Coarse Grid Strategies for Computationally Efficient Flash Flood Simulations

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Keywords: Shallow Water Equations with Porosity, Friction Law-based Upscaling, Forecasting / Nowcasting, Case Studies

Flash floods are localized, sudden events characterized by a high rise of the water table in very short time usually caused by localized rainfall events with very high intensity. The short time period and relative small scale of flash flood events rise challenges in flood protection measures as well as the model-based forecasting of these events. Here, the resolution of large-scale weather prediction and hydrological models often is not sufficient to capture these very short and localized rainfall events and their consequences. In recent years, urban flood modeling is carried out with two-dimensional numerical models, which are able to reproduce the effects of buildings and other obstacles in city environment on the flood wave. These models are computationally expensive and real world applications are usually run on supercomputers. The main factor of the computational cost is the high-resolution required to capture the building and topography effects acccurately. In this presentation, scaling strategies (sometimes referred to as *coarse grid strategies*) for the efficient numerical modeling of flash floods are presented. Coarse grid strategies aim to reduce computational effort by not directly discretizing but conceptually accounting for buildings or topography. This allows coarser grids with less cells, therefore reducing the computational cost while obtaining similar accuracy of the results (water levels, flooding areas, arrival time of flood peaks) when compared to fine grid (high-resolution) simulations. Two approaches are shown: (1) friction-law based approach, wherein the unresolved structures are described with an increased roughness coefficient and (2) anisotropic porosity approach where the fraction of the cell available to flow is described via a porosity term. The models are applied in case studies ranging from laboratory experiments to rainfall-runoff in small natural catchments. The presented case studies show that the friction-law based approach is more suitable for numerical rainfall-runoff simulations in small catchments while the anisotropic porosity approach performs better for flood modeling in urban environment. Limitations and capabilities of both approaches are discussed. Both model concepts speed up the calculation significantly, in most cases at least two orders of magnitude.

Coarse grid methods for computationally efficient flash flood simulations

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25 - 27 October 2016
2nd International Symposium on Flash Floods in Wadi Systems 2016,
TU Berlin Campus El Gouna, Egypt

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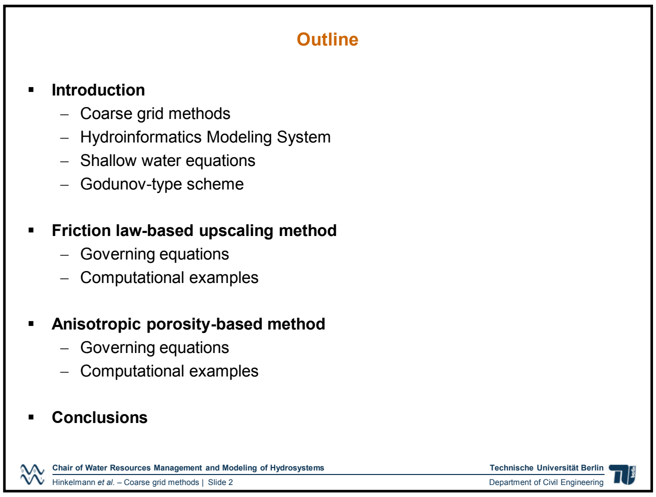
* Alexander von Humboldt Research Fellowship is gratefully acknowledged.



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- **Anisotropic porosity-based method**
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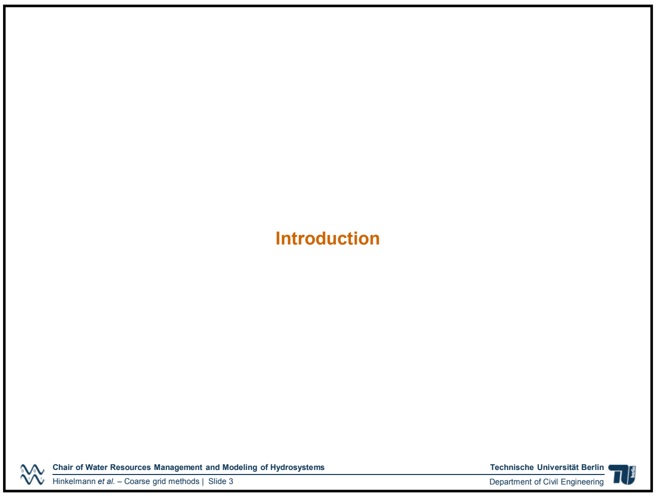
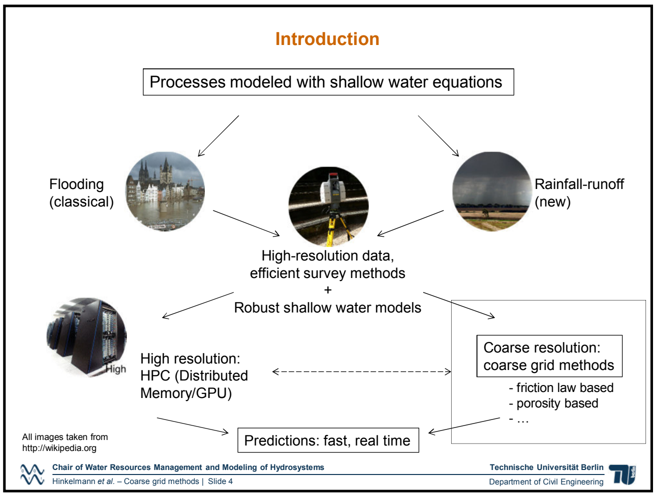


Introduction



Introduction

Processes modeled with shallow water equations

Flooding (classical)

Rainfall-runoff (new)

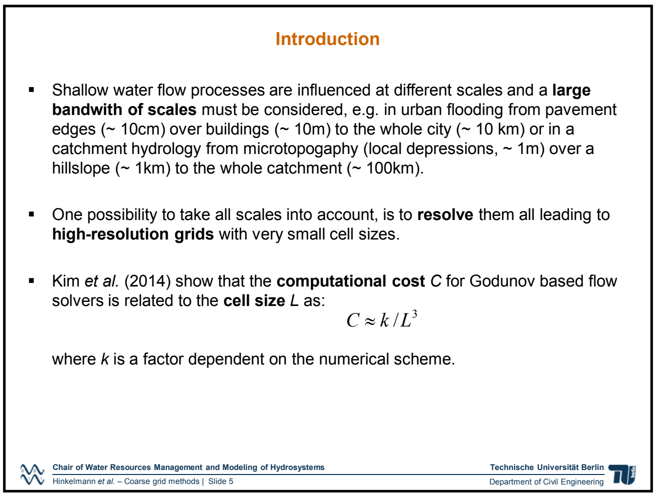
High-resolution data, efficient survey methods
+
Robust shallow water models

High resolution: HPC (Distributed Memory/GPU)

Coarse resolution: coarse grid methods
- friction law based
- porosity based
- …

Predictions: fast, real time

All images taken from http://wikipedia.org

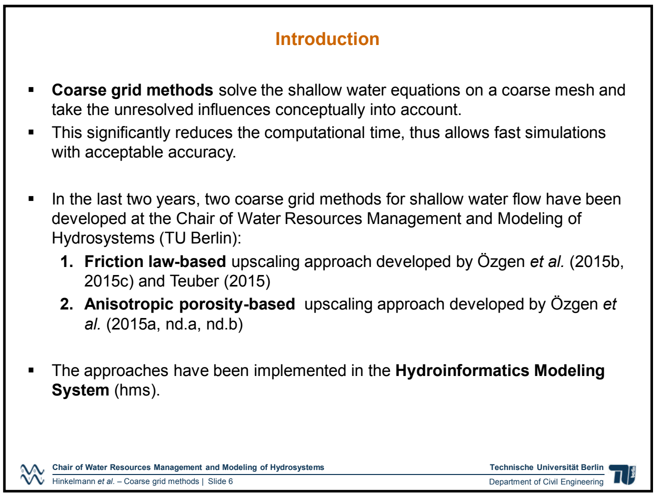


Introduction

- Shallow water flow processes are influenced at different scales and a **large bandwith of scales** must be considered, e.g. in urban flooding from pavement edges (~ 10cm) over buildings (~ 10m) to the whole city (~ 10 km) or in a catchment hydrology from microtopogaphy (local depressions, ~ 1m) over a hillslope (~ 1km) to the whole catchment (~ 100km).
- One possibility to take all scales into account, is to **resolve** them all leading to **high-resolution grids** with very small cell sizes.
- Kim *et al.* (2014) show that the **computational cost** *C* for Godunov based flow solvers is related to the **cell size** *L* as:

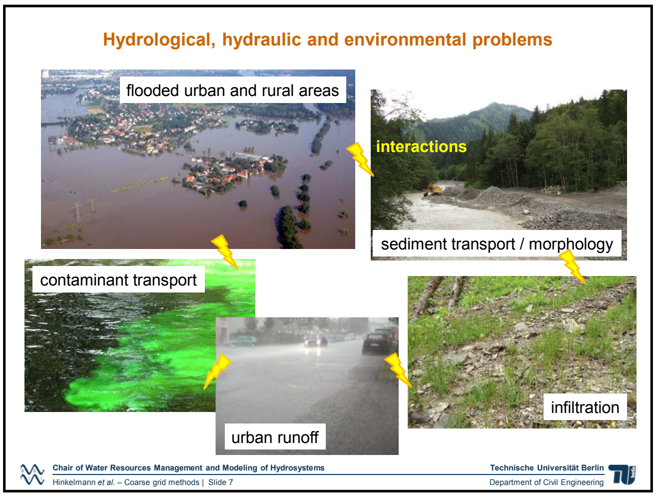
$$C \approx k / L^3$$

where *k* is a factor dependent on the numerical scheme.

