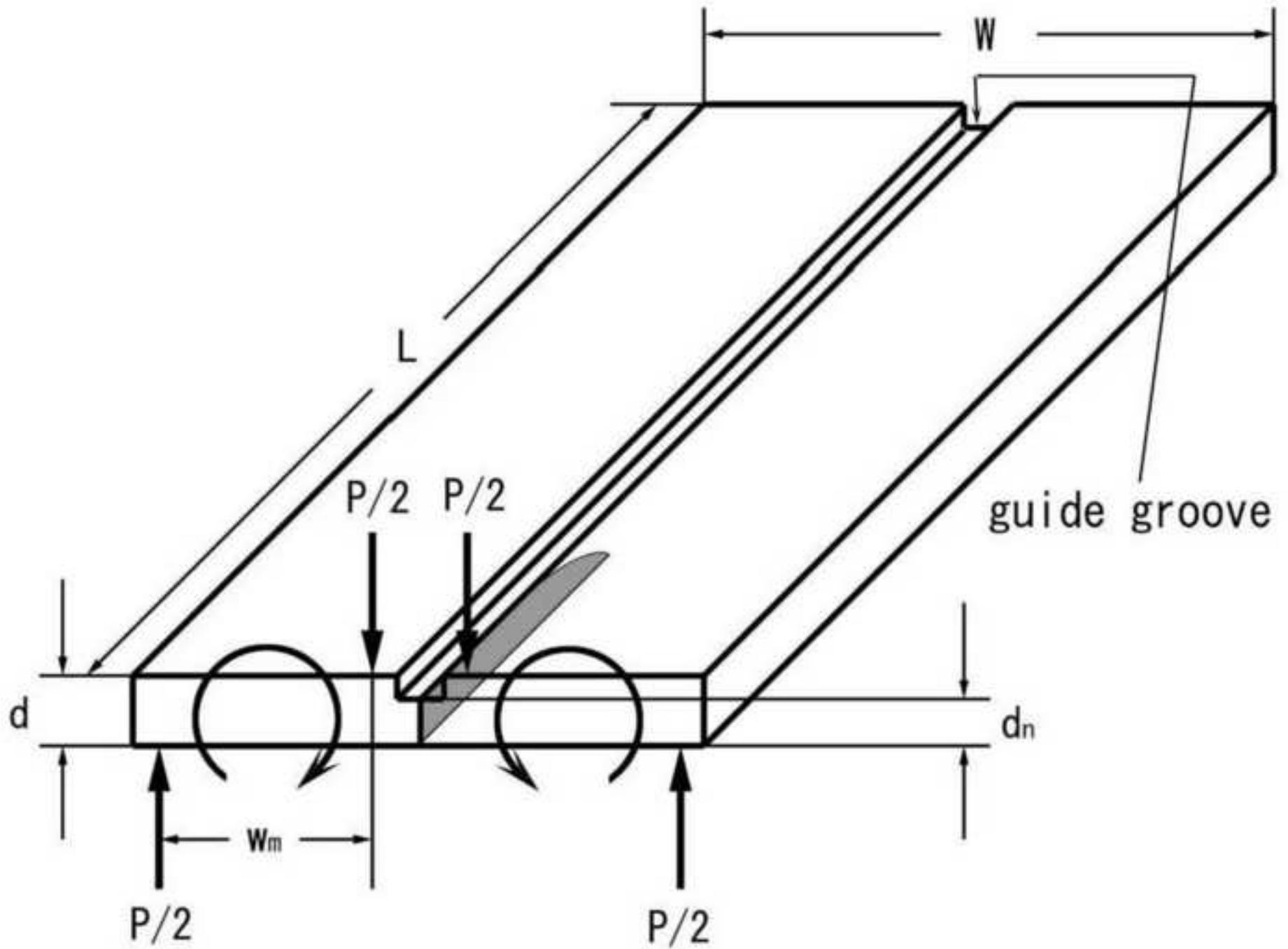


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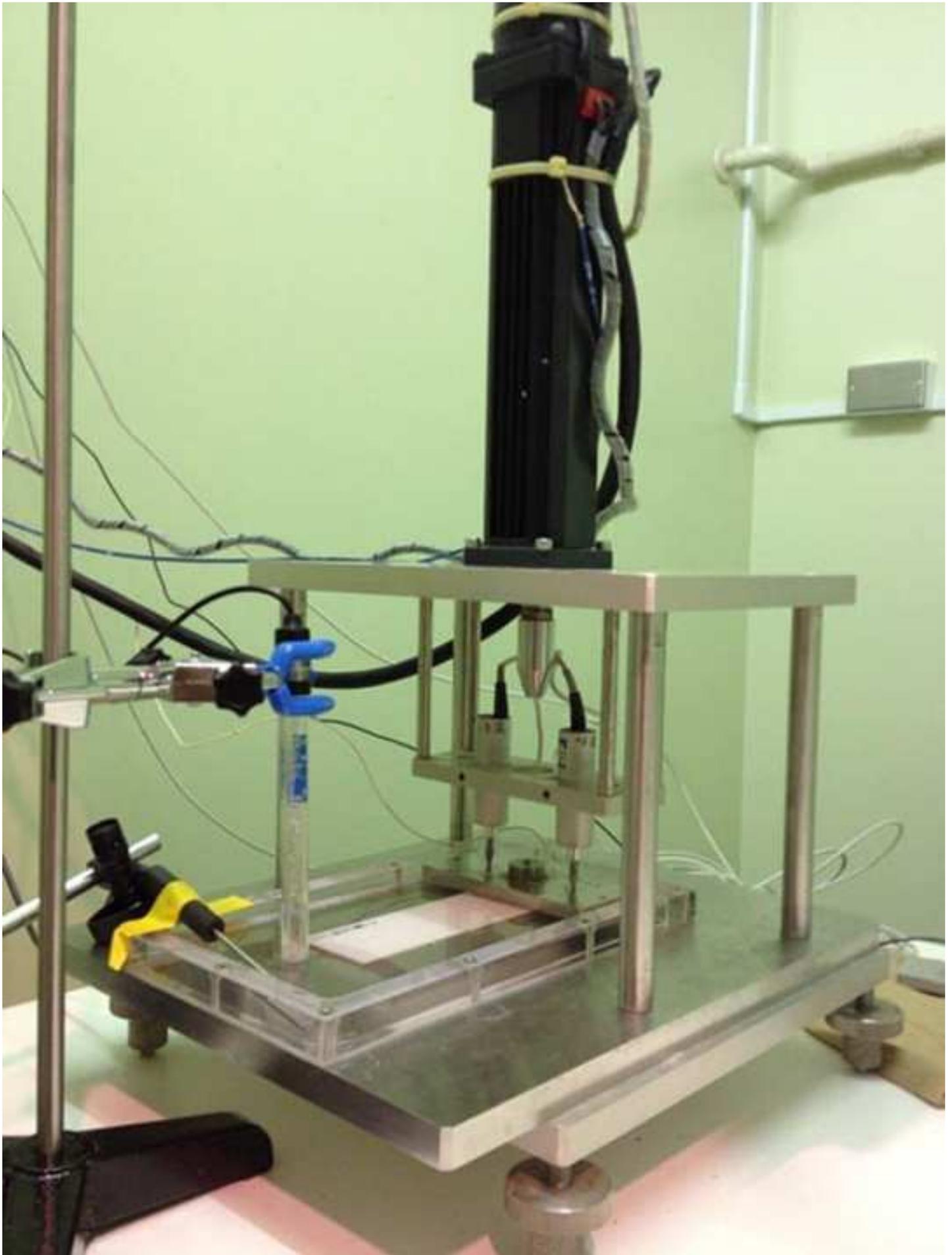


