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under Rollover Risk and Solvency Concern

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Abstract

We consider the effects of the endogenous interaction between rollover risk and solvency concern—generated by not only debt rollover but also by an assessment regarding the firm's solvency risk via a learning process over time—on the decisions of the firm about debt maturity, default, investment, and leverage policies. We distinguish between short-term liquidity uncertainty and long-term solvency uncertainty in order to clarify how the two sources of uncertainty affect such decisions of the firm. If debt maturity is exogenously determined, it is important to note that the effect of long-term solvency uncertainty on the investment policy—debt overhang—is opposite to that of short-term liquidity uncertainty. If debt maturity is endogenously determined, we show that the equilibrium debt maturity increases (decreases) with short-term liquidity (long-term solvency) uncertainty when the chosen debt maturity is sufficiently long, and that for any debt maturity the firm's incentives to default increase (decrease) with short-term liquidity (long-term solvency) uncertainty whereas the firm's incentives to invest decrease (increase) with short-term liquidity (long-term solvency) uncertainty.

JEL Classification Codes: D83, G31, G32, G33.

Keywords: debt maturity, debt overhang, debt rollover, investment, learning.

1. Introduction

This paper studies the effects of the interaction between rollover risk and solvency concern on the debt maturity, default, investment, and leverage policy decisions of the firm under debt rollover and a learning process of the firm's solvency risk. Rollover risk in this paper means not only the possibility of the firm's failure to roll over debt, but also the fluctuations in rollover gains/losses incurred by the firm. The importance of the interaction between rollover risk and solvency concern in financial markets is verified by the financial crisis of 2007–2008: deterioration in rollover risk caused severe financial difficulties for many firms and exacerbated their solvency concerns, whereas the aggravated solvency concerns conversely increased rollover risk. In fact, because economic circumstances changed drastically during the financial crisis, market participants were forced to reconsider the solvency risk of the firm by examining its current and future profitability. This requires us to consider that the interaction between rollover risk and solvency concern over the firm endogenously arises from not only debt rollover but also from the assessment of the firm's solvency risk via the learning process of market participants. This requirement leads us to incorporate the learning process of the firm's solvency risk into the debt rollover model. The investigation of such an endogenous interaction under debt rollover and learning can derive new theoretical predictions and empirical implications about the debt maturity, default, investment, and leverage policies of the firm.

For this purpose, we suppose a firm that continuously evolves capital stock but must continuously roll over maturing bonds. As in Diamond and He (2014), the latter requirement forces equity holders to pay the principal back on maturing bonds by issuing new bonds with the same principal and maturity at market prices, which can be higher or lower than the principal of the maturing bonds. Thus, the firm's equity holders absorb rollover gains/losses. This implies that the more severely the firm's fundamentals have deteriorated, the heavier the rollover losses incurred by equity holders are because of falling prices of newly issued bonds, even though maturing debt holders are paid in full. Hence, this conflict of interest between equity and debt holders may induce equity holders to default optimally when absorbing further losses is unprofitable for equity holders. In this sense, rollover risk is related to solvency concern.

We also introduce two sources of uncertainty in cash flows—short-term liquidity uncertainty (cash flow shock) and long-term solvency uncertainty (profitability uncertainty)—and

incorporate a learning process over time regarding the long-run solvency uncertainty, following the learning models of Gryglewicz (2011), DeMarzo and Sannikov (2017), and He, Wei, Yu, and Gao (2017).[1,2] Liquidity is the ability of a firm to compensate for rollover losses at each point in time, and is short-term in nature. Solvency is the ability of a firm to incur debt obligations over longer periods of time. More specifically, we assume that cash flows follow a Brownian motion with drift that is not directly observable. In this framework, short-term liquidity uncertainty arises from the volatility risk associated with the Brownian motion, whereas long-term solvency uncertainty is represented by the uncertain drift. Investors observe noisy cash flows and learn about the drift through a Bayesian-type updating process. Hence, via the learning process, persistent cash flow shocks affect solvency concern, which has an effect on the debt maturity, default, investment, and leverage policies of the firm. The changes in these policies influence rollover gains/losses and fluctuate rollover risk. Variations in rollover risk, in turn, affect solvency concern. This learning model framework enables us to not only incorporate an endogenized interaction between rollover risk and solvency concern, but also to disentangle the effects on the firm's debt maturity, default, investment, and leverage policies of long-term solvency uncertainty from those of short-term liquidity uncertainty. For the latter perspective, greater short-term liquidity (long-term solvency) uncertainty makes cash flow signals less (more) informative under the learning process. Hence, the two sources of uncertainty have different implications for the informativeness of cash flow signals under the learning process.

The endogenized interaction between rollover risk and solvency concern affects the firm's debt maturity, default, investment, and leverage policies through several channels in our model. For example, the shorter-term debt decreases *default loss* for debt holders because the close-to-maturity portion of the debt is larger under debt rollover. Hence, the shorter-term debt may increase equity holders' incentives to default and make the firm become more insolvent while it may decrease equity holders' incentives to invest. The debt overhang—reduced incentives for equity holders to undertake profitable investments under outstanding debt—decreases the cash flows of the firm and forces equity holders to absorb the greater *rollover losses*; as a result, debt overhang aggravates rollover risk and forces equity holders

[1] Chang, Dasgupta, Wang, and Yao (2014) show empirically the importance of decomposing corporate cash flows into a temporary and a permanent component in order to understand how firms allocate cash flows, and whether financial constraints matter in this allocation decision.

[2] Instead of the capital structure model such as that presented in this paper, DeMarzo and Sannikov (2017) and He, Wei, Yu, and Gao (2017) consider the learning process in the continuous-time agency model.

to choose to default earlier (become more insolvent). The greater possibility of the choice of earlier default induces the firm to make do with the smaller cash flows because there is not as much need for the firm to hedge against negative shocks at any point in time. This effect reduces investment incentives for equity holders further. In addition, the capital structure and debt maturity of the firm need to be selected so as to control exposure to rollover and solvency risk by considering the incentives for equity holders to default and invest as well as the restrictions or costs of adjusting debt maturity. Hence, both of these choices are affected by the endogenized interaction of rollover risk and solvency concern.

However, it is complicated to analyze the endogenized interaction between rollover risk and solvency concern under the learning model because it is not straightforward to disentangle the effects of the default and investment decisions. To make the analysis tractable, we compare our baseline model with two benchmarks. The first benchmark is the "constant capital stock with no investment" model, in which neither investment nor depreciation occurs. In this case, no investment decision is considered. Only the effect of the default decision—the value of the option to default—is investigated. The second benchmark is the "equity finance" model, in which all the required funds are financed by equity. In this case, no default decision is considered because there is no debt. Only the effect of the investment decision—the value of the option to start or terminate investment—is examined.

