Out-of-time-ordered correlator and entanglement scrambling in RCFTs

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In this thesis we consider the dynamics of excited states in rational CFTs. We study so called out-of-time-ordered correlators and entanglement scrambling in rational CFTs. We obtain the results that are different from chaotic CFTs such as holographic CFTs, which reflects the integrability of rational CFTs.

There are two useful quantities which characterize chaotic/integrable behavior in many body systems. First one is character is so called out-of-time-ordered (OTO) correlators. OTO correlators can extract the Lyapunov exponents and the scrambling time if we apply to chaotic systems. Another character is the scrambling of entanglement in excited states. This can be quantify the depth of quasi particle dip in the time evolution of entanglement entropy after global quenches. This dip is related to the number of conserved currents $c_{currents}$, which is an analog of the number of conserved charges in Hamiltonian systems. These two characters do not seem to be orthogonal, though we do not know the precise relation between them. Technically, though both of them are related to two different limits of conformal blocks in CFTs. Therefore, the study of both of OTO correlators and scrambling of entanglement is important.

In chapter 4, we consider the OTO correlators in rational CFTs, which is one of the main part of this thesis. First we explain the way to calculate OTO correlators in two dimensional conformal field theories. Finally, we argue that the monodromy of conformal block are important to calculate OTO correlators $C(t)$ in CFTs. Then, we obtain the universal results on late time value of OTO correlators in rational CFTs:

$$\lim_{t\to\infty} C_{ij}(t) = \frac{S^*_{ij}}{S_{00}}\frac{S_{0i}}{S_{00}}\frac{S_{0j}}{S_{00}}, \tag{1}$$

where S_{ij} is modular S matrix. This formula is one of the main results in this thesis. From this formula, we can see the different behavior from holographic CFTs. In holographic CFTs, OTO correlators decay to 0 in late time. On the other hand, this formula shows that the OTO correlators take some finite values at late time in rational CFTs.

Moreover, we study the large c limit of WZW $SU(N)_k$ models in detail, which have two parameters level k and rank N. To extract the Lyapunov exponents and scrambling time from OTO correlators, we need to take the large c limit. $SU(N)_k$ WZW model is one of

rational CFTs with the large c limit. Keeping the ratio $\lambda = N/k$ we take large N and large k limit, in which the central charge also becomes infinite. We consider the OTO correlators of fundamental representation fields. Then, the late time value becomes 1 regardless of λ. We also study the large λ limit of OTO correlators. In this case, we can approximate the four point function by free fermions. Therefore, we can analyze the OTO correlator at any time. From this expression we can explicitly show that the value of the OTO correlator does not change under the time evolution. This result suggests that the Lyapunov exponent is equal to 0, which is compatible with the expectation that the rational CFTs are integrable. This result also suggest that the actual Lyapunov exponents are 0 in rational CFTs.

In chapter 5, we consider the scattering effect on entanglement propagation in rational CFTs, which is another main part of this thesis. The motivation is to see how the interaction change the structure of entanglement. Generically, the scattering event affects the entanglement propagation. On the other hand in rational CFTs, which have special interaction, we also expect something special on entanglement propagation.

First, we derive the formula for entanglement entropy after the single local excitation:

$$\Delta S_A = \log d_a, \tag{2}$$

where d_a is the quantum dimension of the primary operator $\mathcal{O}_a$ used for the excitation. This is interpreted as the entanglement between quasi particles created by the insertion of the local operator. This is the different result from holographic CFTs, which are chaotic.

Next, we derive the following formula for entanglement entropy after the multiple local excitations by the insertion of primary operators $\mathcal{O}_{a_i}$:

$$S_A^i = S_A^f = \sum_{p=1}^{k} \log d_{a_p}. \tag{3}$$

This formula shows that entanglement entropy is not changed after the scattering event of quasi particles. Therefore, there are no scattering effect on entanglement propagation in rational CFTs. We also argue that the finiteness of the quantum dimension d_a is important to derive the no scattering effect. Therefore, we can expect that there are effect of scattering effect in chaotic CFTs like holographic theories.

From these results, we find both of OTO correlators and entanglement after local excitations shows the integrable behavior.