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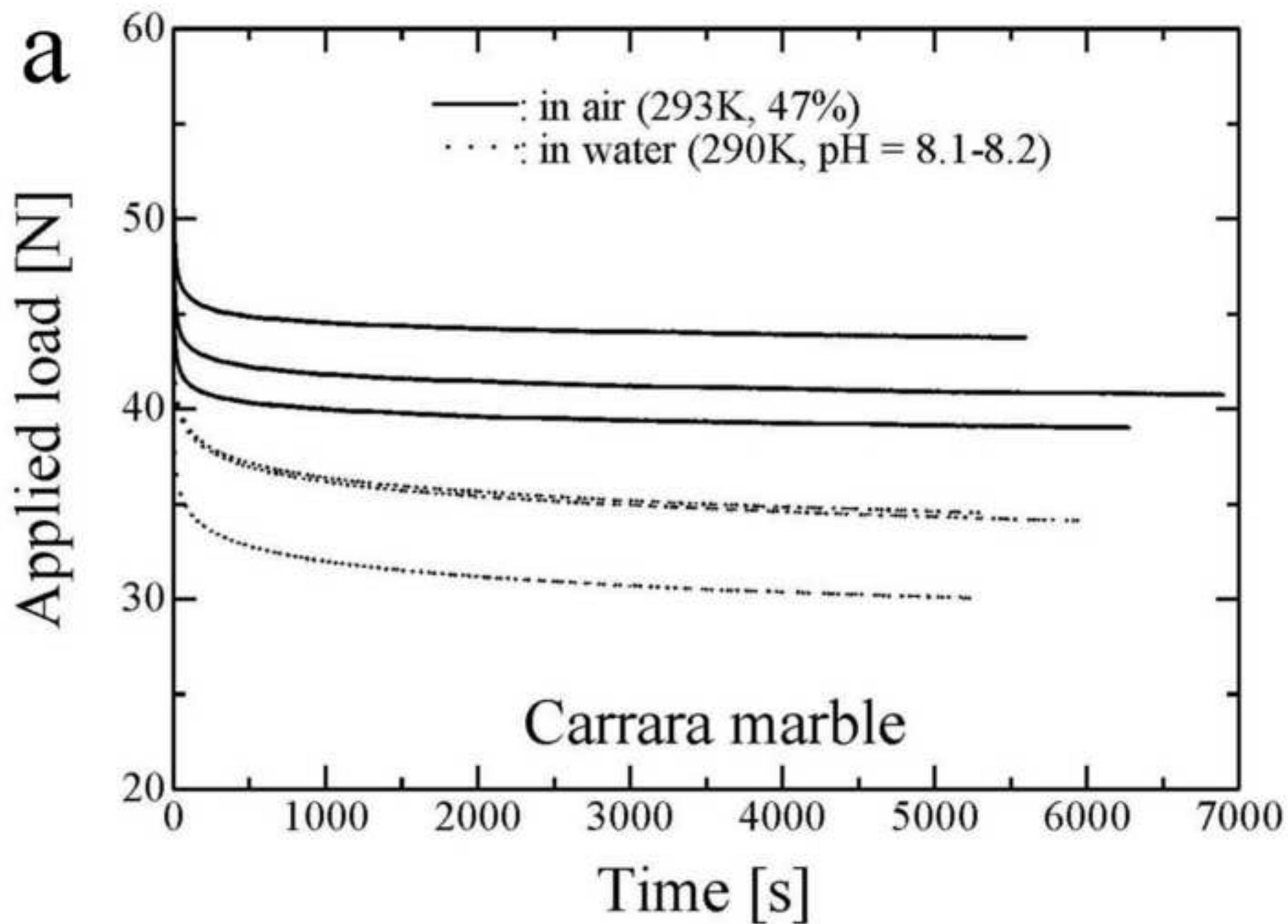
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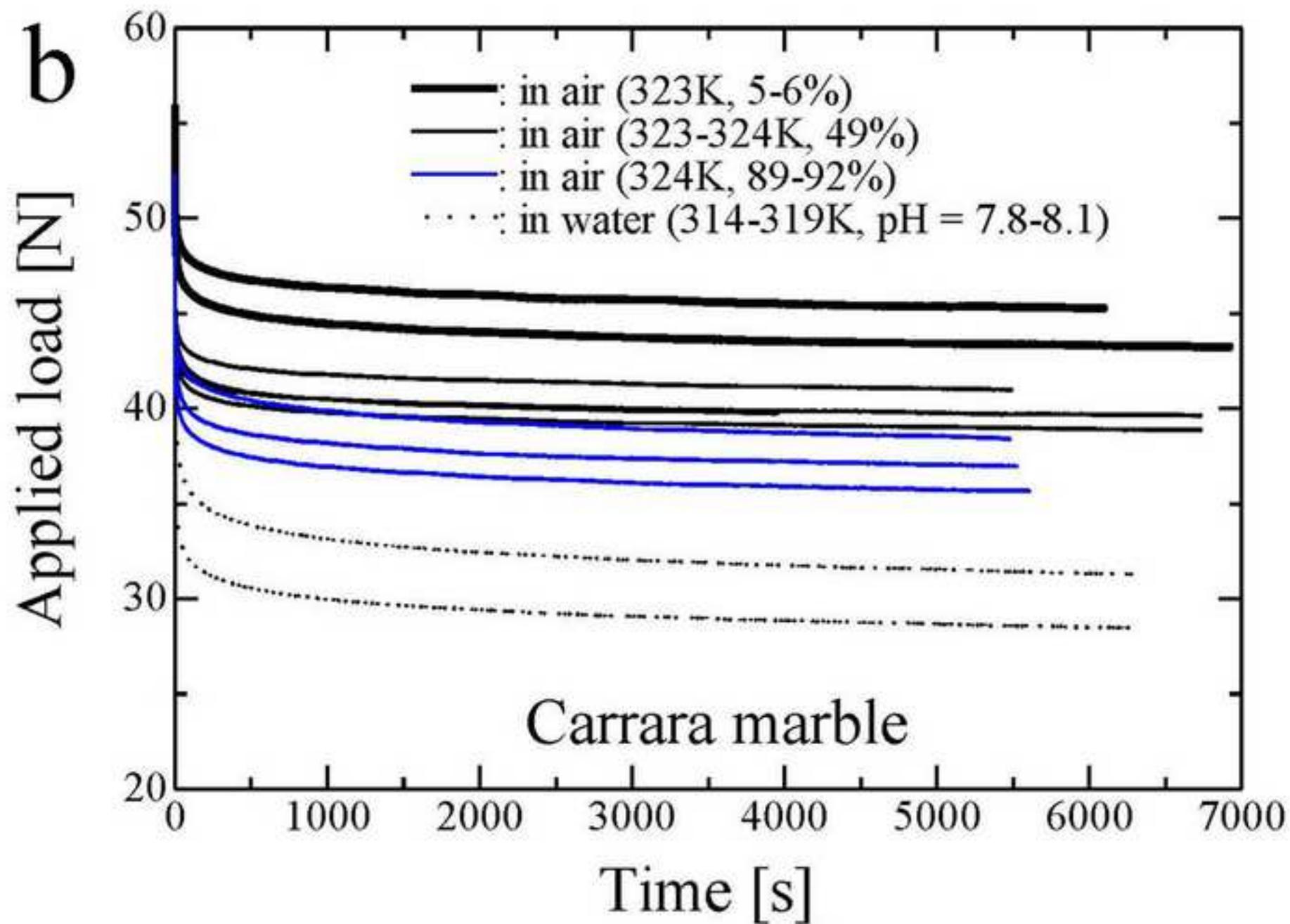


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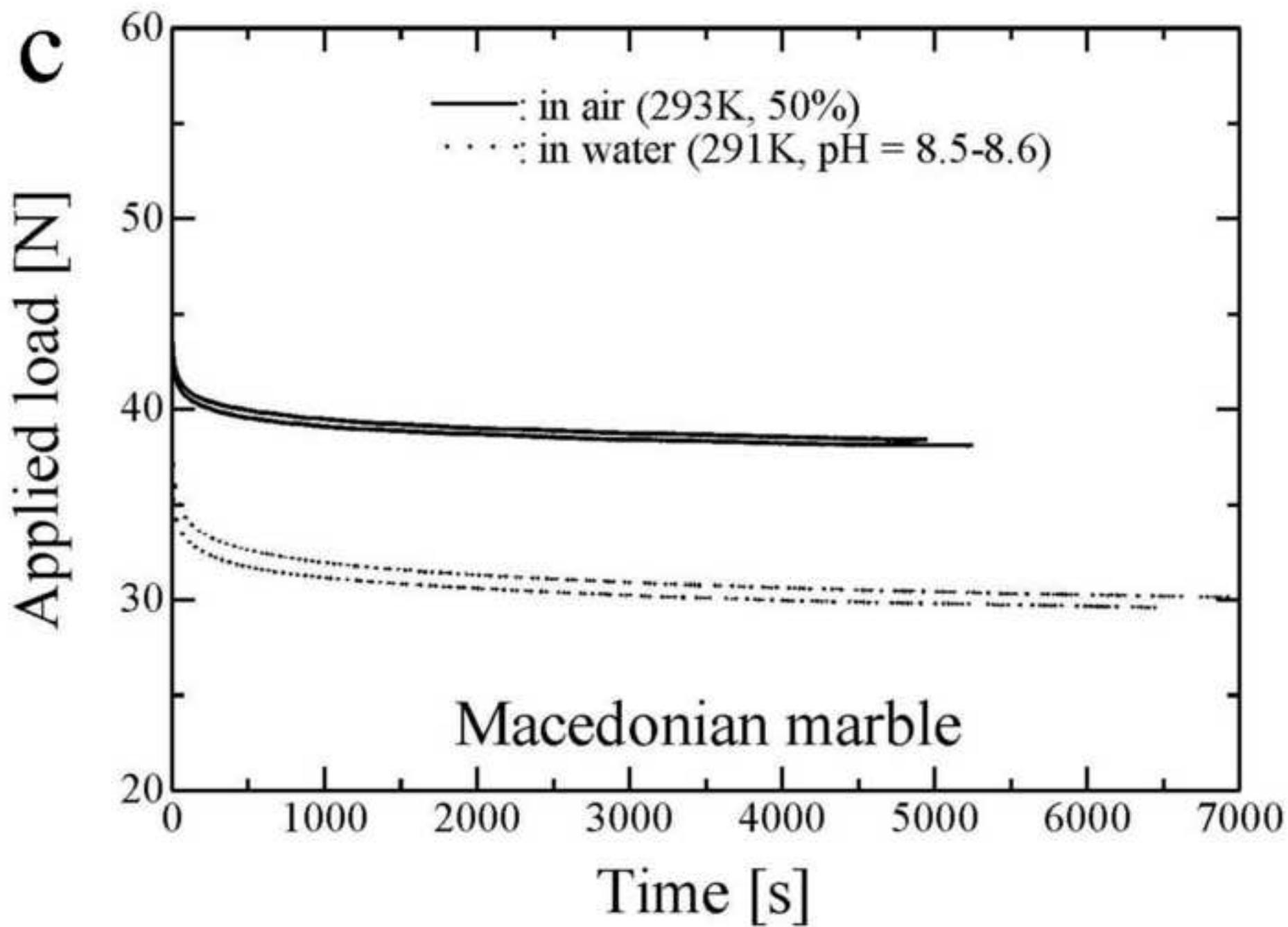