The main results of our model are summarized as follows. Suppose that debt maturity is determined exogenously. Then, default is more likely to occur if debt maturity is shorter, as in He and Xiong (2012a) and Diamond and He (2014). In addition, default is more likely to arise in the baseline model than in the constant capital stock with no investment model. Intuitively, investment consideration induces equity holders to be more likely to choose default earlier. However, unlike Diamond and He (2014), investment incentives for equity holders improve as debt maturity increases. This novel result implies that less debt overhang occurs for longer maturities. However, investment incentives are more aggravated in the baseline model than in the equity finance model. Intuitively, the market value of debt decreases with debt maturity because the future portion of debt is more risky. This leverage effect increases equity holders' incentive to invest when debt maturity is longer. In addition, debt overhang forces equity holders to invest later in the baseline model than in the equity finance model.

Second, when debt maturity is exogenously determined, we also discuss how short-term liquidity uncertainty and long-term solvency uncertainty affect optimal default and invest-

ment policies for different debt maturities. An increase in short-term liquidity (long-term solvency) uncertainty raises (reduces) the firm's incentives to default for any maturity in the baseline model and the constant capital stock with no investment model. However, an increase in short-term liquidity (long-term solvency) uncertainty reduces (raises) the firm's incentives to invest for any maturity in the baseline model and in the equity finance model. Gryglewicz (2011) finds a similar result for the firm's default policy. However, our finding regarding the firm's investment policy provides new results and indicates that the effect of long-term solvency uncertainty on the investment policy—debt overhang—is opposite to that of short-term liquidity uncertainty. Intuitively, greater short-term liquidity (long-term solvency) uncertainty makes cash flow signals less (more) informative through the learning process of profitability uncertainty. Hence, under our learning model framework, the two sources of uncertainty affect the firm's need for cash flows and the market value of debt differently, thereby leading to the different effects on the investment policy.

Third, our model has consequences for capital structure when debt maturity is exogenously determined. The model predicts that an increase in short-term liquidity (long-term solvency) uncertainty raises (reduces) the leverage ratio of the firm, while reducing (raising) credit spreads if debt maturity is sufficiently long. These results again depend on the fact that the greater short-term liquidity (long-term solvency) uncertainty makes cash flow signals less (more) informative through the learning process of profitability.

Now, suppose that debt maturity is endogenously determined so that a part of the required funds must be financed by issuing debt at the initial time. Then, the fourth key result of our paper is that the equilibrium debt maturity increases (decreases) with short-term liquidity (long-term solvency) uncertainty if the firm does not depend on debt financing substantially so that the chosen debt maturity becomes sufficiently long; and that the corresponding default and investment thresholds increase (decrease) with short-term liquidity (long-term solvency) uncertainty. These results imply that the effect of long-term solvency uncertainty on several key financial variables such as debt maturity and the firm's incentives to default and invest (debt overhang) are opposite to those of short-term liquidity uncertainty under certain conditions, even though debt maturity is endogenously determined.

We also show that the more pessimistic the initial belief in long-term solvency uncertainty is, the shorter the equilibrium debt maturity and the higher the corresponding default and investment thresholds are. This is because the initial value of debt decreases when the initial belief in long-term solvency uncertainty is more pessimistic. Thus, the equilibrium

debt maturity must be shorter to recover the reduction in the initial value of debt.

Empirically, our paper recommends differentiation between short-term liquidity uncertainty and long-run solvency uncertainty in the analysis of debt maturity and default and investment decisions under debt rollover. Our prediction about the effect of long-term solvency uncertainty on debt maturity when the firm does not depend on debt financing substantially is supported by the finding of Badoer and James (2016) that firms with higher long-term credit ratings are more likely to issue long-term debt. In addition, our prediction about the effect of a change in the initial belief regarding long-term solvency uncertainty on debt maturity and the firm's investment decisions may be consistent with several empirical studies that debt maturity shortening and investment cuts are more likely to occur in response to deteriorating economic conditions.

If a growth firm is defined as one that has substantial uncertainty about its expected investment performance but achieves higher earnings when the investment succeeds, our results also have important empirical implications. Because a growth firm can then be regarded as one with higher long-run solvency uncertainty, our prediction suggests that such a growth firm should have shorter-term debt if these firms do not depend on debt financing substantially. By contrast, if uncertainty about the expected investment performance of a firm is not large but lower earnings are only attained even when the investment succeeds, our prediction implies that such a mature firm should have longer-term debt if these firms do not depend on debt financing substantially. Although Diamond and He (2014) present a similar result regarding debt maturity, they define a growth firm as one in which there is substantial uncertainty about investment opportunities and the value of the new investment projects is highly correlated with the value of the existing investment projects. In this sense, our result provides clearer empirical implications about debt maturity in the growth firm.

Similarly, if deregulation causes substantial uncertainty about the expected investment performance of regulated firms but brings about higher earnings when the regulated firms' investment succeeds, our model predicts that the regulated firms should have shorter-term debt after deregulation than before deregulation if these firms do not depend on debt financing substantially. This prediction is consistent with the finding of Ovtchinnikov (2016) regarding the effect of deregulation on regulated firms' debt decisions.

This paper is related to several recent studies on default–liquidity interaction under debt rollover.[3] He and Xiong (2012a) consider the setting in which bond investors hit by liquidity

[3] The methodology of these papers extends the constant debt maturity structure model of Leland (1994,

shocks are forced to sell their holdings immediately at an exogenous transaction cost. Because such deterioration in liquidity causes a firm to suffer losses in rolling over its maturing debt, equity holders must absorb the losses when they do not choose to default.[4] However, when the equity value drops to zero, equity holders choose to default optimally. He and Milbradt (2014) endogenize secondary bond market liquidity (or the transaction cost) by modeling bond trading in a search-based secondary market, and examine debt valuations, equity valuations, and the default policy. Unlike the authors of these two papers, Diamond and He (2014) neglect both the liquidity shock of bond investors and transaction costs in secondary bond markets. Instead, they incorporate investment opportunities, determine debt maturity endogenously, and discuss how debt maturity affects debt overhang. In contrast to these three papers, our paper distinguishes between short-term liquidity uncertainty (cash flow shock) and long-term solvency uncertainty (profitability uncertainty) and incorporates into the debt rollover model an assessment regarding the firm's solvency risk via the learning process over time. Relative to models without long-term solvency uncertainty, we can clarify the effects of long-term solvency uncertainty on the decisions of the firm about debt maturity, default, investment, and leverage policies, by separating these effects from the effects of short-term liquidity uncertainty, under the endogenous interaction between rollover risk and solvency concern generated by both debt rollover and learning.

Our analysis is related to several existing studies on the effect of debt overhang first formalized by Myers (1977), which points out that outstanding debt may distort the firm's investment incentives downward. Myers (1977) suggests that the shorter the maturity of debt is, the smaller the ex ante debt overhang. This is because the value of shorter-term debt is less sensitive to the value of the firm and receives a smaller benefit from new investment taken after the debt is issued. Gertner and Scharfstein (1991) incorporate investment opportunities in a two-period model, conditional on ex post financial distress. Holding constant the promised payment to debt holders, they show that shorter-term debt imposes a

1998) and Leland and Toft (1996). By ruling out Brownian cash flow shocks and assuming deterministically decreasing cash flows with a possible terminal upward jump, He and Milbradt (2016) consider how a firm chooses its debt maturity structure and default timing dynamically without any commitment to a policy of constant debt maturity structure. DeMarzo and He (2016) also study a model in which equity holders lack the ability to commit to their future leverage choices and can fully issue or buy back debt at the current market price at any time.