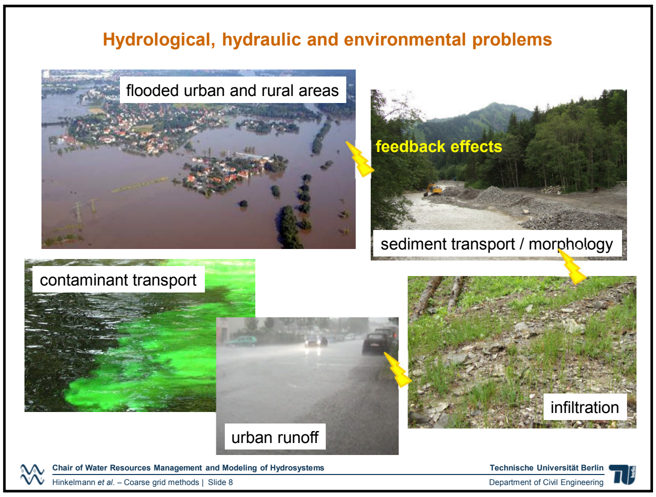


Introduction

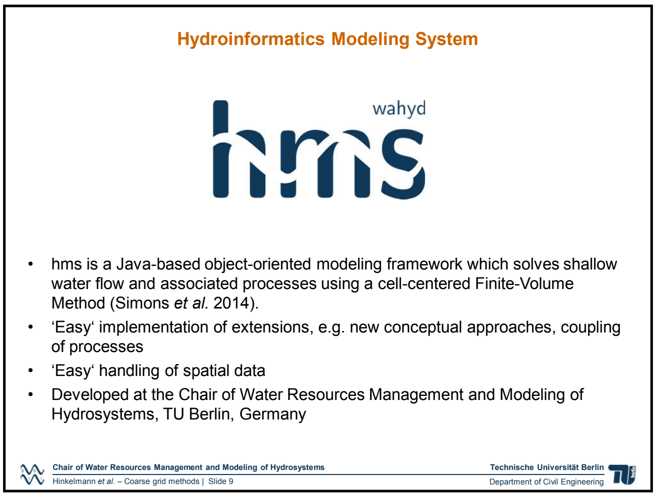
- **Coarse grid methods** solve the shallow water equations on a coarse mesh and take the unresolved influences conceptually into account.
- This significantly reduces the computational time, thus allows fast simulations with acceptable accuracy.
- In the last two years, two coarse grid methods for shallow water flow have been developed at the Chair of Water Resources Management and Modeling of Hydrosystems (TU Berlin):
  1. **Friction law-based** upscaling approach developed by Özgen *et al.* (2015b, 2015c) and Teuber (2015)
  2. **Anisotropic porosity-based** upscaling approach developed by Özgen *et al.* (2015a, nd.a, nd.b)
- The approaches have been implemented in the **Hydroinformatics Modeling System** (hms).



Hydrological, hydraulic and environmental problems



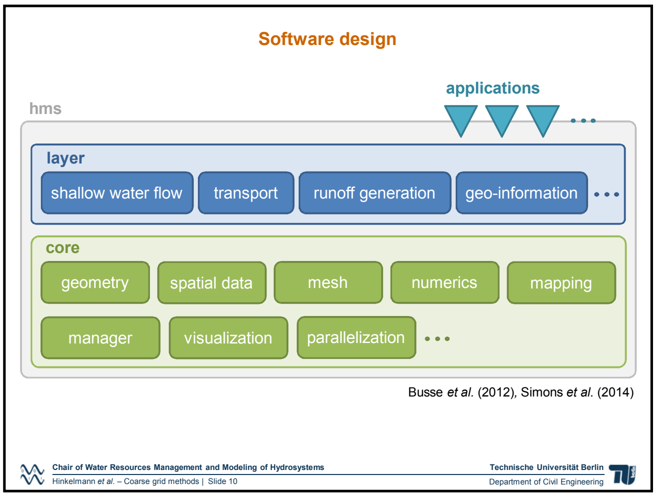
Hydrological, hydraulic and environmental problems



Hydroinformatics Modeling System

wahyd
hms

- hms is a Java-based object-oriented modeling framework which solves shallow water flow and associated processes using a cell-centered Finite-Volume Method (Simons *et al.* 2014).
- 'Easy' implementation of extensions, e.g. new conceptual approaches, coupling of processes
- 'Easy' handling of spatial data
- Developed at the Chair of Water Resources Management and Modeling of Hydrosystems, TU Berlin, Germany



Software design

applications

hms

layer

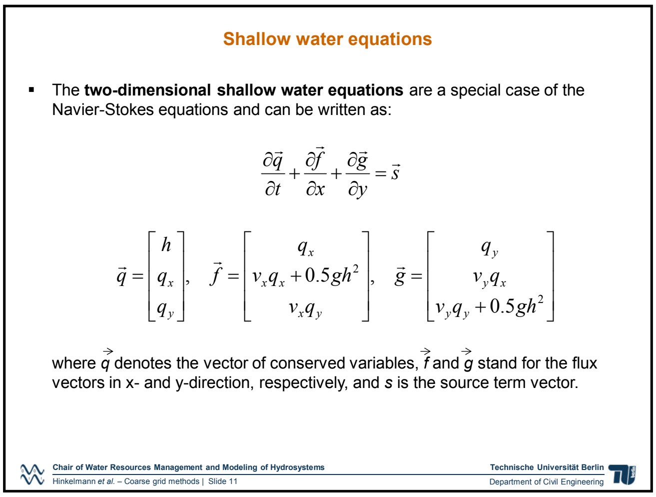
shallow water flow | transport | runoff generation | geo-information | • • •

core

geometry | spatial data | mesh | numerics | mapping

manager | visualization | parallelization | • • •

Busse *et al.* (2012), Simons *et al.* (2014)



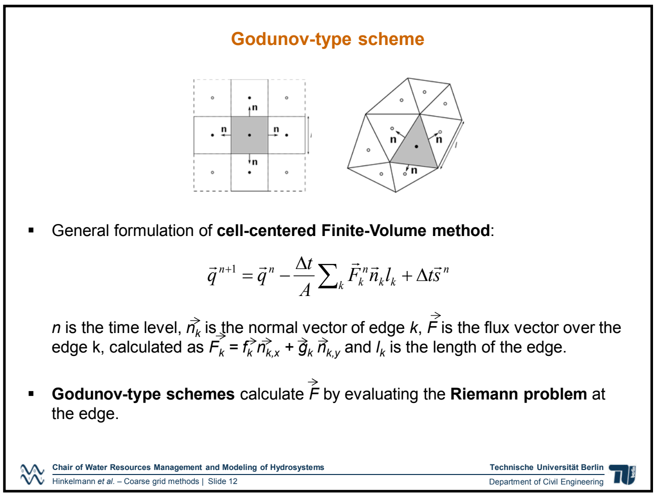
Shallow water equations

- The **two-dimensional shallow water equations** are a special case of the Navier-Stokes equations and can be written as:

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{g}}{\partial y} = \vec{s}$$

$$\vec{q} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix}, \quad \vec{f} = \begin{bmatrix} q_x \\ v_x q_x + 0.5gh^2 \\ v_x q_y \end{bmatrix}, \quad \vec{g} = \begin{bmatrix} q_y \\ v_y q_x \\ v_y q_y + 0.5gh^2 \end{bmatrix}$$

where $\vec{q}$ denotes the vector of conserved variables, $\vec{f}$ and $\vec{g}$ stand for the flux vectors in x- and y-direction, respectively, and *s* is the source term vector.



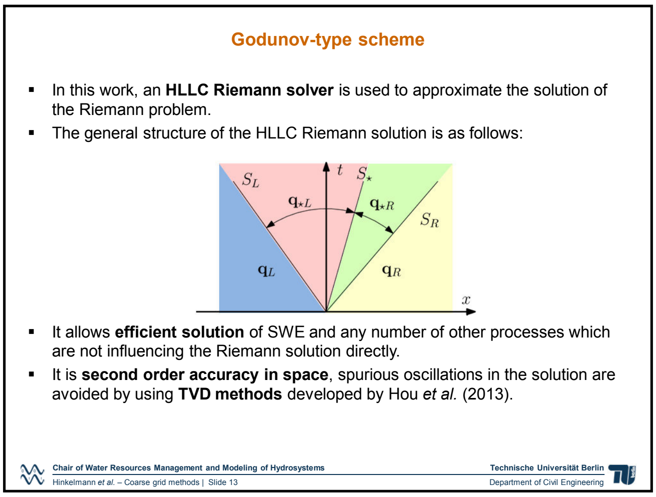
Godunov-type scheme

- General formulation of **cell-centered Finite-Volume method**:

$$\vec{q}^{\,n+1} = \vec{q}^{\,n} - \frac{\Delta t}{A} \sum_k \vec{F}_k^{\,n} \vec{n}_k l_k + \Delta t \vec{s}^{\,n}$$

n is the time level, $\vec{n}_k$ is the normal vector of edge *k*, $\vec{F}$ is the flux vector over the edge k, calculated as $\vec{F}_k = \vec{f}_k \vec{n}_{k,x} + \vec{g}_k \vec{n}_{k,y}$ and l_k is the length of the edge.

- **Godunov-type schemes** calculate $\vec{F}$ by evaluating the **Riemann problem** at the edge.