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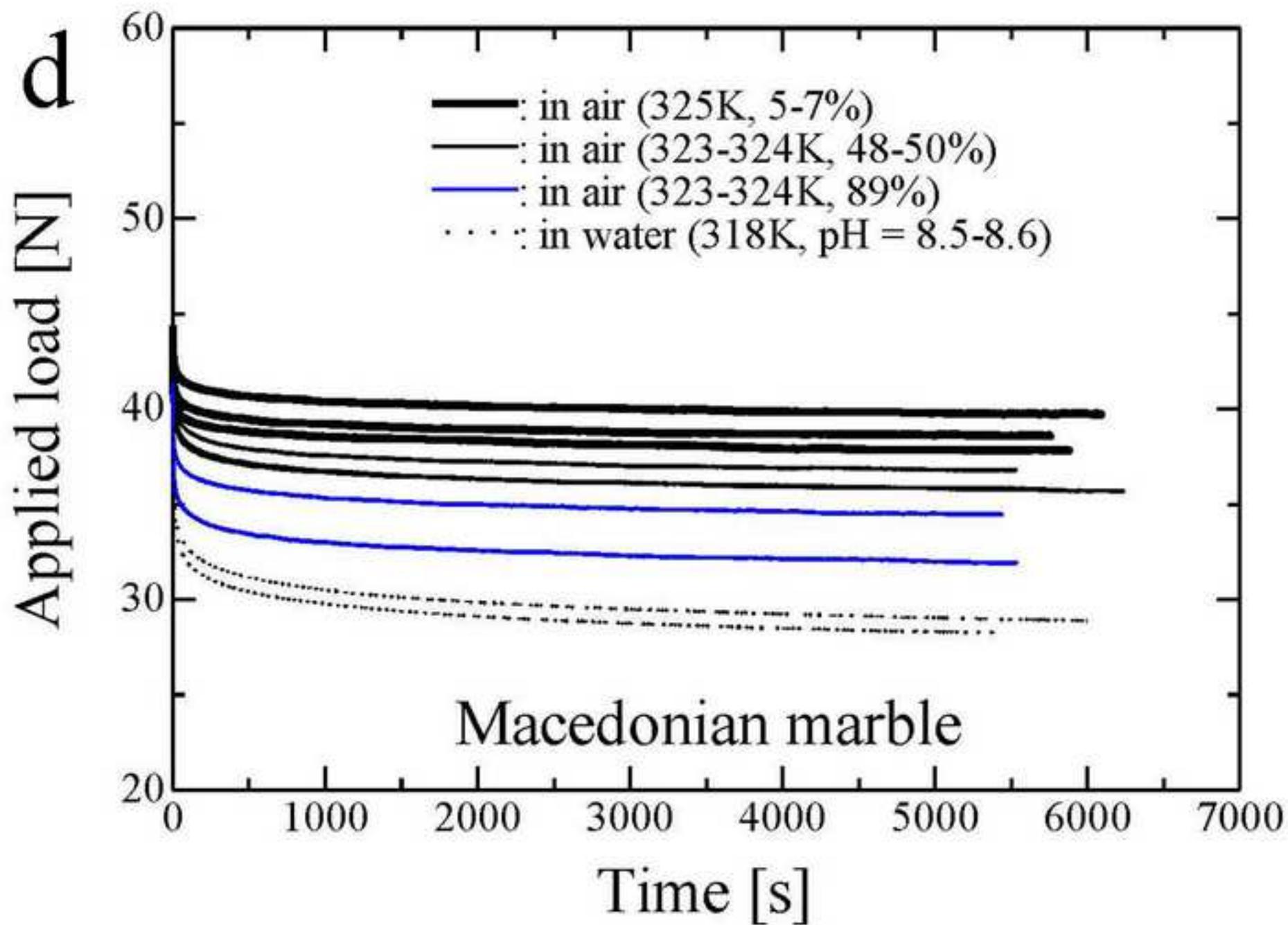


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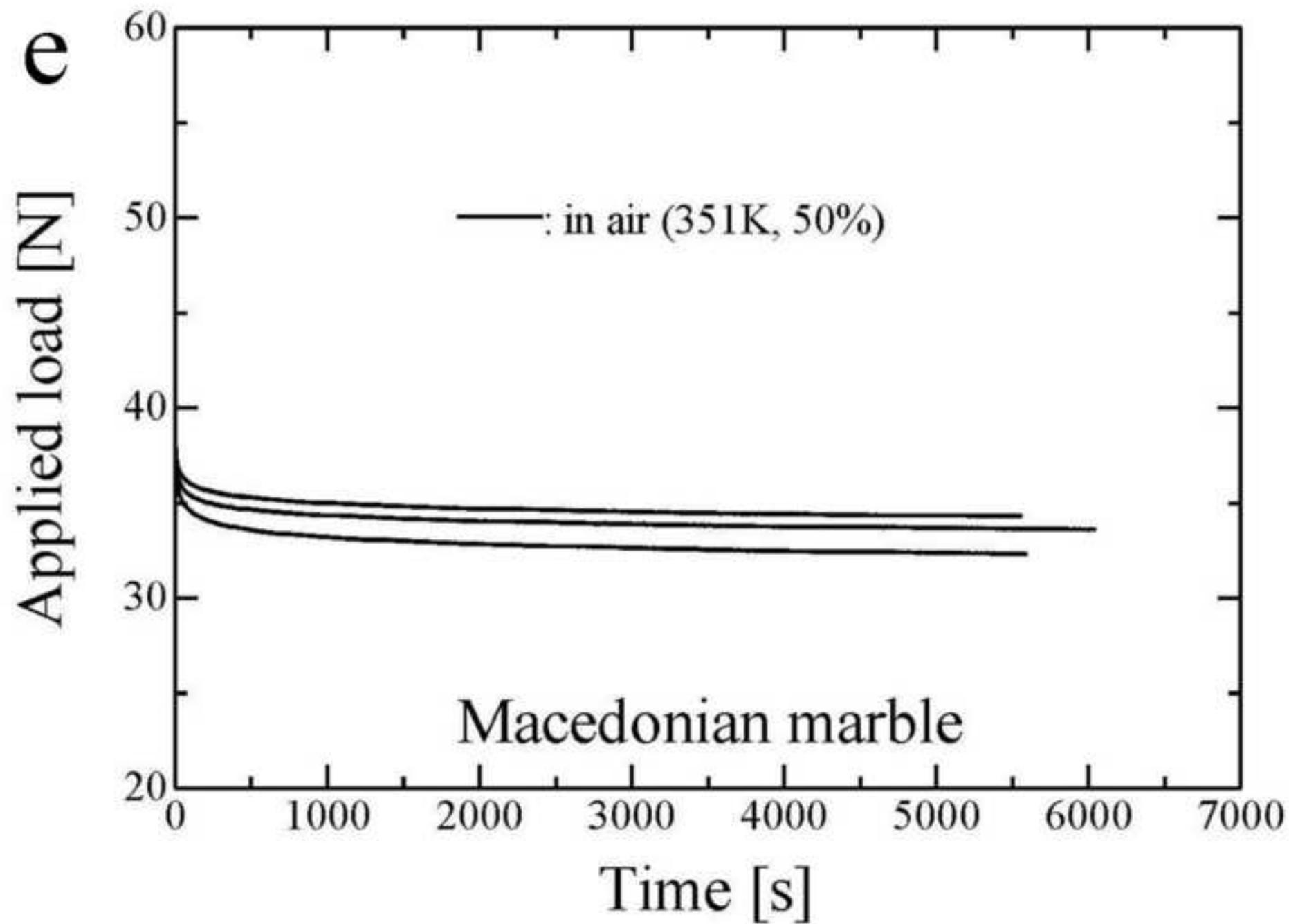