[4] He and Xiong (2012b) focus on rollover risk originated from coordination problems between debt holders in firms that cannot raise funds by issuing new equity. Cheng and Milbradt (2012) also discuss a similar coordination problem under the risk-shifting incentive of the manager. However, neither the investment decision nor the learning process is considered in these models.

stronger overhang ex post because making an early fixed-promised debt payment causes debt to be safer and raises the market value of the debt (the firm's market leverage), increases transfers to debt holders, and thus causes ex post debt overhang. The quantitative study of Titman and Tsyplakov (2007) based on Leland (1998) also fixes the promised debt payment and focuses on leverage adjustments by incorporating a tax shield and physical costs of default and adjusting leverage. They suggest that the shorter-term debt improves investment incentives further but triggers the earlier default. In the analysis of the effect of maturity on debt overhang, Diamond and He (2014) stress the timing of investment decisions and hold the initial market value of debt constant by varying the promised debt payment. As in Titman and Tsyplakov (2007), they develop a dynamic model based on Leland (1994, 1998) in which firms have many investment opportunities in the present and future, and show that investment incentives first increase with debt maturity for very short maturity, but then decrease with debt maturity for longer maturities. By contrast, our paper indicates that investment incentives improve with debt maturity as debt maturity increases. The difference between the results of our paper and Diamond and He (2014) reflects the following: in our model, the possibility of debt rollover in the continuous time model creates leverage effects through the changes not only in the debt value when rolling over debt but also in the default loss for debt holders because the initial market value of debt per capital stock varies while the promised payment to debt holders per capital stock is held constant. Note that our leverage effect does not arise in the two-period framework of Gertner and Scharfstein (1991) because they do not consider the possibility of debt rollover so that there is no default loss in the date-1 portion of debt in their model.

Our paper also complements the recent study of Gryglewicz (2011) on solvency–liquidity interaction under a learning process.[5] In his model, the firm faces both short-term liquidity uncertainty and long-term solvency uncertainty. In addition, at each point in time, the positive net earnings of the firm can be distributed as dividends or retained to increase cash holdings, although the firm cannot issue new equity. As a result, losses and dividends must be covered from cash reserves instead of new equity issues in his model, unlike He and Xiong (2012a), He and Milbradt (2014), Diamond and He (2014), DeMarzo and He (2016), and our

[5] A different learning model is also developed using the continuous-time agency framework of DeMarzo and Sannikov (2017), in which the firm's expected cash flows are controlled through costly effort observed by the agent alone and the firm's expected profitability is learned over time. Grenadier and Malenko (2010) examine a real options model in which firms face uncertainty about the permanency of past shocks and have an additional option to learn before investing.

model.[6] Using this framework, Gryglewicz (2011) focuses on the role of initial cash holdings under liquidity and solvency concerns by prohibiting new external financing and ruling out debt rollover and the investment decision of the firm, although he assumes that the firm has sufficient cash holdings to avoid liquidity default (liquidity concern) when he analyzes capital structure, default, and credit spread. The main difference between our paper and Gryglewicz (2011) is that we investigate the effect of the interaction between rollover risk and solvency concern, generated by both debt rollover and learning, when newly issued equity covers the net loss under debt rollover and when debt maturity and investment decisions are endogenously determined. We can therefore derive the effects of long-term solvency uncertainty on the debt maturity, default, investment, and leverage policy decisions of the firm under the endogenous interaction between rollover risk and solvency concern.

The paper is organized as follows. Section 2 presents the model setting. Section 3 derives the debt and equity valuations when debt maturity is exogenously determined. Section 4 investigates the optimal default, investment, and leverage policies of the firm when debt maturity is exogenously determined. Section 5 examines the optimal levels of default, investment, and leverage when debt maturity is endogenously determined. Section 6 concludes. All proofs are in the Appendix.

2. The Model

2.1. Outline of the model.—

We consider a firm that generates uncertain cash flows and selects not only initial capital and debt maturity structures at $t = 0$ but also investment and default policies at $t \geq 0$ after the determination of the initial capital and debt maturity structures. We assume that all of the agents in the model are risk neutral and discount cash flows at a constant risk-free rate r, and that management acts in the interest of equity holders after the determination of the initial capital and debt maturity structures.

The firm's financing comes from a combination of equity and multiple debt issues. The initial leverage choice of the firm at $t = 0$ including the debt maturity choice will be discussed in Section 5. Because it is difficult to analyze dynamic models of multiple debt issues, we use a

[6] In fact, the benchmark model of Gryglewicz (2011) follows the framework of Leland (1994). As a result, losses and dividends can be covered from new equity issues in his benchmark model. However, the valuations of equity, debt, and the firm in his benchmark model are different from those derived in our model because he assumes perpetual debt and neglects the investment decision problem.

framework based on He and Xiong (2012a) and Diamond and He (2014) by extending it with the incorporation of capital stock accumulation.[7] In this framework, even though conditions change, the firm keeps constant the total amount promised to debt holders per capital stock at each refinancing, and does not adjust this amount in response to new conditions. To satisfy this requirement, we assume that the firm can always raise equity as needed whenever the value of equity is positive. Then, equity holders are willing and able to inject any funds necessary to cover investment costs or losses at refinancing. To focus on the effect of external market liquidity, we also assume that internal liquidity such as cash holdings and credit lines is unavailable.

The assumption on the financing of the firm allows us to eliminate the possibility of the firm defaulting because of illiquidity, that is, liquidity default. Instead, this assumption enables us to focus on solvency default, which is defined as a situation in which the firm voluntarily defaults if the value of equity falls below zero.

2.2. Earnings and learning.—

At each time t, a firm produces output by employing capital. The firm's capital stock K_t evolves according to

$$dK_t = (i_t - \delta)K_t dt, \tag{1}$$

where i_t is the firm's growth investment rate controlled by equity holders and $\delta \geq 0$ is the rate of depreciation. We assume that $i_t \in \{0, i\}$ takes a binary value, and that the investment cost is $\lambda i_t K_t$. We also assume that $r + \delta > i > \delta$. The firm generates a stochastic flow of earnings:

$$dX_t = \overline{\mu} K_t dt + \sigma K_t d\overline{Z}_t - \lambda i_t K_t, \tag{2}$$

where $\overline{\mu}$ is the true mean value of earnings per capital stock, σ is the constant volatility, and $\{\overline{Z}_t : 0 \leq t \leq \infty\}$ is a standard Brownian motion.

The firm faces two sources of uncertainty about the instantaneous flow of earnings. The first uncertainty arises from Brownian shocks $d\overline{Z}_t$, whereas the second uncertainty comes from the fact that the true $\overline{\mu}$ is ex ante unknown to all parties. We assume that $\overline{\mu}$ is a fixed parameter and can take either of the two values μ_L or μ_H, with $\mu_L < \mu_H$.