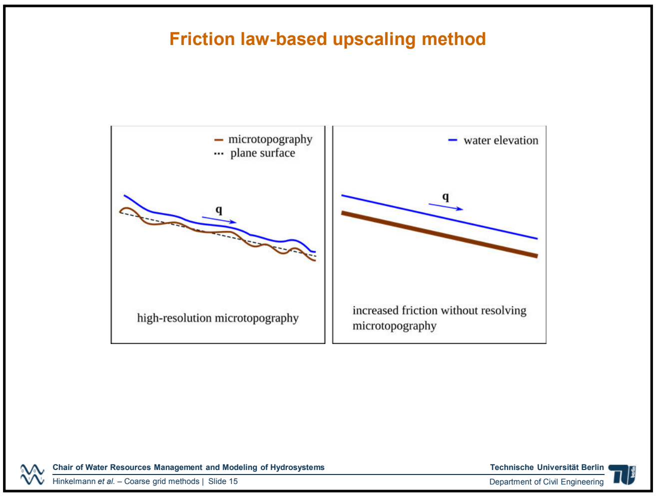


Godunov-type scheme

- In this work, an **HLLC Riemann solver** is used to approximate the solution of the Riemann problem.
- The general structure of the HLLC Riemann solution is as follows:

- It allows **efficient solution** of SWE and any number of other processes which are not influencing the Riemann solution directly.
- It is **second order accuracy in space**, spurious oscillations in the solution are avoided by using **TVD methods** developed by Hou *et al.* (2013).

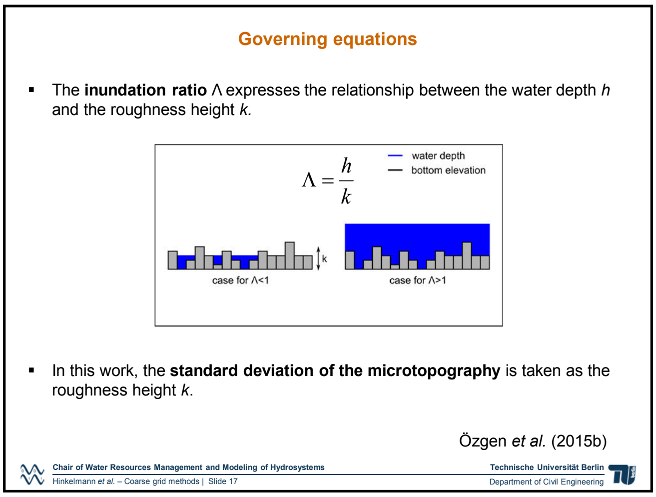
Friction law-based upscaling method



Friction law-based upscaling method

Governing equations

- Many traditional friction laws relate the friction to the water depth *h*.
- The developed upscaling approach uses a dimensionless variable Λ, called **inundation ratio**, instead of the water depth.
- The inundation ratio was initially introduced by Lawrence (1997).



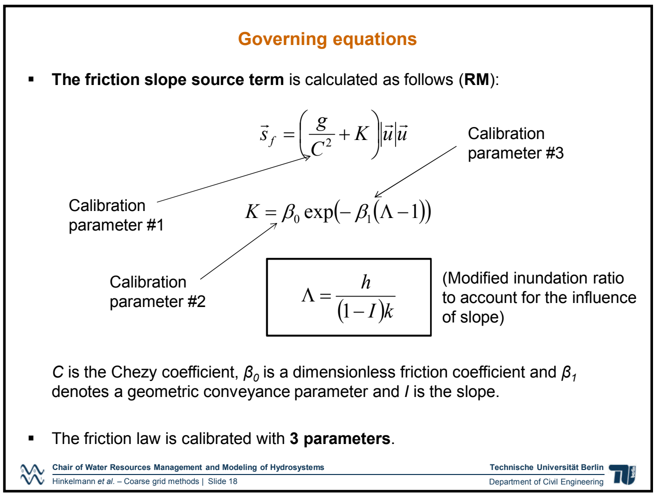
Governing equations

- The **inundation ratio** Λ expresses the relationship between the water depth *h* and the roughness height *k*.

$$\Lambda = \frac{h}{k}$$

- In this work, the **standard deviation of the microtopography** is taken as the roughness height *k*.

Özgen *et al.* (2015b)



Governing equations

- **The friction slope source term** is calculated as follows (**RM**):

$$\vec{s}_f = \left(\frac{g}{C^2} + K\right)|\vec{u}|\vec{u}$$

Calibration parameter #3

Calibration parameter #1

$$K = \beta_0 \exp\left(-\beta_1\left(\Lambda - 1\right)\right)$$

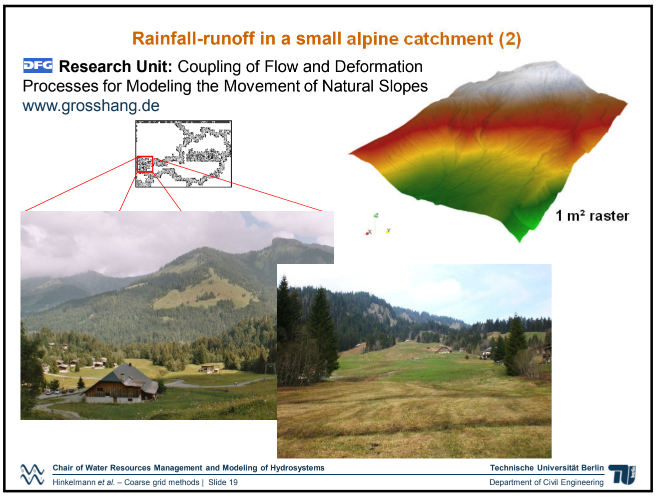
Calibration parameter #2

$$\boxed{\Lambda = \frac{h}{\left(1 - I\right)k}}$$

(Modified inundation ratio to account for the influence of slope)

C is the Chezy coefficient, β_0 is a dimensionless friction coefficient and β_1 denotes a geometric conveyance parameter and *I* is the slope.

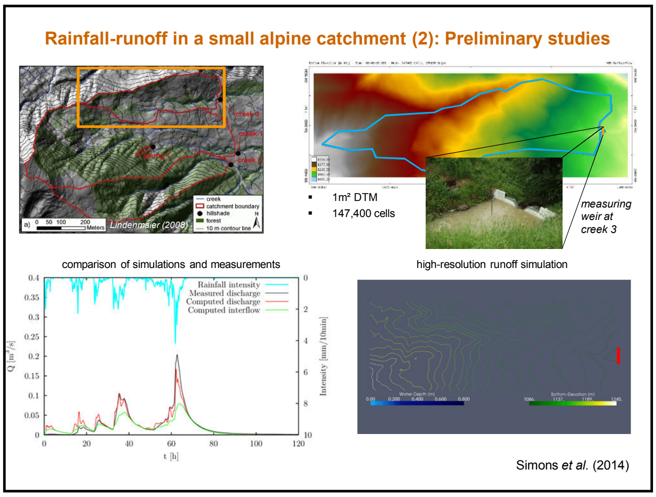
- The friction law is calibrated with **3 parameters**.



Rainfall-runoff in a small alpine catchment (2)

DFG **Research Unit:** Coupling of Flow and Deformation Processes for Modeling the Movement of Natural Slopes
www.grosshang.de

1 m² raster



Rainfall-runoff in a small alpine catchment (2): Preliminary studies

Lindenmaier (2008)

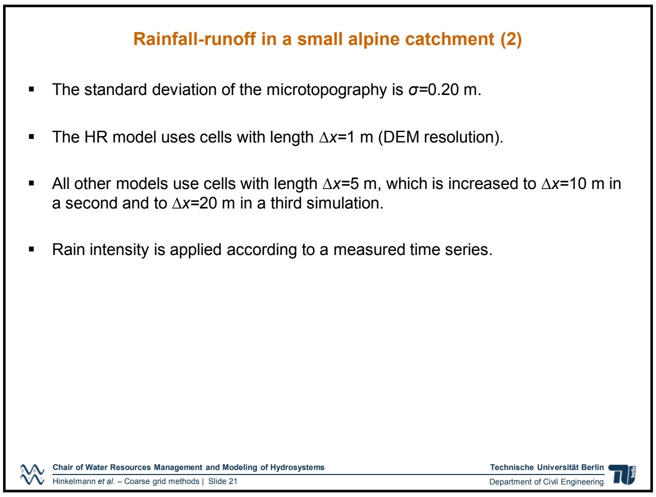
- 1m² DTM
- 147,400 cells

measuring weir at creek 3

comparison of simulations and measurements

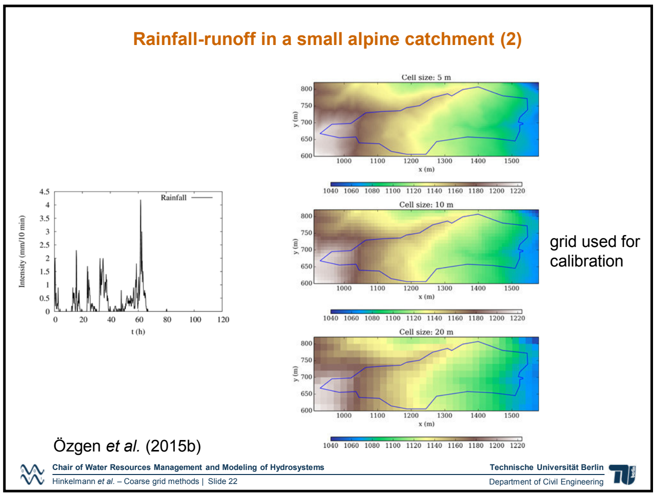
high-resolution runoff simulation

Simons *et al.* (2014)