Figure6a

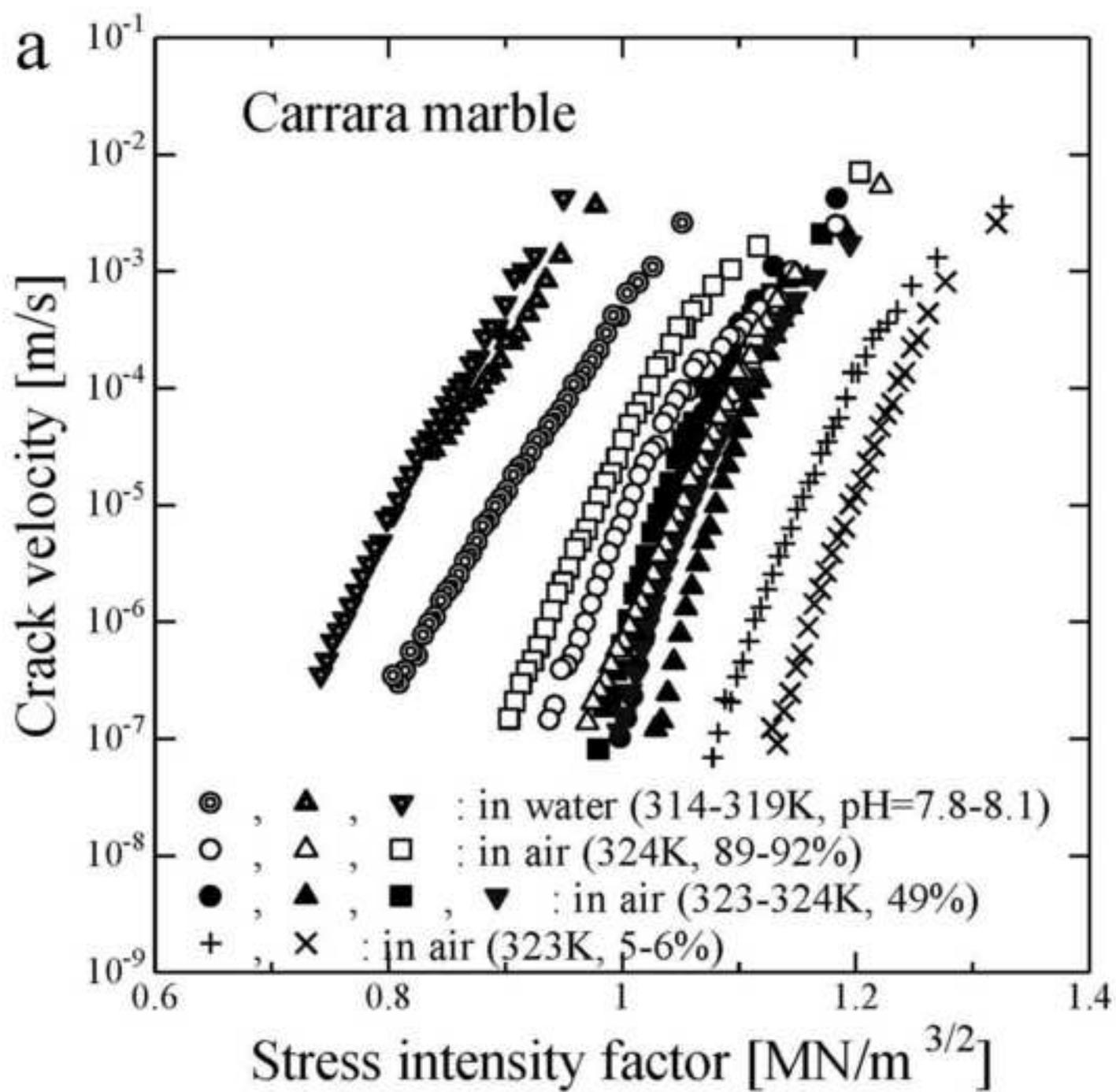
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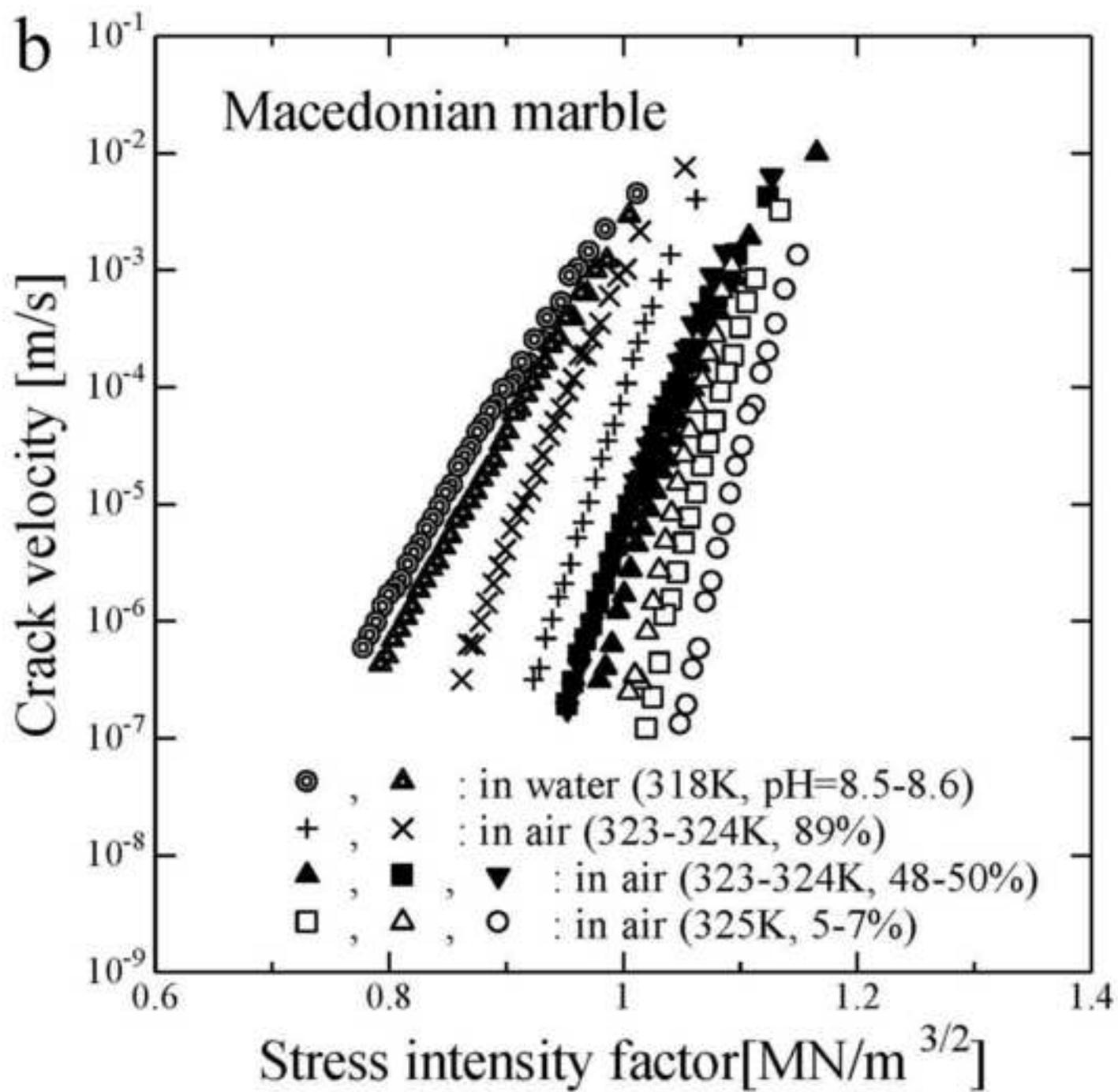