The information structure of our model is as follows. We assume that all parties have

[7] He and Xiong (2012a) and Diamond and He (2014) use the framework of Leland (1994, 1998) and Leland and Toft (1996). The latter three papers take as fixed parameters both the frequency of refinancing and the total amount of promised repayments of debt.

the same information at each time t. More specifically, at the initial time 0, all parties share a common prior expectation μ_0 about $\overline{\mu}$, with $\mu_0 \in (\mu_L, \mu_H)$. As time evolves, more information generated by X_t becomes available. Thus, all parties update their expectation of $\overline{\mu}$. Let $\mathcal{F}_t$ denote the current set of information generated by X_t. Then, the posterior expectation of the mean earnings based on information up to time t, μ_t, is given by $\mu_t = \boldsymbol{E}[\overline{\mu} \mid \mathcal{F}_t]$.

Let dZ_t denote the difference between the realized and expected earnings. Then, the dynamics of X_t in terms of observables are represented as follows:

$$dX_t = (\mu_t - \lambda i_t)K_t dt + \sigma K_t dZ_t. \tag{3}$$

Note that the process $\{Z_t : 0 \leq t \leq \infty\}$ is a Brownian motion adapted to filtration $\mathcal{F}_t$.

The posterior expectation of the mean earnings per capital stock, μ_t, evolves as (see Lipster and Shiryaev (2001))[8]

$$d\mu_t = \frac{1}{\sigma}(\mu_t - \mu_L)(\mu_H - \mu_t)dZ_t. \tag{4}$$

The key point of equation (4) is that expectations adjust more rapidly if σ is small, while learning slows down if μ_t is close to either μ_L or μ_H. This feature will help us understand our results regarding the differences in the effects of short-term liquidity uncertainty and long-term solvency uncertainty in the subsequent analysis.

As discussed in Gryglewicz (2011), the specification written as (3) and (4) elucidates a close relation between cash flow shocks and solvency. Compared with equation (2), we find that in equation (3), short-term negative (positive) cash flow shocks $dZ_t < 0$ ($dZ_t > 0$) are more likely to occur if the firm is of low (high) expected long-term profitability $\overline{\mu}$. The reason is that $dZ_t = \frac{dX_t - (\mu_t - \lambda i_t)K_t dt}{\sigma K_t} < 0$ is more likely to arise if dX_t is more likely to fall below $(\mu_t - \lambda i_t)K_t dt$. Because (2) implies that dX_t is more likely to be small if the true $\overline{\mu}$ is low ($\overline{\mu} = \mu_L$), we show that $dZ_t < 0$ is more likely to occur if $\overline{\mu} = \mu_L$. Similarly, $dZ_t > 0$ is more likely if $\overline{\mu} = \mu_H$. Hence, this specification indicates that cash flow shocks and solvency are closely interrelated.

Suppose that equity holders always invest ($i_t = i$) and the firm does not default. Given the

[8] DeMarzo and Sannikov (2017) and He, Wei, Yu, and Gao (2017) show that the evolution equation of the posterior expectation of the mean earnings can be represented independently of an unobservable effort level of the agent, even though the stochastic flow of cash earnings depends on the unobservable effort level. Similarly, our evolving equation of μ can be represented independently of the observable investment level, although the stochastic flow of cash earnings depends on the observable investment level.

current value of the posterior expectation μ_t, the present value of the firm per capital stock, which is equal to the expected discounted future cash flows per capital stock, is represented by $\frac{\mu_t - \lambda i}{r - i + \delta}$. Because the present value of the firm per capital stock without investment is $\frac{\mu_t}{r+\delta}$, investment is always profitable for the firm if $\frac{\mu_t - \lambda i}{r - i + \delta} \geq \frac{\mu_t}{r+\delta}$. Hence, if $\mu_t \geq (r+\delta)\lambda$, investment at time t maximizes the total value of the firm. We also assume that investment can be undertaken only by equity holders, and that future investment policies are lost when debt holders take over the firm at default. Thus, if default occurs, the first-best policy that investment occurs at every instant when $\mu_t \geq (r+\delta)\lambda$ cannot be achieved.

2.3. Stationary debt structure and rolling over debt.—

According to Leland (1994, 1998) and Leland and Toft (1998), we assume that the firm has one unit of debt per capital stock with a constant aggregate principal face value of debt p and maintains a stationary debt structure per capital stock under a refinancing policy in which at each instant a constant fraction of debt per capital stock, fdt, becomes due and must be refinanced to keep the amount of total debt outstanding per capital stock constant.[9] Thus, given refinancing frequency f, the average debt maturity is $m \equiv \frac{1}{f}$. In addition, because each debt is retired exponentially, the firm's existing debt per capital stock is identical at any point in time.

Let $D(R_t, K_t)$ denote the market value of the firm's debt, where $R_t = \mu_t K_t$ is the posterior expectation of the mean earnings. In issuing new debt to replace maturing debt, the firm receives total proceeds $\frac{D(R_t,K_t)}{m}dt$ by issuing $\frac{K_t}{m}dt$ units of new debt and pays $\frac{pK_t}{m}dt$ to replace maturing debt. Because the market price of newly issued debt fluctuates with the posterior expectation of the mean earnings μ_t, the net payments to bond holders lead to rollover gains/losses, which are represented by $\frac{1}{m}\left[D(R_t, K_t) - pK_t\right]dt$.[10] The rollover gains or losses are received or paid by equity holders. This implies that any gain will be paid out to equity holders immediately, whereas any loss will be paid off by issuing more equity at the market price. Thus, for μ_t, the expected net cash flow to equity holders is

$$(\mu_t - \lambda i_t)K_t dt + \frac{1}{m}\left[D(R_t, K_t) - pK_t\right]dt, \tag{5}$$

[9] We assume that there is no coupon payment. Thus, debt in our model can be interpreted as zero-coupon debt.

[10] As we assume zero-coupon debt, it follows from discounting that the firm always incurs rollover losses. However, whether rollover gains are possible or not is not essential to our analysis, as discussed in He and Xiong (2012a) and Diamond and He (2014).

where the first term indicates the firm's expected net cash flows and the second term the rollover gains or losses.

When the firm issues additional equity to absorb rollover losses, the equity issuance dilutes the value of the existing shares. Hence, rollover losses affect the equity value. In fact, as investment can only be undertaken by equity holders, future investment opportunities are lost when debt holders take over the firm from bankruptcy. Thus, equity holders are willing and able to pay off rollover losses to keep the firm's operations running whenever the equity value is positive, that is, whenever the option value of keeping the firm alive justifies expected rollover losses. This means that insolvency default is triggered by equity holders when the equity value drops to zero.