Rainfall-runoff in a small alpine catchment (2)

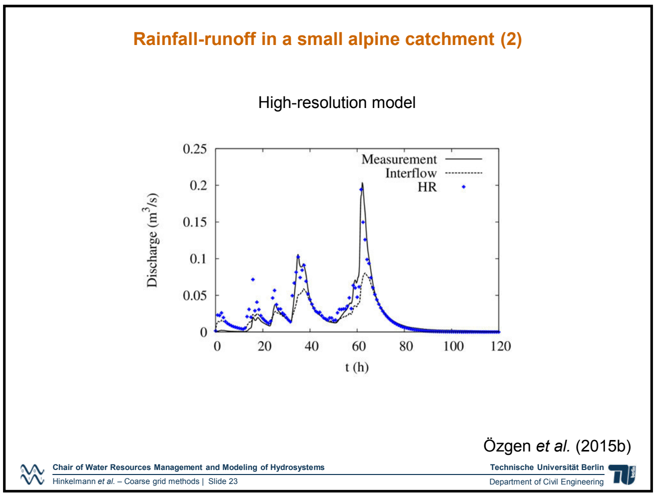
- The standard deviation of the microtopography is σ=0.20 m.
- The HR model uses cells with length Δx=1 m (DEM resolution).
- All other models use cells with length Δx=5 m, which is increased to Δx=10 m in a second and to Δx=20 m in a third simulation.
- Rain intensity is applied according to a measured time series.



Rainfall-runoff in a small alpine catchment (2)

grid used for calibration

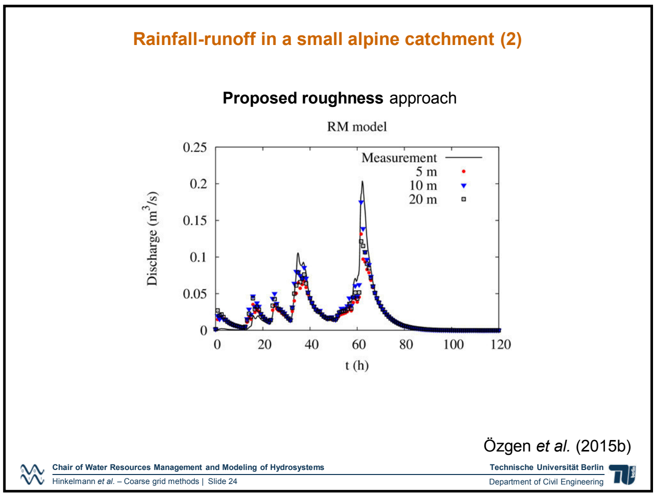
Özgen *et al.* (2015b)



Rainfall-runoff in a small alpine catchment (2)

High-resolution model

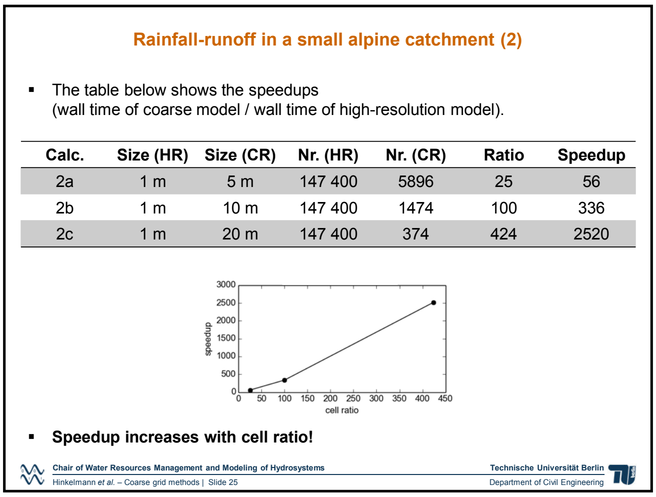
Özgen *et al.* (2015b)



Rainfall-runoff in a small alpine catchment (2)

Proposed roughness approach

Özgen *et al.* (2015b)

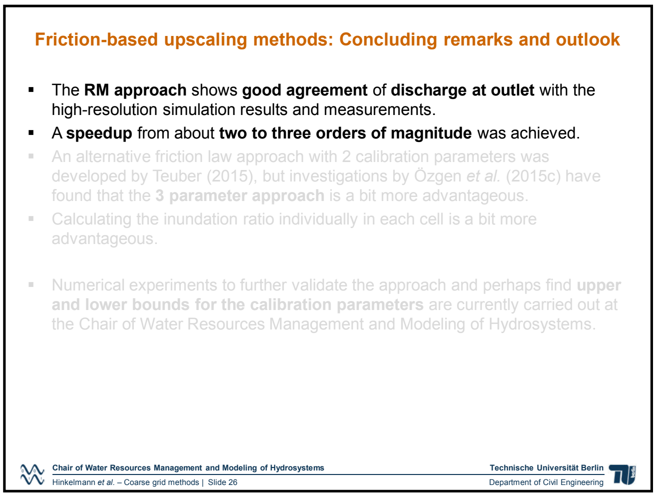


Rainfall-runoff in a small alpine catchment (2)

- The table below shows the speedups
  (wall time of coarse model / wall time of high-resolution model).

Calc.	Size (HR)	Size (CR)	Nr. (HR)	Nr. (CR)	Ratio	Speedup
2a	1 m	5 m	147 400	5896	25	56
2b	1 m	10 m	147 400	1474	100	336
2c	1 m	20 m	147 400	374	424	2520

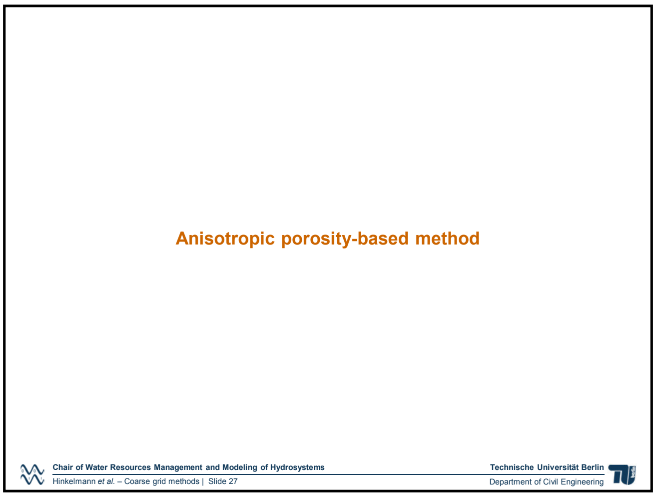
- **Speedup increases with cell ratio!**



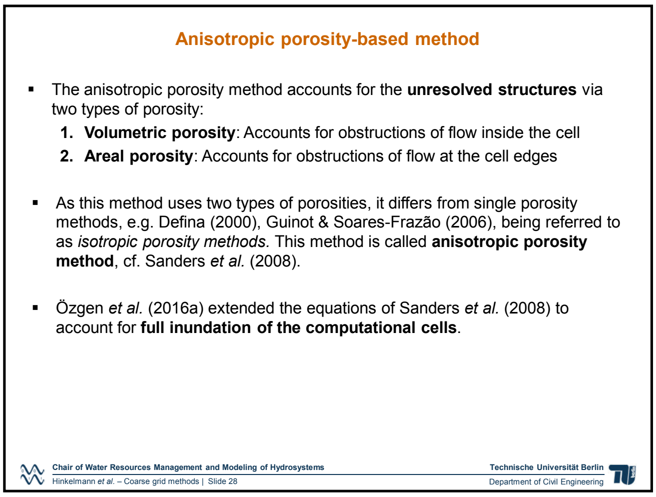
Friction-based upscaling methods: Concluding remarks and outlook

- The **RM approach** shows **good agreement** of **discharge at outlet** with the high-resolution simulation results and measurements.
- A **speedup** from about **two to three orders of magnitude** was achieved.
- An alternative friction law approach with 2 calibration parameters was developed by Teuber (2015), but investigations by Özgen *et al.* (2015c) have found that the **3 parameter approach** is a bit more advantageous.
- Calculating the inundation ratio individually in each cell is a bit more advantageous.

- Numerical experiments to further validate the approach and perhaps find **upper and lower bounds for the calibration parameters** are currently carried out at the Chair of Water Resources Management and Modeling of Hydrosystems.



Anisotropic porosity-based method

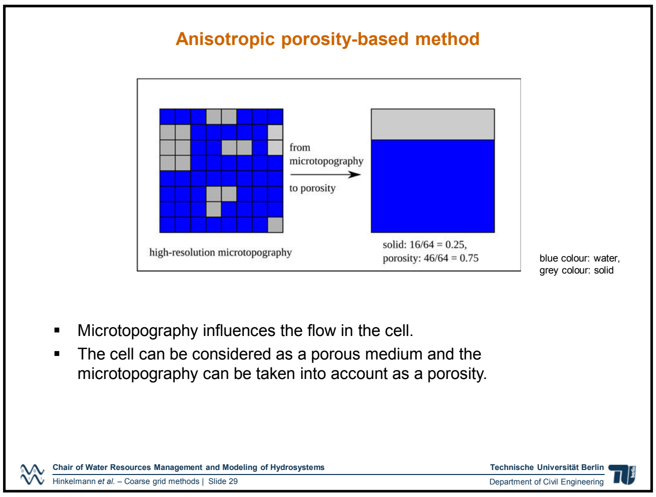


Anisotropic porosity-based method

- The anisotropic porosity method accounts for the **unresolved structures** via two types of porosity:
  1. **Volumetric porosity**: Accounts for obstructions of flow inside the cell
  2. **Areal porosity**: Accounts for obstructions of flow at the cell edges

- As this method uses two types of porosities, it differs from single porosity methods, e.g. Defina (2000), Guinot & Soares-Frazão (2006), being referred to as *isotropic porosity methods*. This method is called **anisotropic porosity method**, cf. Sanders *et al.* (2008).