Figure7a

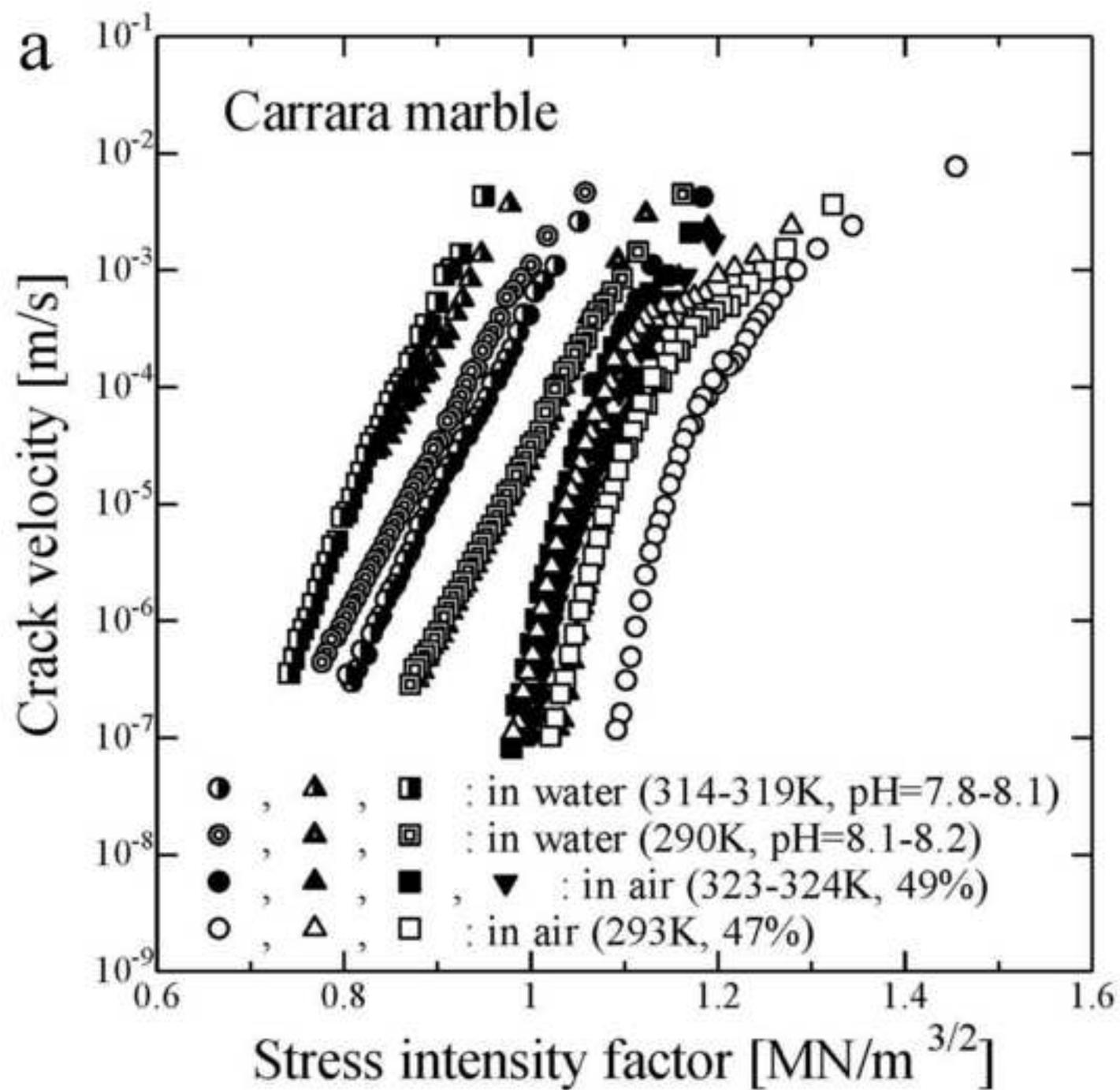
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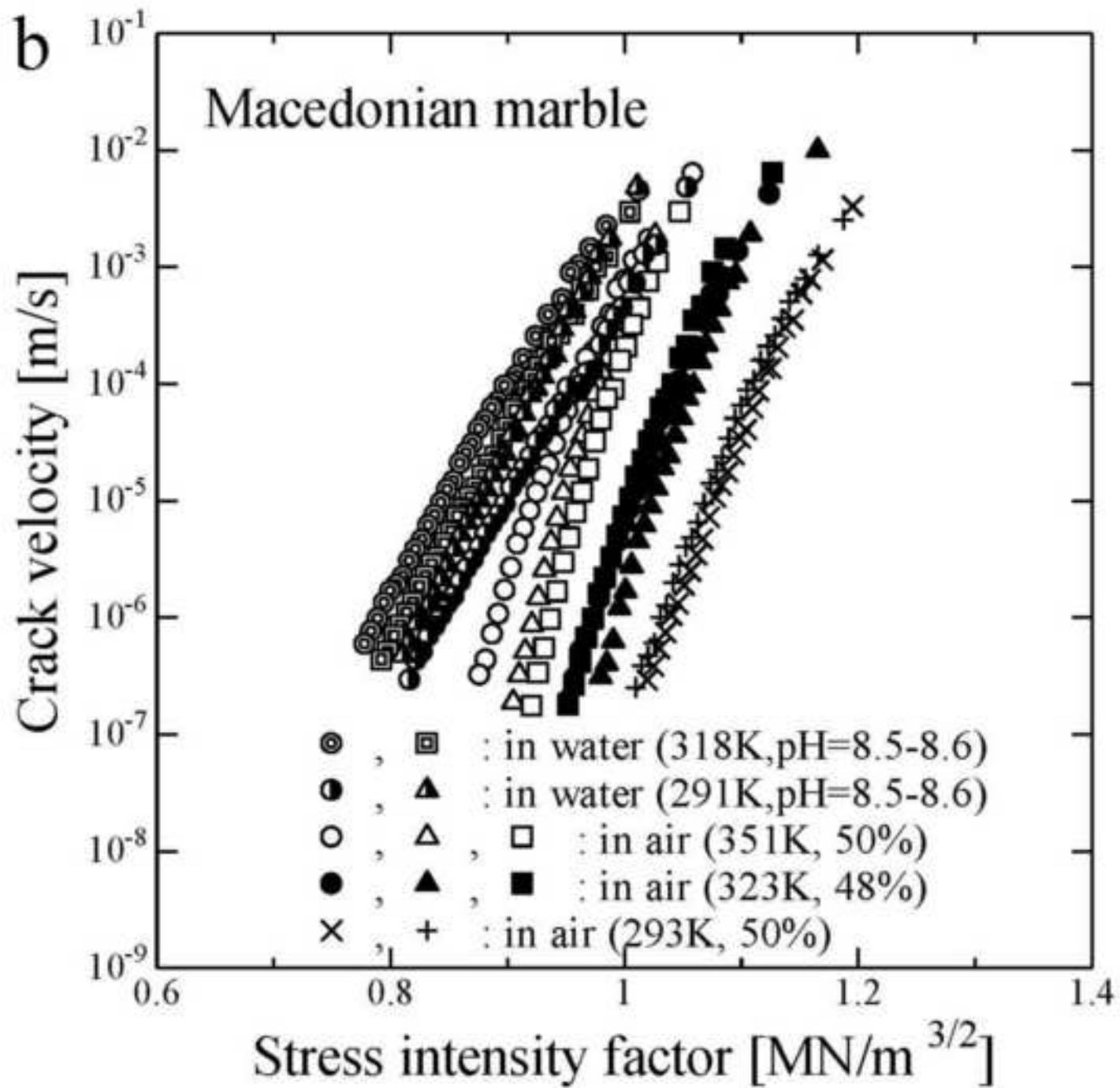


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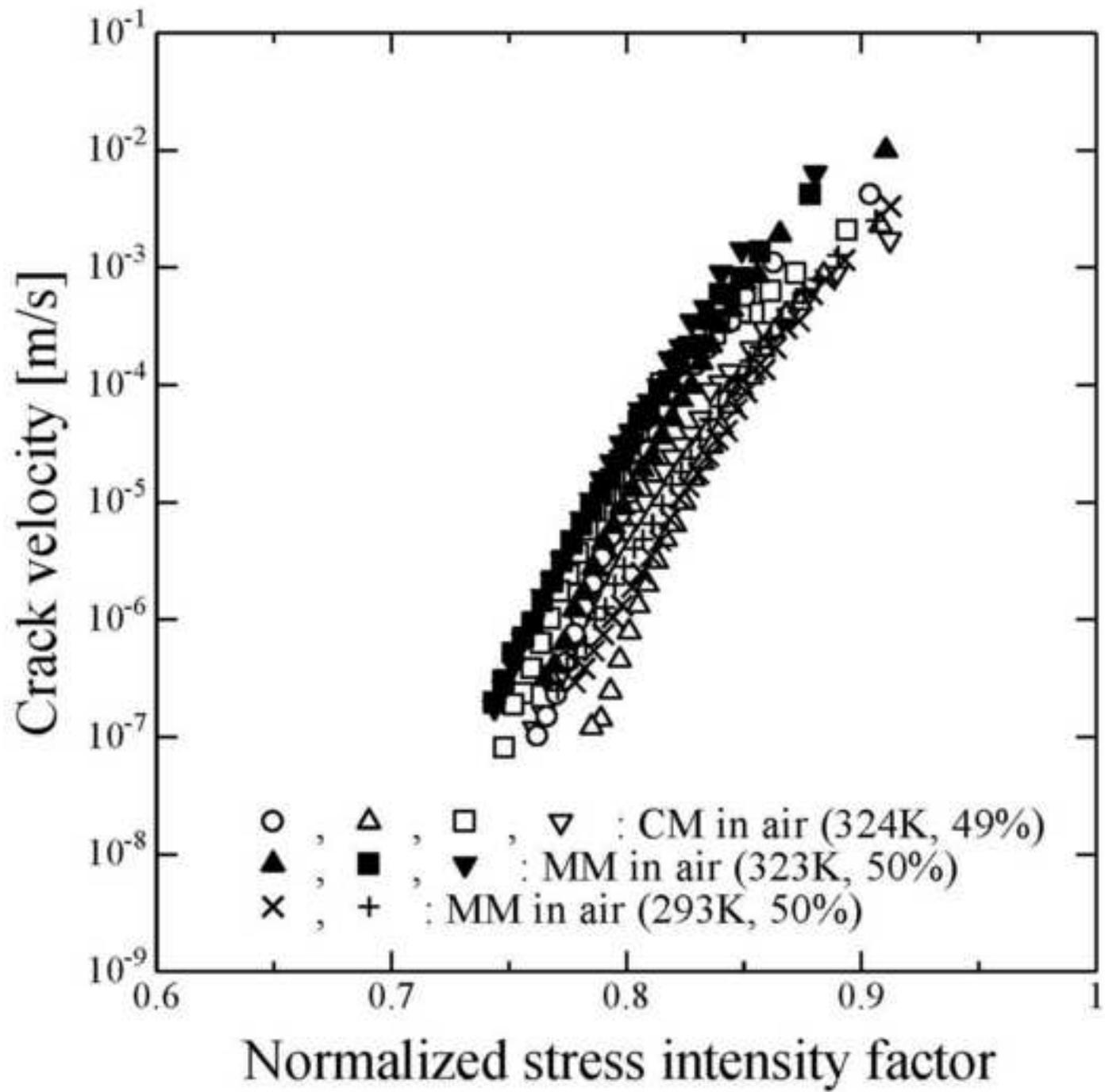