3. Valuations of Debt and Equity for Different Debt Maturities

We now determine the values of claims held by debt and equity holders for different debt maturities. These values depend on the flows to the claimants and on the insolvency default and investment times chosen by equity holders. The insolvency default occurs when the posterior expectation μ_t drops to an endogenously determined threshold μ_B. In the Appendix, we assume that the total value of the firm increases in μ_t,[11] and can show that the optimal investment time is determined by an endogenous investment threshold μ_I.

3.1. Debt and equity values with evolution of capital stock.—

First, using (1) and (4), note that

$$dR_t = d(\mu_t K_t) = \mu_t(i_t - \delta)K_t dt + \frac{K_t}{\sigma}(\mu_t - \mu_L)(\mu_H - \mu_t)dZ_t. \tag{6}$$

Then, because equity holders use the investment threshold policy, the debt value before default satisfies the following ordinary differential equations:

$$rD(R,K) = \begin{cases} \frac{1}{2\sigma^2}(\mu-\mu_L)^2(\mu_H-\mu)^2(K)^2 D_{RR}(R,K) + \mu(i-\delta)KD_R(R,K) \\ \quad +(i-\delta)KD_K(R,K) + \frac{1}{m}\left[pK - D(R,K)\right], \quad \text{if } \mu \geq \mu_I, \\ \frac{1}{2\sigma^2}(\mu-\mu_L)^2(\mu_H-\mu)^2(K)^2 D_{RR}(R,K) - \mu\delta KD_R(R,K) - \delta KD_K(R,K) \\ \quad +\frac{1}{m}\left[pK - D(R,K)\right], \quad \text{if } \mu_I > \mu \geq \mu_B. \end{cases} \tag{7}$$

[11] Our numerical calculation ensures that this assumption holds in our parameter set.

The first two terms (third term) on the right-hand side of (7) capture(s) the expected change in the debt value from a change in R in equation (6) (K in equation (1)), and the final term is the change in the debt value caused by rolling over debt.

Using the scale invariance of the firm's technology arising from the homogeneity assumption, we write $D(R,K) \equiv d(\frac{R}{K},1)K \equiv d(\mu)K$. Hence, we can reduce (7) to the following equations with a single state variable μ.

$$\left(r - i + \delta + \frac{1}{m}\right) d(\mu) = \frac{p}{m} + \frac{1}{2\sigma^2}(\mu - \mu_L)^2(\mu_H - \mu)^2 d''(\mu), \quad \text{if } \mu \geq \mu_I, \tag{8a}$$

$$\left(r + \delta + \frac{1}{m}\right) d(\mu) = \frac{p}{m} + \frac{1}{2\sigma^2}(\mu - \mu_L)^2(\mu_H - \mu)^2 d''(\mu), \quad \text{if } \mu_I > \mu \geq \mu_B. \tag{8b}$$

We need several boundary conditions to solve equation (8). Equity holders choose default at $\mu = \mu_B$. Then, we assume that the liquidation value per capital stock is equal to the value per capital stock of the all-equity firm at the moment of default, $\frac{\mu_B}{r+\delta}$. Furthermore, debt holders take over the firm with the value $\frac{\mu_B}{r+\delta}$ per capital stock without future investment.[12] This requirement is represented by the following value-matching condition:

$$d(\mu_B) = \frac{\mu_B}{r+\delta}. \tag{9}$$

At the investment boundary μ_I, the boundary conditions are also needed, that is,

$$\lim_{\mu \uparrow \mu_I} d(\mu) = \lim_{\mu \downarrow \mu_I} d(\mu), \tag{10}$$

$$\lim_{\mu \uparrow \mu_I} d'(\mu) = \lim_{\mu \downarrow \mu_I} d'(\mu). \tag{11}$$

Finally, if μ hits μ_H, we need the condition that $d(\mu_H)$ is bounded and is equal to the default-free debt value per capital stock, $\frac{p}{1+m(r-i+\delta)}$, as imposed in Gryglewicz (2011) and Diamond and He (2014). Thus,

$$d(\mu_H) = \frac{p}{1 + m(r - i + \delta)}. \tag{12}$$

This condition implies that μ_H is an absorbing state for μ.

We now consider the equity value. Using (1) and (6), the equity value must satisfy the

[12] This assumption implies that debt holders can sell the firm to other investors without any liquidation costs in default. Gryglewicz (2011) imposes a similar assumption although he takes account of liquidation costs. However, even though liquidation costs are considered, our main results are unaffected.

following differential equation:

$$rE(R,K) = \max_{i\in\{0,i\}} \frac{1}{2\sigma^2}(\mu-\mu_L)^2(\mu_H-\mu)^2(K)^2 E_{RR}(R,K) + \mu(i-\delta)KE_R(R,K)$$
$$+ (i-\delta)KE_K(R,K) + (\mu-\lambda i)K - \frac{1}{m}\left[pK - D(R,K)\right]. \tag{13}$$

The first two terms (third term) on the right-hand side of (13) capture(s) the expected change in the equity value caused by a change in R in equation (6) (K in equation (1)), and the final term is the rollover gain/loss of equity holders.

Again, using the scale invariance of the firm's technology arising from the homogeneity assumption, we write $E(R,K) \equiv e(\frac{R}{K},1)K \equiv e(\mu)K$. Then,

$$re(\mu) = \max_{i\in\{0,i\}} \mu - \lambda i + (i-\delta)e(\mu) + \frac{1}{2\sigma^2}(\mu-\mu_L)^2(\mu_H-\mu)^2 e''(\mu) - \frac{1}{m}[p-d(\mu)]. \tag{14}$$

We specify boundary conditions to solve equation (14). In the Appendix, we can show that the threshold investment strategy is optimal. Thus, it follows from the maximization of the right-hand side of (14) with respect to i that

$$i(\mu) = \begin{cases} i, & \text{if } \mu \geq \mu_I, \\ 0, & \text{if } \mu_I > \mu \geq \mu_B, \end{cases}$$

and that the endogenous investment threshold μ_I chosen by equity holders must satisfy the following value-matching condition:[13]

$$e(\mu_I) = \lambda. \tag{15}$$

At the boundary μ_H, the debt value per capital stock is equal to the default-free debt value per capital stock $\frac{p}{1+m(r-i+\delta)}$. Thus, for consistency, we need to have

$$e(\mu_H) = \frac{\mu_H - \lambda i}{r-i+\delta} - \frac{p}{1+m(r-i+\delta)}, \tag{16}$$

[13] In Diamond and He (2014), this condition is represented by the smooth-pasting condition. The difference depends on the difference between the formulation of the two models. More specifically, Diamond and He (2014) suppose that cash flows follow a geometric Brownian motion, whereas they do not consider capital accumulation. By contrast, we suppose that cash flows per capital stock follow an arithmetic Brownian motion by incorporating the capital accumulation process.

where the firm value per capital stock at $\mu = \mu_H$ is equal to the expected discounted future cash flows per capital stock that prevail if the firm always invests and does not default. To ensure this, we assume that $e(\mu_H) > \lambda$, that is, $\frac{\mu_H - \lambda(r+\delta)}{r-i+\delta} > \frac{p}{1+m(r-i+\delta)}$.