- Özgen *et al.* (2016a) extended the equations of Sanders *et al.* (2008) to account for **full inundation of the computational cells**.

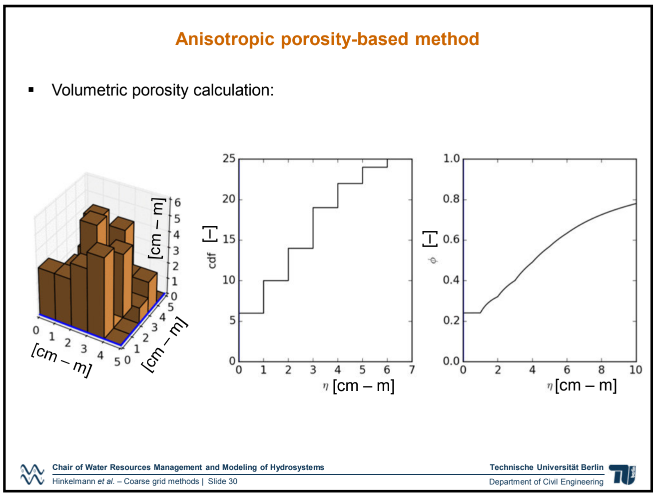


Anisotropic porosity-based method

solid: 16/64 = 0.25,
porosity: 46/64 = 0.75

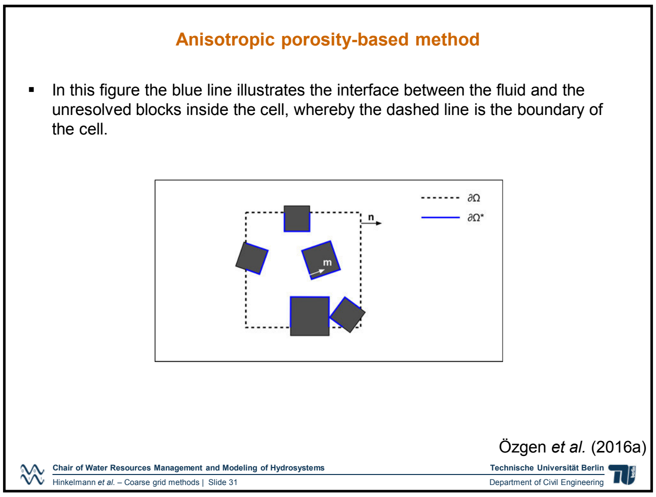
blue colour: water,
grey colour: solid

- Microtopography influences the flow in the cell.
- The cell can be considered as a porous medium and the microtopography can be taken into account as a porosity.



Anisotropic porosity-based method

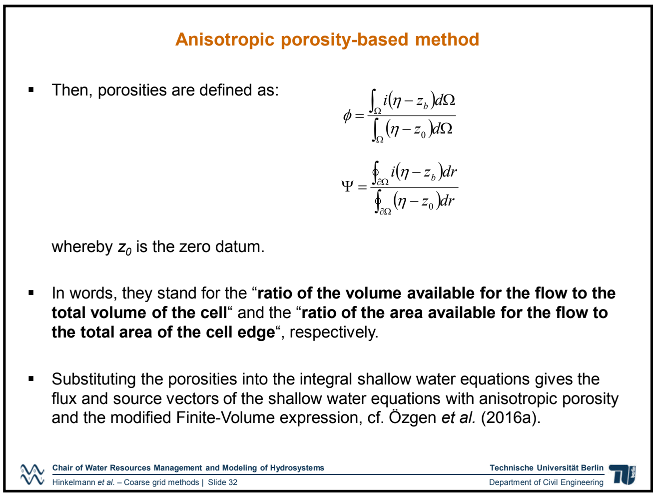
- Volumetric porosity calculation:



Anisotropic porosity-based method

- In this figure the blue line illustrates the interface between the fluid and the unresolved blocks inside the cell, whereby the dashed line is the boundary of the cell.

Özgen *et al.* (2016a)



Anisotropic porosity-based method

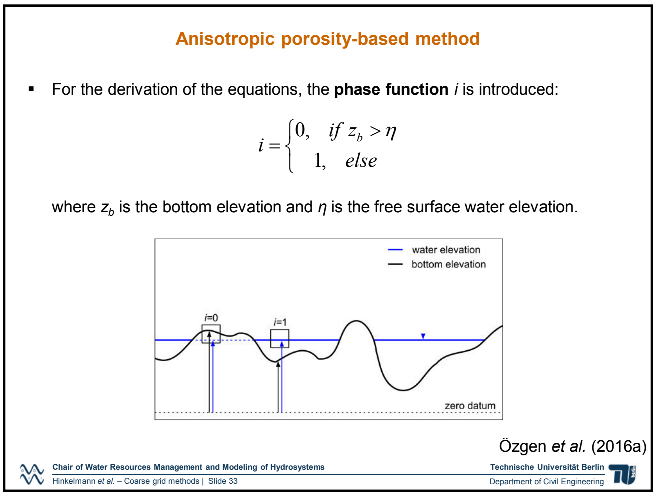
- Then, porosities are defined as:

$$\phi = \frac{\int_{\Omega} i(\eta - z_b)\,d\Omega}{\int_{\Omega} (\eta - z_0)\,d\Omega}$$

$$\Psi = \frac{\oint_{\partial\Omega} i(\eta - z_b)\,dr}{\oint_{\partial\Omega} (\eta - z_0)\,dr}$$

whereby z_0 is the zero datum.

- In words, they stand for the "**ratio of the volume available for the flow to the total volume of the cell**" and the "**ratio of the area available for the flow to the total area of the cell edge**", respectively.
- Substituting the porosities into the integral shallow water equations gives the flux and source vectors of the shallow water equations with anisotropic porosity and the modified Finite-Volume expression, cf. Özgen *et al.* (2016a).



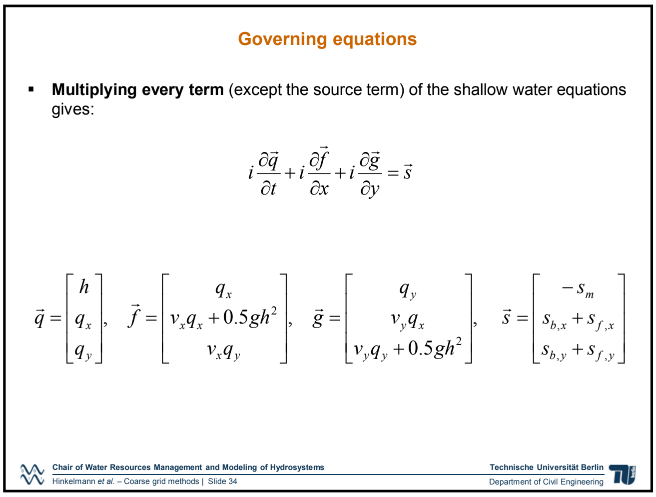
Anisotropic porosity-based method

- For the derivation of the equations, the **phase function** *i* is introduced:

$$i = \begin{cases} 0, & if\ z_b > \eta \\ 1, & else \end{cases}$$

where z_b is the bottom elevation and η is the free surface water elevation.

Özgen *et al.* (2016a)

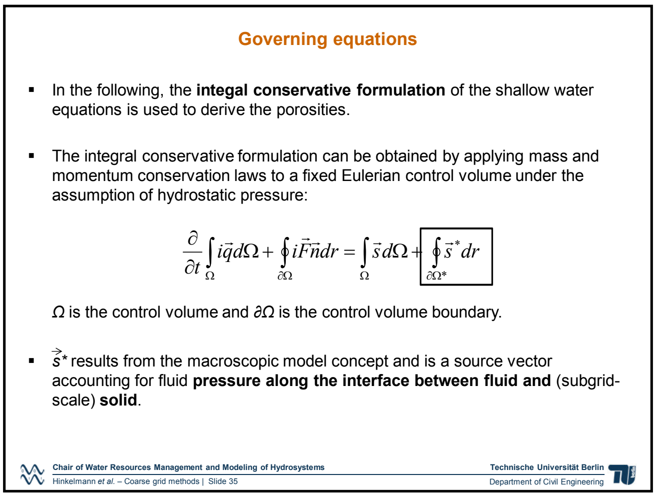


Governing equations

- **Multiplying every term** (except the source term) of the shallow water equations gives:

$$i\frac{\partial \vec{q}}{\partial t} + i\frac{\partial \vec{f}}{\partial x} + i\frac{\partial \vec{g}}{\partial y} = \vec{s}$$

$$\vec{q} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix}, \quad \vec{f} = \begin{bmatrix} q_x \\ v_x q_x + 0.5gh^2 \\ v_x q_y \end{bmatrix}, \quad \vec{g} = \begin{bmatrix} q_y \\ v_y q_x \\ v_y q_y + 0.5gh^2 \end{bmatrix}, \quad \vec{s} = \begin{bmatrix} -s_m \\ s_{b,x} + s_{f,x} \\ s_{b,y} + s_{f,y} \end{bmatrix}$$