Figure9

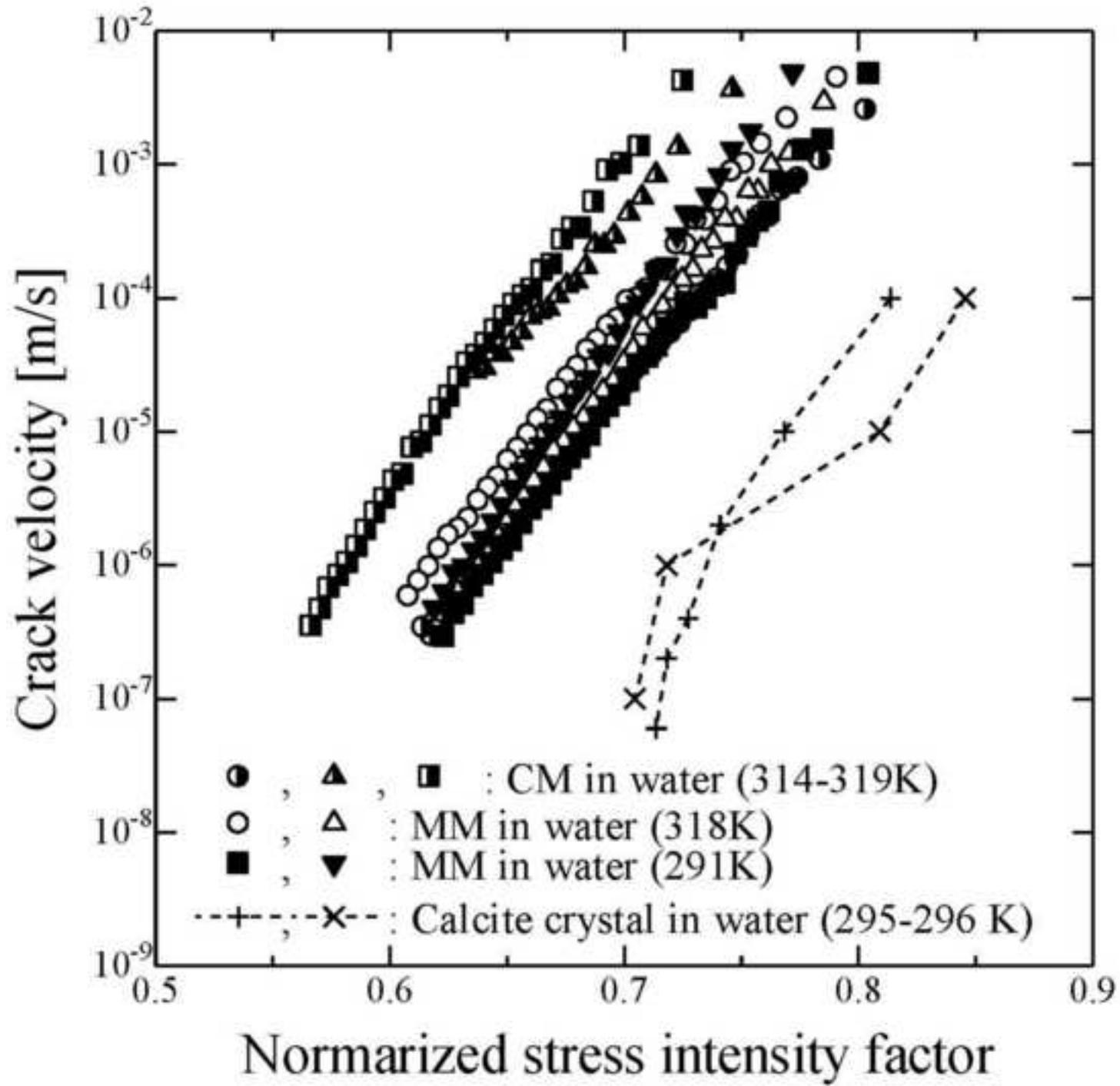
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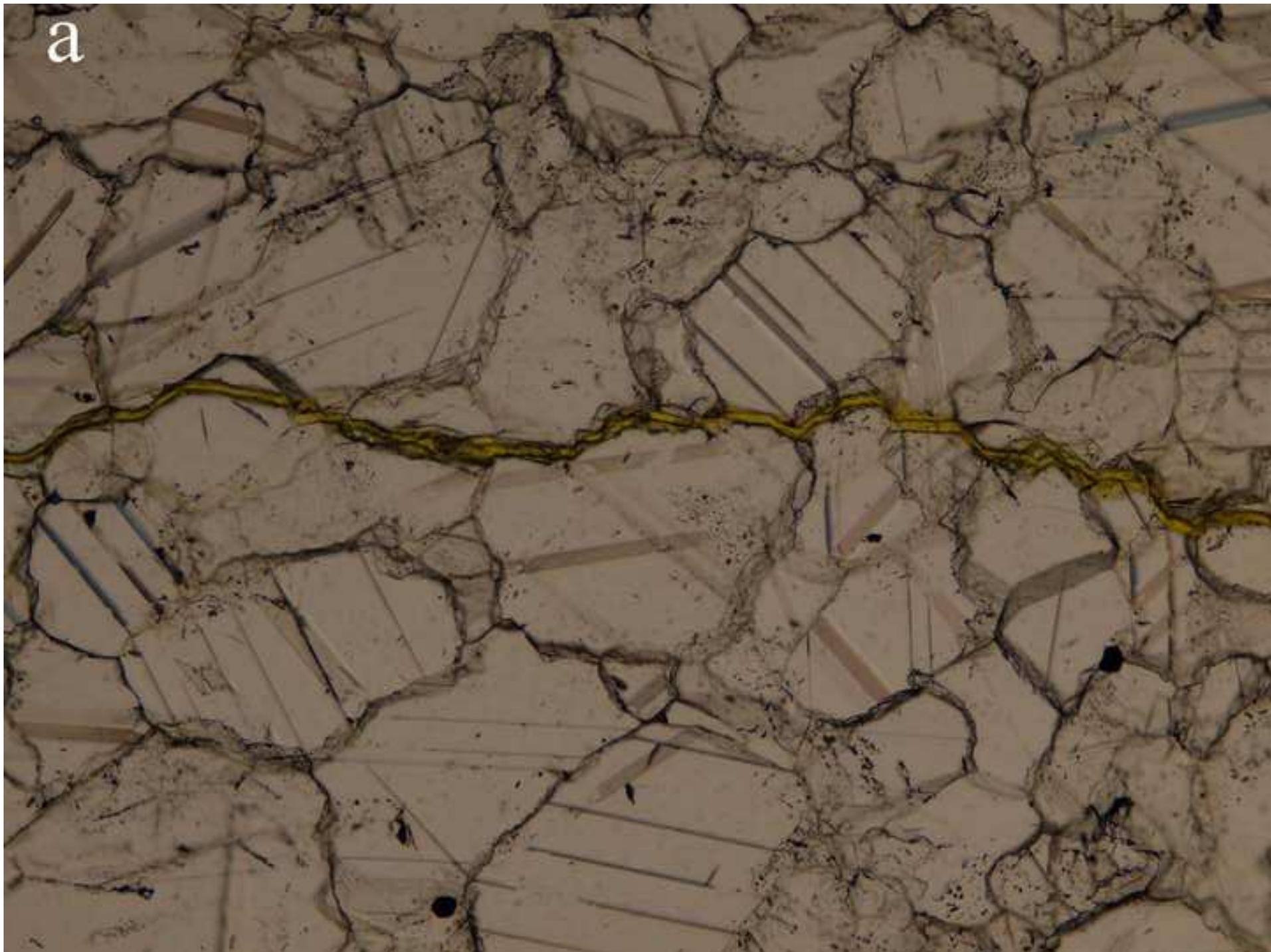
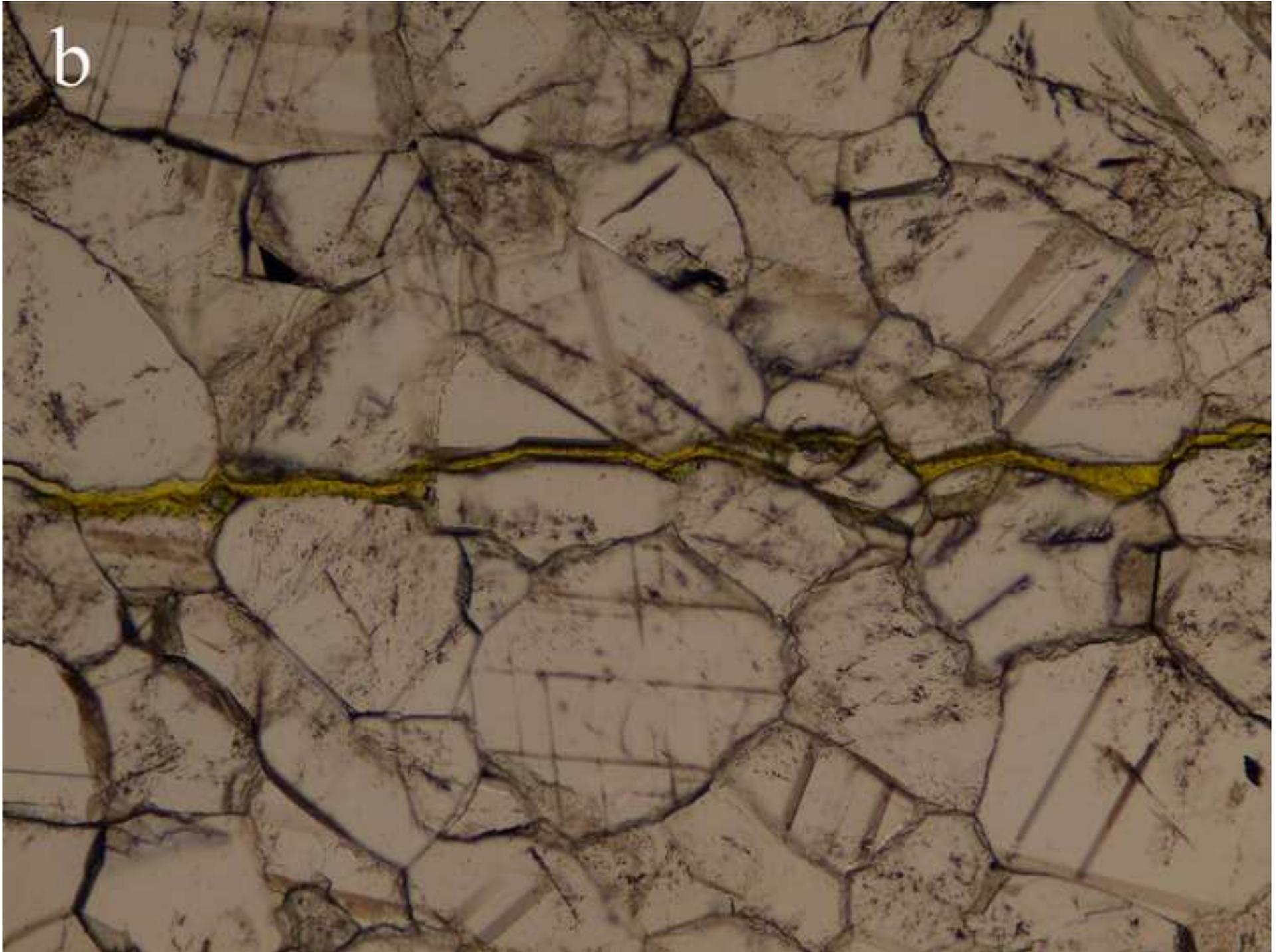