However, equity holders default at μ_B and receive zero under limited liability, which implies

$$e(\mu_B) = 0. \tag{17}$$

The endogenous default boundary also needs to satisfy the smooth-pasting condition:

$$e'(\mu_B) = 0. \tag{18}$$

To ensure the existence of μ_B ($\geq \mu_L$) and the immediate default for $\mu < \mu_B$, we assume that $e(\mu_L) < 0$, that is, $\frac{\mu_L}{r+\delta} < \frac{p}{1+m(r+\delta)}$. Finally, the following boundary conditions at the investment boundary are required:

$$\lim_{\mu \uparrow \mu_I} e(\mu) = \lim_{\mu \downarrow \mu_I} e(\mu), \tag{19}$$

$$\lim_{\mu \uparrow \mu_I} e'(\mu) = \lim_{\mu \downarrow \mu_I} e'(\mu). \tag{20}$$

Now, we provide the following proposition that clarifies the debt and equity values per capital stock as solutions to equations (8) and (14) together with boundary conditions (9)–(12) and (15)–(20). We also verify the optimality of the threshold investment strategy in the Appendix by assuming that $v(\mu)$ is increasing in μ.

Proposition 1: *There exists a unique μ_I that satisfies (14). Thus, the optimal investment policy for each different debt maturity is given by the investment threshold policy: given μ_B and μ_I, equity holders invest as long as* the *posterior expectation μ exceeds a critical value μ_I:*

$$i(\mu) = \begin{cases} i, & \text{if } \mu \geq \mu_I, \\ 0, & \text{if } \mu_I > \mu \geq \mu_B. \end{cases}$$

The debt value per capital stock is: if $\mu \geq \mu_I$,

$$d(\mu) = \frac{p}{1+m(r-i+\delta)} + A_1(\mu - \mu_L)^{1-\beta_1}(\mu_H - \mu)^{\beta_1}; \tag{21a}$$

and if $\mu_I > \mu \geq \mu_B$,

$$d(\mu) = \frac{p}{1+m(r+\delta)} + A_3(\mu-\mu_L)^{1-\beta_2}(\mu_H-\mu)^{\beta_2} + A_4(\mu-\mu_L)^{\beta_2}(\mu_H-\mu)^{1-\beta_2}. \tag{21b}$$

The equity value per capital stock is: if $\mu \geq \mu_I$,

$$e(\mu) = \frac{\mu-\lambda i}{r-i+\delta} - \frac{p}{1+m(r-i+\delta)} + B_1(\mu-\mu_L)^{1-\gamma_1}(\mu_H-\mu)^{\gamma_1} - A_1(\mu-\mu_L)^{1-\beta_1}(\mu_H-\mu)^{\beta_1}; \tag{22a}$$

and if $\mu_I > \mu \geq \mu_B$,

$$\begin{aligned} e(\mu) = {} & \frac{\mu}{r+\delta} - \frac{p}{1+m(r+\delta)} + B_2(\mu-\mu_L)^{1-\gamma_2}(\mu_H-\mu)^{\gamma_2} + B_3(\mu-\mu_L)^{\gamma_2}(\mu_H-\mu)^{1-\gamma_2} \\ & - A_3(\mu-\mu_L)^{1-\beta_2}(\mu_H-\mu)^{\beta_2} - A_4(\mu-\mu_L)^{\beta_2}(\mu_H-\mu)^{1-\beta_2}. \end{aligned} \tag{22b}$$

The constants β_1, β_2, γ_1, γ_2, A_1, A_3, A_4, B_1, B_2, *and* B_3 *are given by (A1), (A3), (A17), (A19), (A5)–(A7), and (A23)–(A25), respectively, in the Appendix. The endogenous boundaries* μ_B *and* μ_I *are also given by (A29) and (A30) in the Appendix. In particular,* μ_B *satisfies* $\frac{\mu_B}{r+\delta} < \frac{p}{1+m(r+\delta)}$.

The value functions of $d(\mu)$ and $e(\mu)$ are interpreted as follows. If μ exceeds the investment threshold μ_I, the debt value per capital stock is equal to the value of default-free debt per capital stock minus the impact of potential future default, that is, the present value of cash flows lost by debt per capital stock in the case of default (the potential loss because of the option to default for debt holders). However, outside the investment region $\mu_I > \mu \geq \mu_B$, the debt value per capital stock includes an additional term that captures the adjustment for entering the investment region in the future (the potential loss because of the option to invest for debt holders). This additional term represents the loss of the debt value per capital stock caused by the execution of investment. This loss arises because the investment cost expense reduces both the cash flows to equity holders and the expectation of the firm's profitability so that potential future default is more likely to occur, thereby moving the equity value per capital stock close to zero. Regarding the equity value per capital stock, if μ exceeds μ_I, it is comprised of four terms. The first and second terms are equal to the firm value per capital stock that would prevail if the firm always invested and did not default, minus the

default-free debt value per capital stock. The remaining two terms reflect the adjustment for stopping investment at least temporarily (the value of the option to terminate investment for equity holders) and for potential future default (the value of the option to default for equity holders). Outside the investment region $\mu_I > \mu \geq \mu_B$, the equity value per capital stock now consists of six terms. The first and second terms are the firm value per capital stock without investment in the case of no default, minus the default-free debt value per capital stock. The remaining four terms capture the adjustment for both entering the investment region in the future (the value of the option to invest for equity holders) and potential future default (the value of the option to default for equity holders).

Proposition 1 indicates that $\frac{\mu_B}{r+\delta} < \frac{p}{1+m(r+\delta)}$. This implies that on the date of default, there is a loss to debt holders.

3.2. Debt and equity values and default and investment boundaries under benchmark models.—

To facilitate discussion, we now solve two other cases that serve as benchmark models. We first consider a benchmark case in which neither investment nor depreciation occurs ($i = \delta = 0$) so that the firm's capital stock is constant ($K_t = const$ for any $t \geq 0$). This corresponds to the case in which no investment decision is considered. We call this the "constant capital stock with no investment" case, and indicate the corresponding solution by a superscript "c".

Then, ignoring μ_I and using $i = \delta = 0$, we rearrange Proposition 1 as follows.

Proposition 2: *Suppose that $i = \delta = 0$ and the capital stock is constant for any $t \geq 0$. Then, the debt value per capital stock is:*

$$d^c(\mu) = \frac{p}{1+mr} - \left(\frac{p}{1+mr} - \frac{\mu_B^c}{r}\right)\left(\frac{\mu-\mu_L}{\mu_B^c-\mu_L}\right)^{1-\beta^c}\left(\frac{\mu_H-\mu}{\mu_H-\mu_B^c}\right)^{\beta^c}, \tag{23}$$

and the equity value per capital stock is:

$$e^c(\mu) = \frac{\mu}{r} - \frac{p}{1+mr} + \left(\frac{p}{1+mr} - \frac{\mu_B^c}{r}\right)\left(\frac{\mu-\mu_L}{\mu_B^c-\mu_L}\right)^{1-\beta^c}\left(\frac{\mu_H-\mu}{\mu_H-\mu_B^c}\right)^{\beta^c}, \tag{24}$$

where $\beta^c > 1$ is the positive root of $(\beta^c)^2 - \beta^c - \frac{2(1+mr)\sigma^2}{m(\mu_H-\mu_L)^2} = 0$. The default threshold is

given by

$$\mu_B^c = \frac{\mu_H \mu_L + [(\beta^c - 1)\mu_H - \beta^c \mu_L]\frac{rp}{1+mr}}{(1-\beta^c)\mu_L + \beta^c \mu_H - \frac{rp}{1+mr}}. \tag{25}$$

Note that μ_B^c *satisfies* $\frac{\mu_B^c}{r} < \frac{p}{1+mr}$.