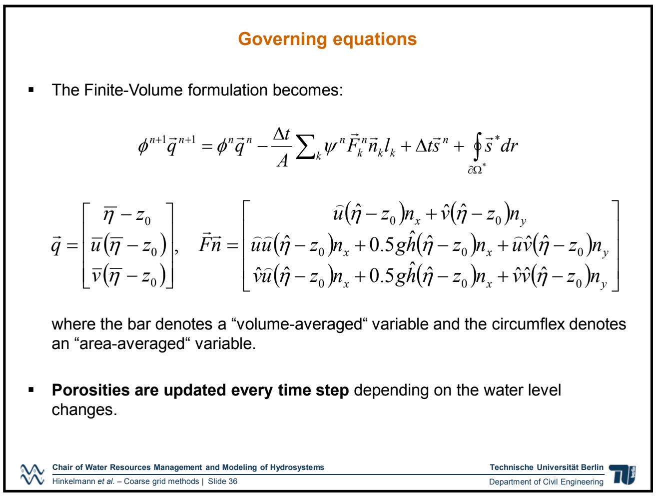
Governing equations

- In the following, the **integal conservative formulation** of the shallow water equations is used to derive the porosities.
- The integral conservative formulation can be obtained by applying mass and momentum conservation laws to a fixed Eulerian control volume under the assumption of hydrostatic pressure:

$$\frac{\partial}{\partial t}\int_{\Omega} i\vec{q}\,d\Omega + \oint_{\partial\Omega} i\vec{F}\vec{n}\,dr = \int_{\Omega} \vec{s}\,d\Omega + \boxed{\oint_{\partial\Omega^*} \vec{s}^*\,dr}$$

Ω is the control volume and ∂Ω is the control volume boundary.

- $\vec{s}^*$ results from the macroscopic model concept and is a source vector accounting for fluid **pressure along the interface between fluid and** (subgrid-scale) **solid**.



Governing equations

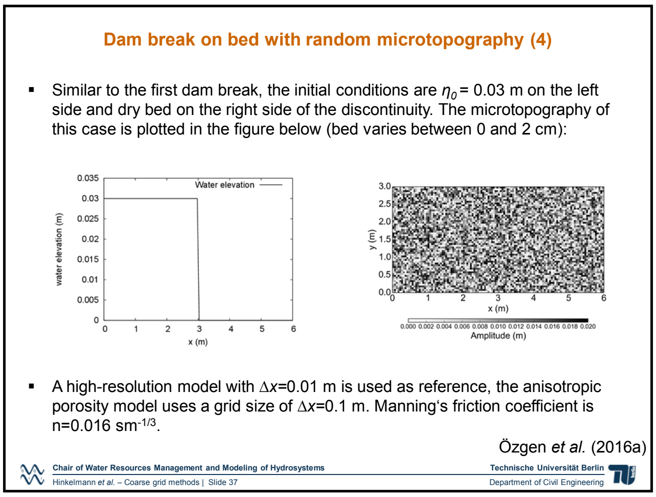
- The Finite-Volume formulation becomes:

$$\phi^{n+1}\vec{q}^{\,n+1} = \phi^{n}\vec{q}^{\,n} - \frac{\Delta t}{A}\sum_k \psi^{n}\vec{F}_k^{n}\vec{n}_k l_k + \Delta t\vec{s}^{\,n} + \oint_{\partial\Omega^*} \vec{s}^*\,dr$$

$$\vec{q} = \begin{bmatrix} \bar{\eta} - z_0 \\ \bar{u}(\bar{\eta} - z_0) \\ \bar{v}(\bar{\eta} - z_0) \end{bmatrix}, \quad \vec{F}\vec{n} = \begin{bmatrix} \hat{u}(\hat{\eta} - z_0)n_x + \hat{v}(\hat{\eta} - z_0)n_y \\ \hat{u}\hat{u}(\hat{\eta} - z_0)n_x + 0.5g\hat{h}(\hat{\eta} - z_0)n_x + \hat{u}\hat{v}(\hat{\eta} - z_0)n_y \\ \hat{v}\hat{u}(\hat{\eta} - z_0)n_x + 0.5g\hat{h}(\hat{\eta} - z_0)n_x + \hat{v}\hat{v}(\hat{\eta} - z_0)n_y \end{bmatrix}$$

where the bar denotes a "volume-averaged" variable and the circumflex denotes an "area-averaged" variable.

- **Porosities are updated every time step** depending on the water level changes.

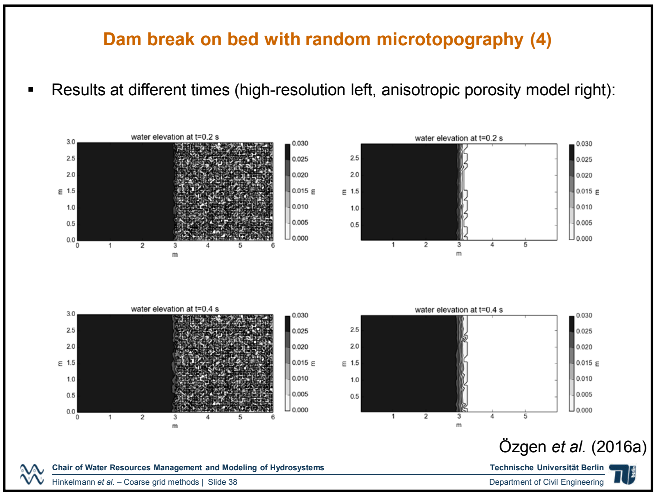


Dam break on bed with random microtopography (4)

- Similar to the first dam break, the initial conditions are η_0 = 0.03 m on the left side and dry bed on the right side of the discontinuity. The microtopography of this case is plotted in the figure below (bed varies between 0 and 2 cm):

- A high-resolution model with Δ*x*=0.01 m is used as reference, the anisotropic porosity model uses a grid size of Δ*x*=0.1 m. Manning‘s friction coefficient is n=0.016 sm$^{-1/3}$.

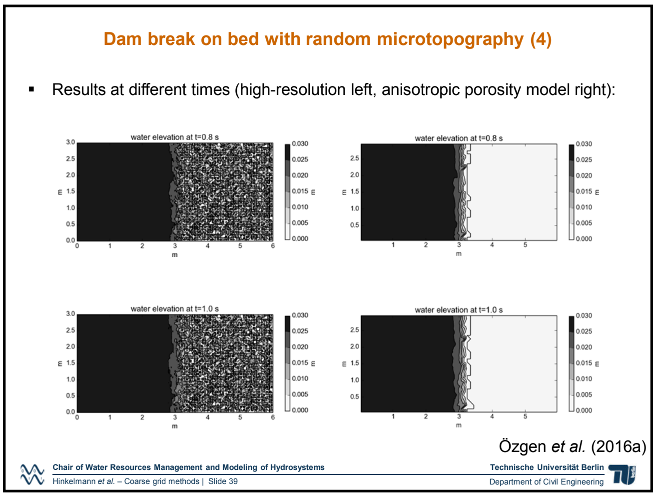
Özgen *et al.* (2016a)



Dam break on bed with random microtopography (4)

- Results at different times (high-resolution left, anisotropic porosity model right):

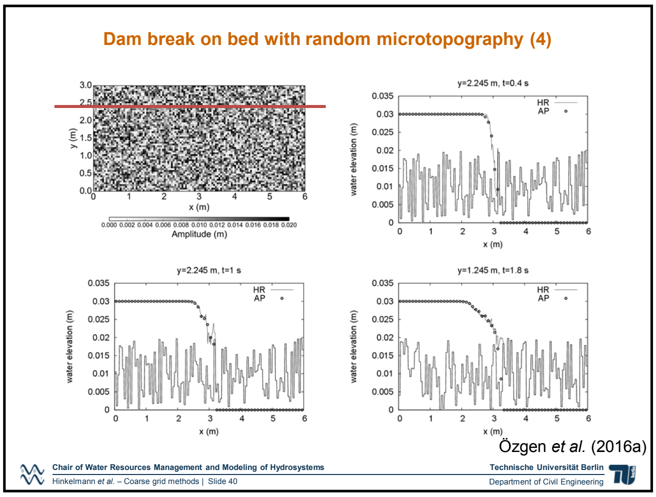
Özgen *et al.* (2016a)



Dam break on bed with random microtopography (4)

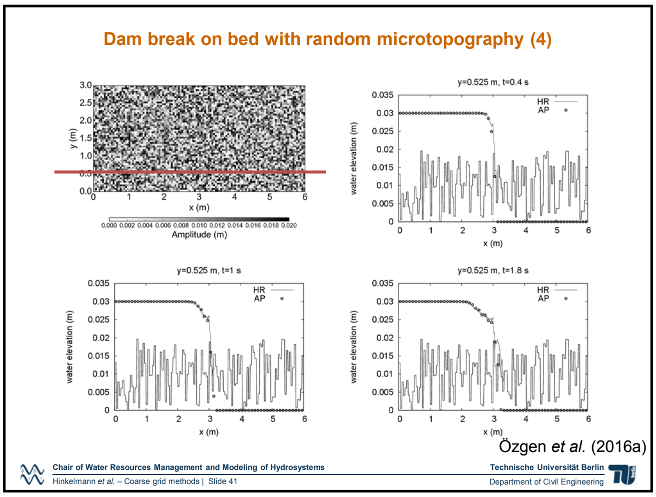
- Results at different times (high-resolution left, anisotropic porosity model right):

Özgen *et al.* (2016a)