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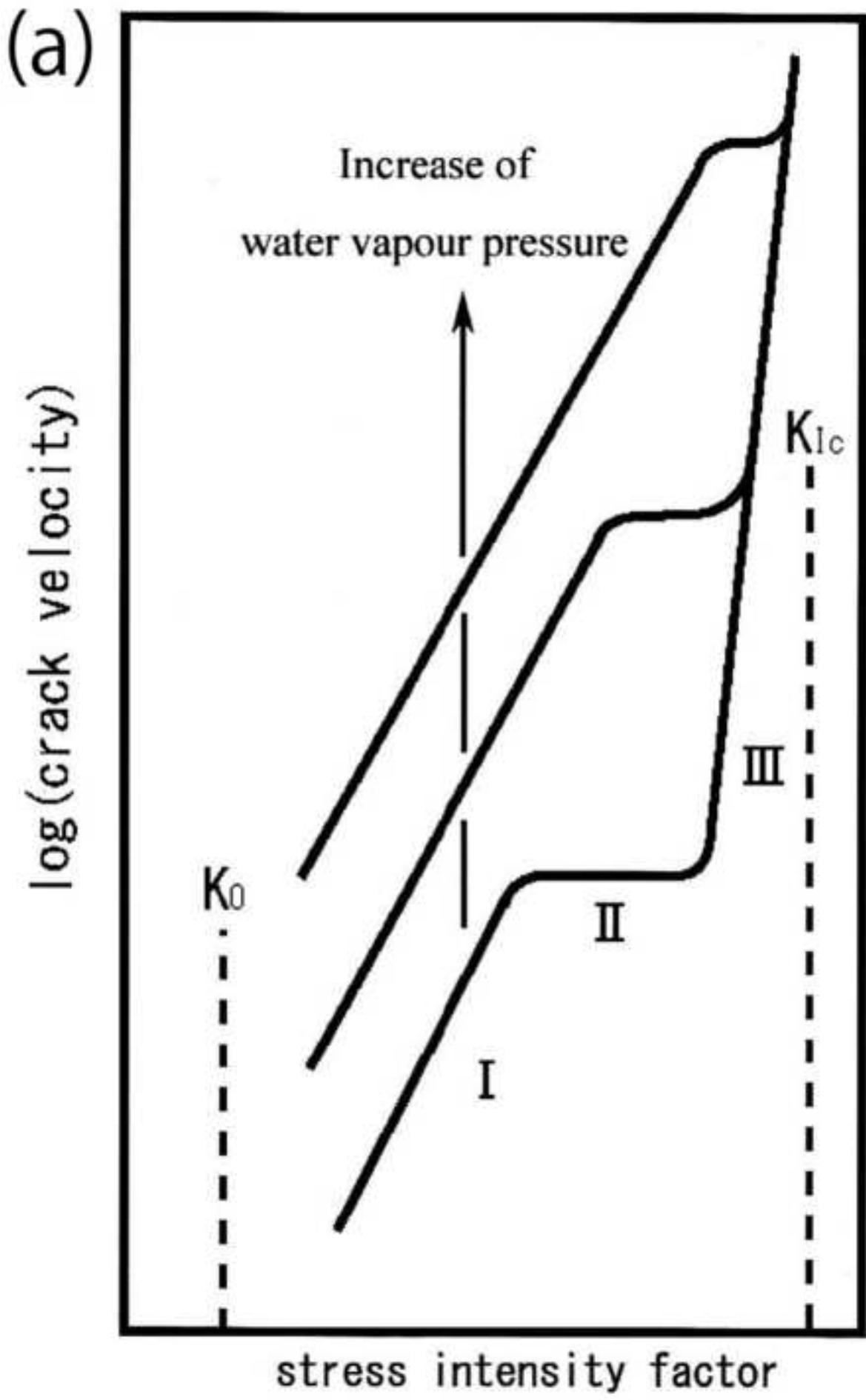
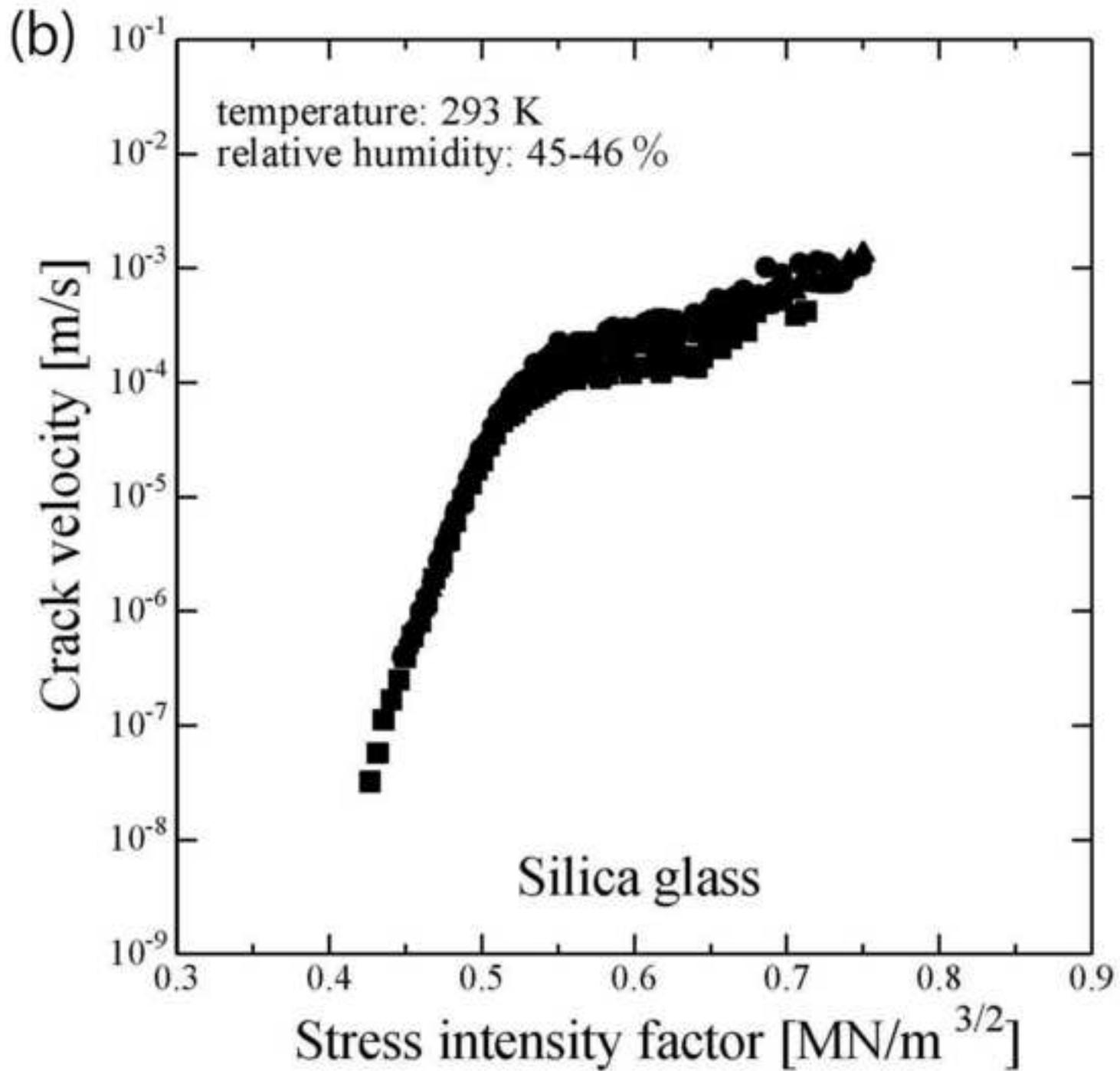
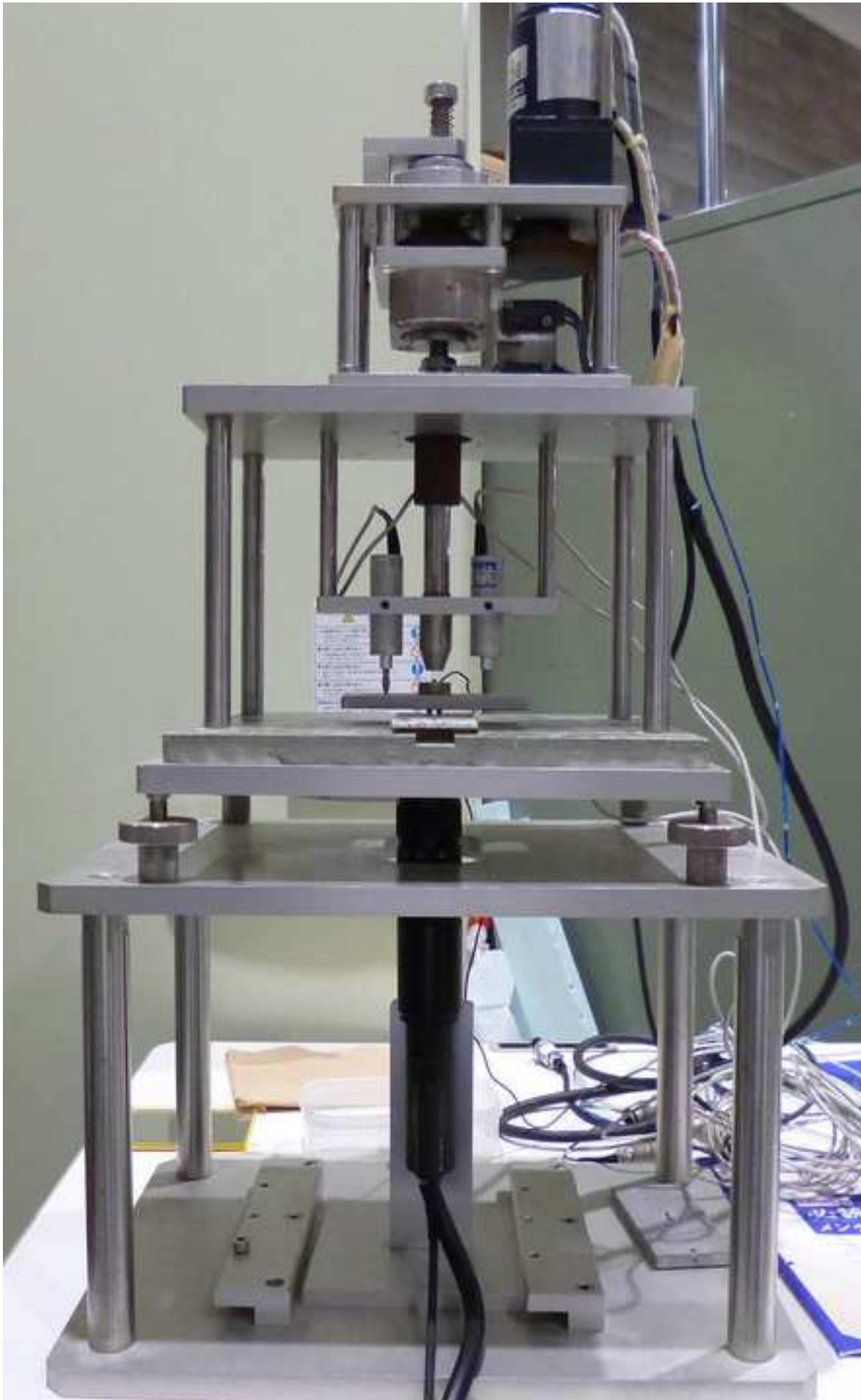