Several remarks are in order. First, as we do not consider the investment threshold μ_I, neither the debt value function per capital stock nor the equity value function per capital stock is changed regardless of $\mu \gtreqless \mu_I$. In addition, except for the term regarding the value of default-free debt per capital stock (the firm value per capital stock that would prevail if the firm always invested and did not default, minus the default-free debt value per capital stock), the debt (equity) value per capital stock consists of only the term involved in the value of the option to default for debt (equity) holders; in other words, it does not include any terms involved in the value of the option to start or terminate investment for debt (equity) holders. Second, the default threshold has the closed-form expression (25) in this case.

Gryglewicz (2011) derives the debt and equity value equations by assuming that new external financing is available only at the initial time while the firm makes the initial financing to cover cash holdings that enable the firm to avoid liquidity default (see Proposition 4 in Gryglewicz (2011)). This assumption allows only solvency default by excluding liquidity default. Because he does not consider debt rollover or the investment decision of the firm, the debt and equity value equations in his model are essentially similar to (23) and (24) if we rule out debt rollover ($m \to \infty$) and make debt perpetual.[14] Note that there is no coupon payment in our model.

More formally, we provide the following corollary to clarify the relationship between our model and the benchmark model of Gryglewicz (2011, Proposition 1).

Corollary to Proposition 2: *Suppose that* $i = \delta = 0$ *and capital stock is constant for any* $t \geq 0$. *If we rule out debt rollover* ($m \to \infty$), *the debt value per capital stock is:*

$$d^c(\mu) = \frac{\mu_B^c}{r}\left(\frac{\mu - \mu_L}{\mu_B^c - \mu_L}\right)^{1-\beta^c}\left(\frac{\mu_H - \mu}{\mu_H - \mu_B^c}\right)^{\beta^c}, \tag{23$'$}$$

[14] Note that β^c in our model converges to β given in Gryglewicz (2011) as $m \to \infty$ (see the following corollary to Proposition 2).

and the equity value per capital stock is:

$$e^c(\mu) = \frac{\mu}{r} - \frac{\mu_B^c}{r}\left(\frac{\mu - \mu_L}{\mu_B^c - \mu_L}\right)^{1-\beta^c}\left(\frac{\mu_H - \mu}{\mu_H - \mu_B^c}\right)^{\beta^c}, \tag{24'}$$

where $\beta^c > 1$ *is the positive root of* $(\beta^c)^2 - \beta^c - \frac{2r\sigma^2}{(\mu_H - \mu_L)^2} = 0$. *The default threshold is given by*

$$\mu_B^c = \frac{\mu_H \mu_L}{(1-\beta^c)\mu_L + \beta^c \mu_H}. \tag{25'}$$

We next discuss the other benchmark case in which equity holders do not issue any debt ($d(\mu) = 0$ for any $t \geq 0$) and finance all the required funds by issuing equity. This corresponds to the case in which neither endogenous default decisions nor debt overhang problems are investigated. We denote this case as the "equity finance" case, and indicate the corresponding solution with a superscript "e". Then, ignoring μ_B and applying a proof procedure similar to that of Proposition 1, we obtain the following proposition.

Proposition 3: *Suppose that equity holders do not issue any debt* ($d(\mu) = 0$ *for any* $t \geq 0$). *Then, the equity value per capital stock is: if* $\mu \geq \mu_I^e$,

$$e^e(\mu) = \frac{\mu - \lambda i}{r - i + \delta} + \left[\frac{\lambda(r+\delta) - \mu_I^e}{r - i + \delta}\right]\left(\frac{\mu - \mu_L}{\mu_I^e - \mu_L}\right)^{1-\gamma_1^e}\left(\frac{\mu_H - \mu}{\mu_H - \mu_I^e}\right)^{\gamma_1^e}, \tag{26a}$$

and if $\mu_I^e > \mu$,

$$e^e(\mu) = \frac{\mu}{r + \delta} + \left[\frac{\lambda(r+\delta) - \mu_I^e}{r + \delta}\right]\left(\frac{\mu - \mu_L}{\mu_I^e - \mu_L}\right)^{\gamma_2^e}\left(\frac{\mu_H - \mu}{\mu_H - \mu_I^e}\right)^{1-\gamma_2^e}, \tag{26b}$$

where $\gamma_1^e > 1$ *and* $\gamma_2^e > 1$ *are the positive roots of* $(\gamma_1^e)^2 - \gamma_1^e - \frac{2(r-i+\delta)\sigma^2}{(\mu_H - \mu_L)^2} = 0$ *and* $(\gamma_2^e)^2 - \gamma_2^e - \frac{2(r+\delta)\sigma^2}{(\mu_H - \mu_L)^2} = 0$, *respectively. The investment threshold is given by*

$$\mu_I^e = -\frac{\begin{array}{c}(r-i+\delta)\left[\mu_L\lambda(r+\delta) - \mu_H\mu_L + \gamma_2^e\lambda(r+\delta)(\mu_H - \mu_L)\right] \\ -(r+\delta)\left[(\mu_H - \gamma_1^e(\mu_H - \mu_L))\lambda(r+\delta) - \mu_H\mu_L\right]\end{array}}{(r-i+\delta)\left[\mu_H - \gamma_2^e(\mu_H - \mu_L) - \lambda(r+\delta)\right] - (r+\delta)\left[\mu_L + \gamma_1^e(\mu_H - \mu_L) - \lambda(r+\delta)\right]}. \tag{27}$$

We provide several comments. First, this benchmark model looks like a standard real option model of investment à la Dixit and Pindyck (1994), except that in our model the investment opportunities for equity holders successively occur after one investment has been made. Second, as equity holders do not need to take account of future default in the absence of debt, there is no default effect in this case. Thus, the debt overhang problem discussed in the next section does not arise in this case. Indeed, it follows from the proof of this proposition that the second term on the right-hand side of (26a) ((26b)) corresponds to $B_1(\mu - \mu_L)^{1-\gamma_1}(\mu_H - \mu)^{\gamma_1}$ $(B_3(\mu - \mu_L)^{\gamma_2}(\mu_H - \mu)^{1-\gamma_2})$ on the right-hand side of (22a) ((22b)), which can be interpreted as the value of the option to stop (start) investment for equity holders. Thus, comparing between the results of Propositions 1 and 3 ((22) and (26)), we see that the equity value per capital stock in this case includes the terms involved in the value of the option to start or stop investment for equity holders, whereas it does not include any terms involved in the value of default-free debt per capital stock or in the adjustment for potential future default (the value of the option to default for equity holders).