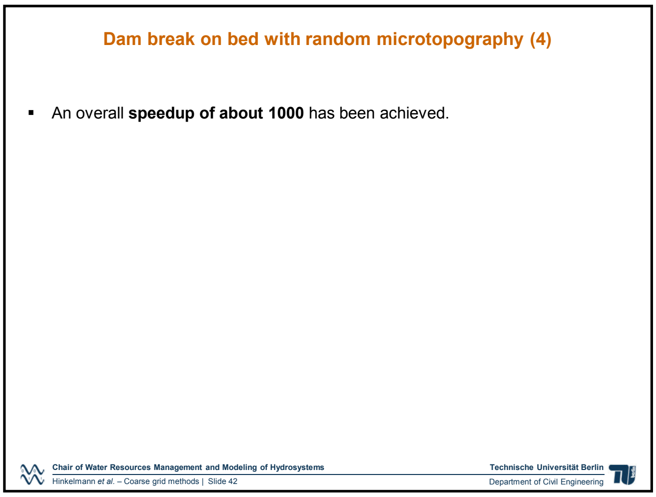
Dam break on bed with random microtopography (4)

Özgen *et al.* (2016a)



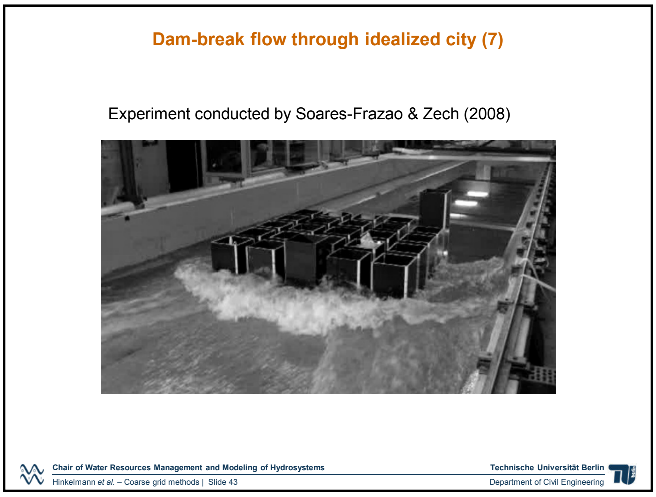
Dam break on bed with random microtopography (4)

Özgen *et al.* (2016a)



Dam break on bed with random microtopography (4)

- An overall **speedup of about 1000** has been achieved.



Dam-break flow through idealized city (7)

Experiment conducted by Soares-Frazao & Zech (2008)

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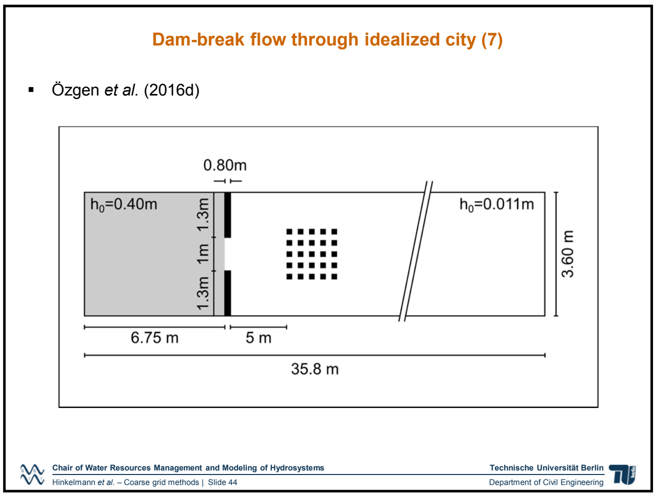

Dam-break flow through idealized city (7)

- Özgen *et al.* (2016d)

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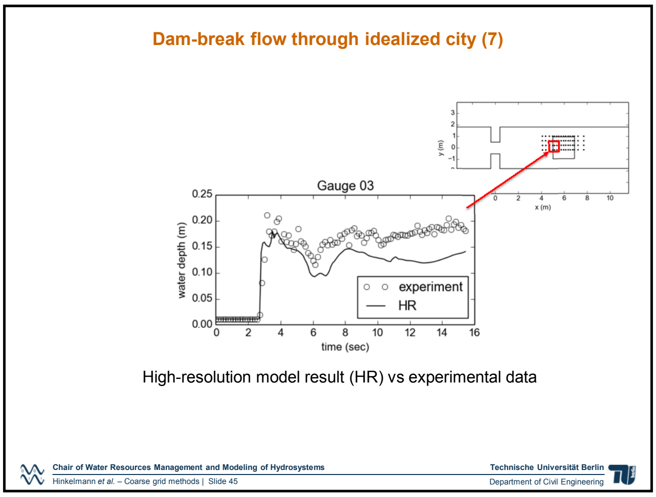

Dam-break flow through idealized city (7)

High-resolution model result (HR) vs experimental data

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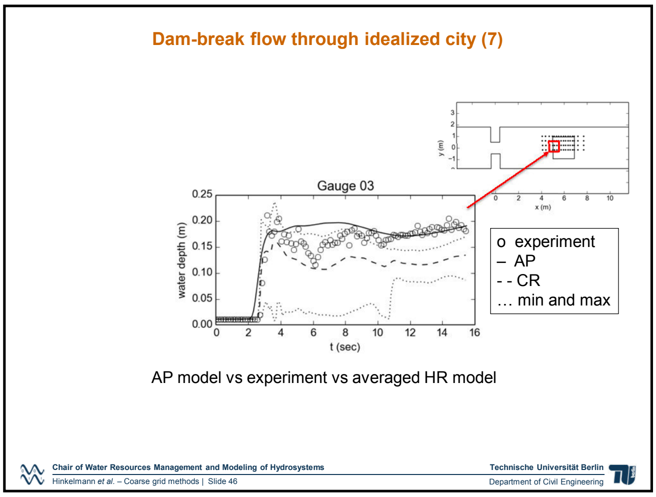

Dam-break flow through idealized city (7)

AP model vs experiment vs averaged HR model

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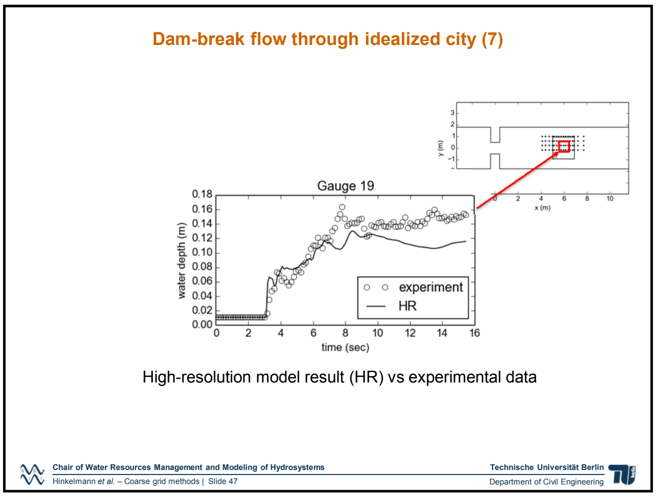

Dam-break flow through idealized city (7)

High-resolution model result (HR) vs experimental data

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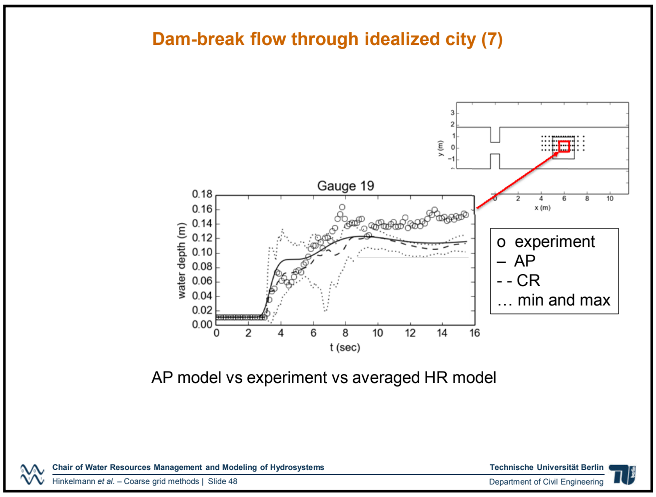

Dam-break flow through idealized city (7)

AP model vs experiment vs averaged HR model

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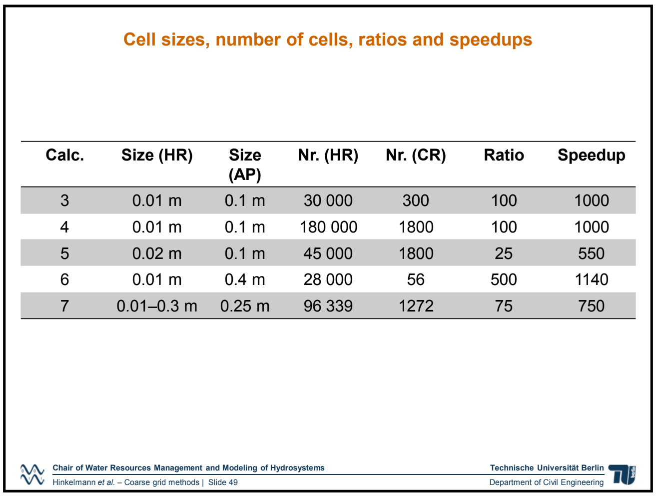
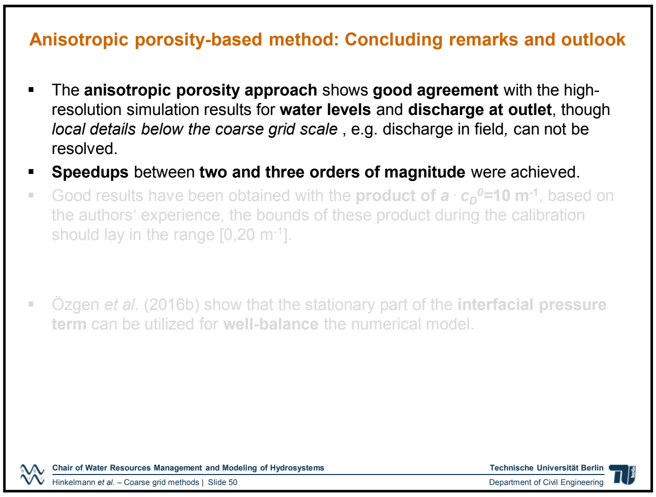