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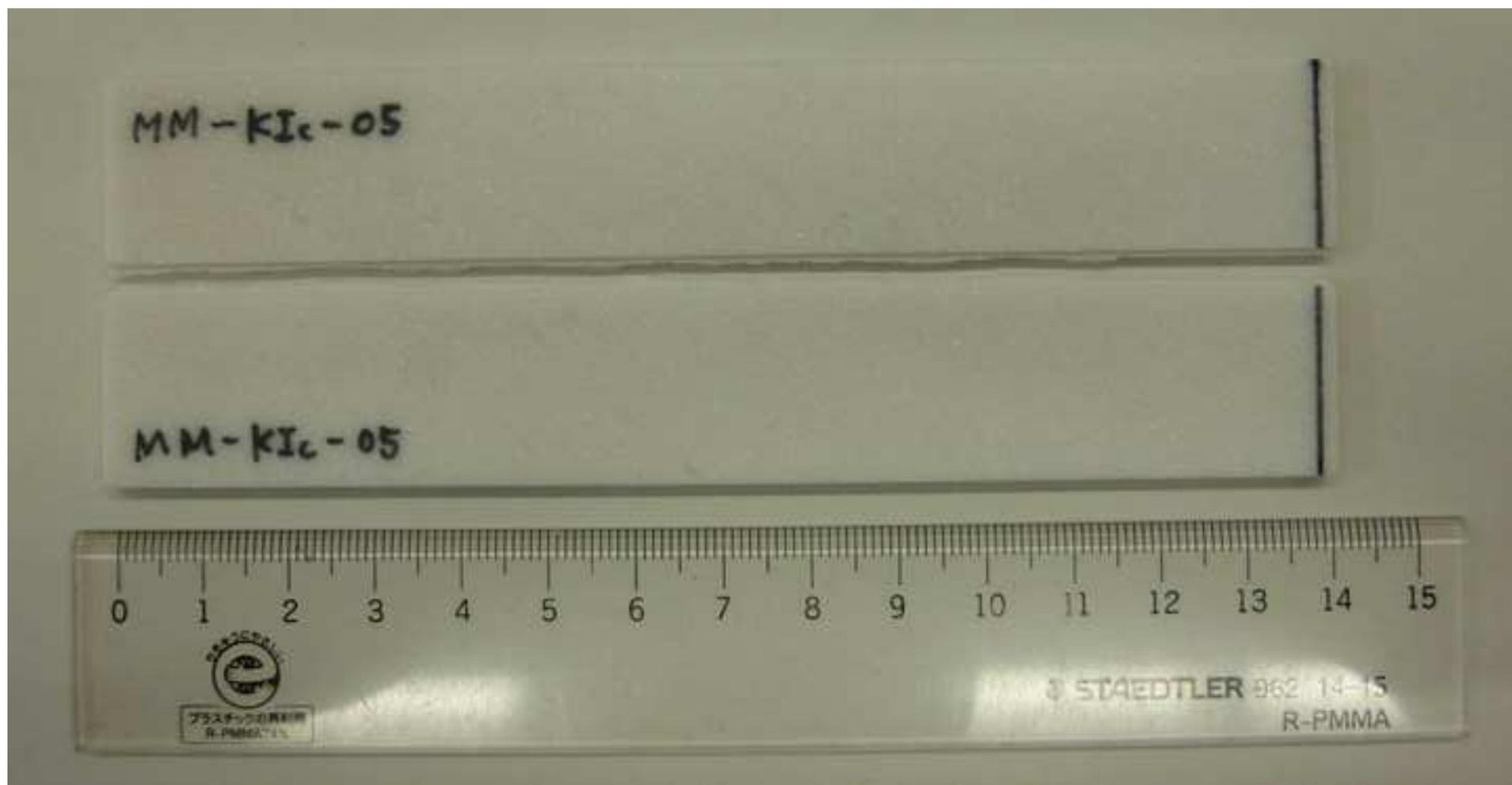


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FigureA2

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