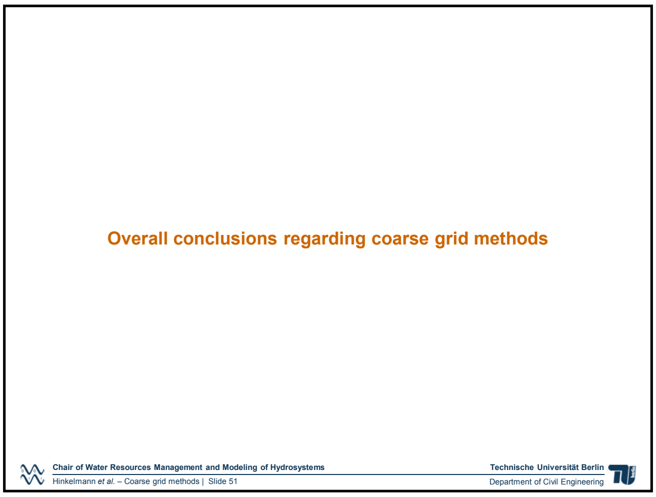
Cell sizes, number of cells, ratios and speedups

Calc.	Size (HR)	Size (AP)	Nr. (HR)	Nr. (CR)	Ratio	Speedup
3	0.01 m	0.1 m	30 000	300	100	1000
4	0.01 m	0.1 m	180 000	1800	100	1000
5	0.02 m	0.1 m	45 000	1800	25	550
6	0.01 m	0.4 m	28 000	56	500	1140
7	0.01–0.3 m	0.25 m	96 339	1272	75	750

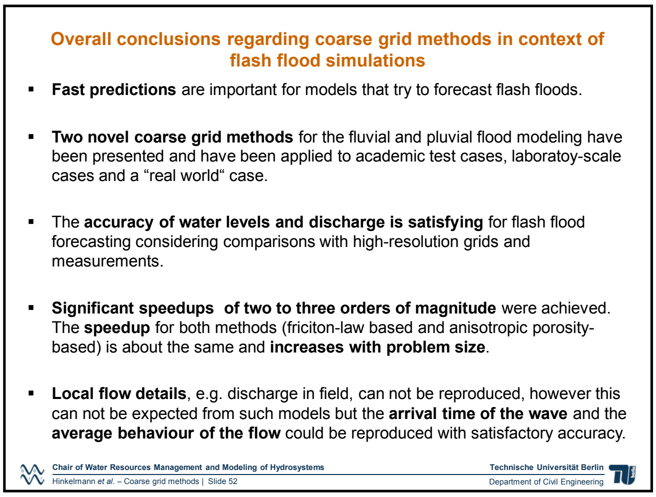


Anisotropic porosity-based method: Concluding remarks and outlook

- The **anisotropic porosity approach** shows **good agreement** with the high-resolution simulation results for **water levels** and **discharge at outlet**, though *local details below the coarse grid scale* , e.g. discharge in field, can not be resolved.
- **Speedups** between **two and three orders of magnitude** were achieved.
- Good results have been obtained with the **product of $a \cdot c_D^0$=10 m^{-1}**, based on the authors' experience, the bounds of these product during the calibration should lay in the range [0,20 m^{-1}].
- Özgen *et al.* (2016b) show that the stationary part of the **interfacial pressure term** can be utilized for **well-balance** the numerical model.

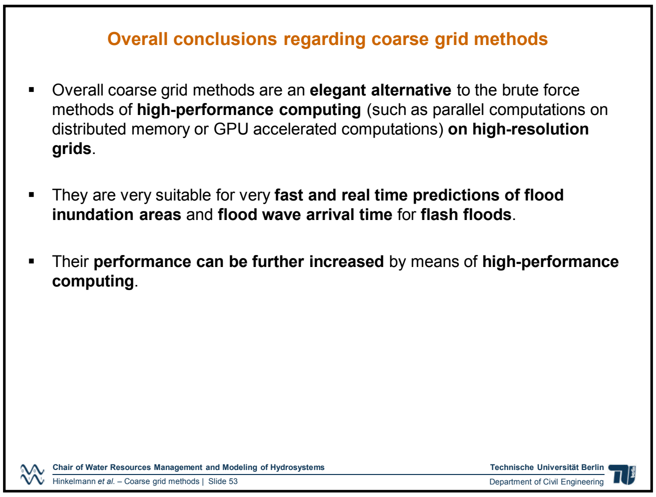


Overall conclusions regarding coarse grid methods



Overall conclusions regarding coarse grid methods in context of flash flood simulations

- **Fast predictions** are important for models that try to forecast flash floods.
- **Two novel coarse grid methods** for the fluvial and pluvial flood modeling have been presented and have been applied to academic test cases, laboratoy-scale cases and a "real world" case.
- The **accuracy of water levels and discharge is satisfying** for flash flood forecasting considering comparisons with high-resolution grids and measurements.
- **Significant speedups of two to three orders of magnitude** were achieved. The **speedup** for both methods (friciton-law based and anisotropic porosity-based) is about the same and **increases with problem size**.
- **Local flow details**, e.g. discharge in field, can not be reproduced, however this can not be expected from such models but the **arrival time of the wave** and the **average behaviour of the flow** could be reproduced with satisfactory accuracy.



Overall conclusions regarding coarse grid methods

- Overall coarse grid methods are an **elegant alternative** to the brute force methods of **high-performance computing** (such as parallel computations on distributed memory or GPU accelerated computations) **on high-resolution grids**.
- They are very suitable for very **fast and real time predictions of flood inundation areas** and **flood wave arrival time** for **flash floods**.
- Their **performance can be further increased** by means of **high-performance computing**.

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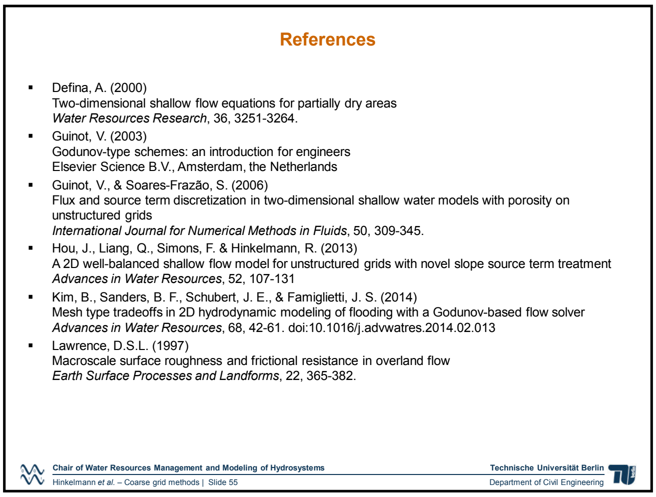
Coarse grid methods for computationally efficient flash flood simulations

END

R. Hinkelmann, I. Özgen, J.Zhao, K. Teuber & D. Liang



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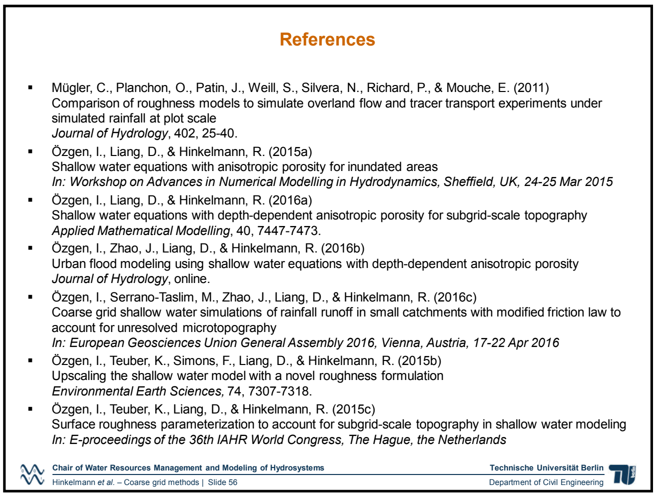

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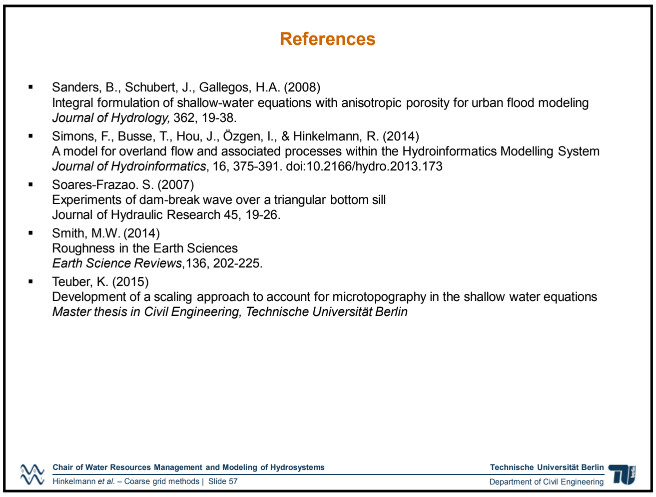

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