4. Optimal Default and Investment Policies for Different Debt Maturities

Comparing the baseline model of Proposition 1 with the other two benchmark models of Propositions 2 and 3, we now consider how changes in m, σ, and $\mu_H - \mu_L$ affect the optimal default and investment policies when m is exogenously determined.

4.1. The effect of debt maturity on default and investment policies.—

Debt overhang, first studied by Myers (1977), captures the idea that equity holders underinvest relative to the level that maximizes the total value of the firm, because a part of investment benefits accrue to the firm's debt claims. In this subsection, we provide numerical examples to illustrate a new insight offered by our paper into the effect of debt maturity on default and debt overhang when debt maturity is exogenously determined.

For this purpose, we use the following set of basic parameters: $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$. However, the initial market value of debt per capital stock varies with maturity m relative to $p = 0.8$.

By contrast, Diamond and He (2014) hold the initial market value of debt constant by varying the face value of debt. We do not follow the setting of Diamond and He (2014), not only because the default and investment policies of the firm strongly depend on the *leverage*

effect caused by the effect of debt maturity on the market value of debt but also because we need to investigate the *leverage effect* created by learning when we discuss how short-term liquidity uncertainty and long-term solvency uncertainty affect the default and investment policies of the firm. In addition, because our *leverage* effect is derived from the possibility of debt rollover and learning, unlike Gertner and Scharfstein (1991), it would seem interesting to consider this effect fully under our framework.

Figure 1 illustrates the optimal default policies when varying debt maturity m for the two models: the baseline model and the constant capital stock with no investment ($i = \delta = 0$) model.

In the case of constant capital stock with no investment, it follows from (24) that the equity value per capital stock includes the term involved in the value of the option to default for equity holders, whereas it does not include any terms involved in the value of the option to start or terminate investment for equity holders. In this case, Figure 1 indicates that the default threshold μ_B^c decreases with m: default is more likely to occur if debt maturity is shorter. In the baseline model case, it follows from (22b) that the equity value per capital stock additionally includes the term involved in the value of the option to start investment for equity holders, as well as the terms observed in the case of constant capital stock with investment. Then, Figure 1 shows that the default threshold μ_B in this case still decreases with m; and it is higher than μ_B^c for any m. Hence, consideration of the value of the option to start investment is more likely to induce equity holders to default for any m.

The reason why μ_B decreases with m is that equity holders default earlier in order to refuse to subsidize debt holders as m is shorter. This is because shorter-term debt requires equity holders to absorb greater rollover losses when the firm's future prospects deteriorate.[15] Such a relation between the default threshold and debt maturity is also obtained in He and Xiong (2012a) and Diamond and He (2014), although their model frameworks are different from ours. Our analysis also indicates that for any m, the default threshold is higher in the baseline model than in the constant capital stock with no investment model. As the firm has an option to invest in the former case but not in the latter case, equity holders are more likely to choose default in the former case than in the latter case, when the firm's future prospects deteriorate. Consequently, for any m, the default threshold is higher in the baseline model than in the constant capital stock with no investment model.

Figure 1 also illustrates the optimal investment policies when varying debt maturity m for

[15] See the latter part of the discussion in footnote 18.

the two models: the baseline model and equity finance ($d = 0$ for any $t \geq 0$) model.

In the equity finance model, the firm does not default because it does not issue any debt. Hence, it follows from (26a) and (26b) that the equity value per capital stock includes the term involved in the value of the option to invest for equity holders, but does not include any terms involved in the value of the option to default for equity holders. In addition, the equity value per capital stock does not depend on m. Consequently, the investment threshold μ_I^e does not depend on m. In the baseline model, it follows from (22a) and (22b) that the equity value per capital stock additionally includes the terms involved in the value of the option to default for equity holders. Then, Figure 1 indicates that the investment threshold μ_I is higher than μ_I^e for any m. Hence, consideration of the value of the option to default for equity holders is more likely to induce equity holders to delay investment for any m. Furthermore, Figure 1 also illustrates that the investment threshold μ_I in this case decreases with m. This implies that debt has less overhang when the debt maturity is long.

Intuitively, consideration of the value of the option to default for equity holders forces the firm to take account of the likelihood of failing to receive cash flows as a result of investment. Hence, it is not surprising that this consideration is more likely to induce equity holders to delay the investment timing for any m. However, the reason why μ_I in the baseline model decreases with m is explained as follows. As shorter-term debt induces more frequent repricing, shorter-term debt holders share less gains given good news. Hence, shorter-term debt may improve equity holders' incentives to invest. However, our model also incorporates the leverage effect caused by debt rollover, through which the market value of debt per capital stock decreases with debt maturity (see Figures 9 and 10) because the future portion of debt is more risky because of the greater possibility of default. Thus, the leverage effect decreases the transfer to debt holders and increases equity holders' incentives to invest, as debt maturity is longer. In addition, the other effect by which longer-term debt increases equity holders' incentives to invest results from the above finding that μ_B decreases with debt maturity. This is because the decrease in μ_B is more likely to delay default and induces the firm to obtain larger cash flows as a result of the greater likelihood of being able to receive cash flows when investing or the greater need to hedge negative shocks at each point in time. As the second and third effects are stronger than the first effect, longer-term debt has less debt overhang.

In contrast, holding the initial market value of debt constant by varying the promised debt payment, Diamond and He (2014) suggest that investment incentives first increase with

debt maturity for very short maturities, then decrease with debt maturity as debt maturity increases. By varying the initial market value of debt per capital stock, we consider the leverage effect caused by debt rollover, which is not derived by Gertner and Scharfstein (1991), although Diamond and He (2014) rule out this effect by holding the initial market value of debt constant. Thus, investment incentives increase with debt maturity. In fact, holding the promised debt payment per capital stock constant by varying the initial market value of debt per capital stock, we might choose a considerably higher or lower leverage ratio, which would affect the results of this paper. To check this problem, we report the default and investment thresholds, the debt value per capital stock, and the firm value per capital stock at the end of the Appendix and in Figure A1 using our basic parameter set for $m = 5$. The results confirm that our choice of the value of the promised debt payment per capital stock does not lead to extraordinary results.

4.2. The effects of short-term liquidity uncertainty and long-term solvency uncertainty on default and investment policies.—

We now discuss how the two sources of uncertainty affect optimal default and investment policies for fixed debt maturity. In this model, short-term liquidity uncertainty arises from the unpredictable immediate earnings because of Brownian shocks. Thus, short-term liquidity uncertainty is the liquidity shock generated by the volatility risk σ of the Brownian motion. However, long-term solvency uncertainty is profitable uncertainty represented by the uncertain drift $\overline{\mu}$ that may cause the firm to undergo solvency distress. We analyze the effects of these two uncertainties on default and investment policies using the basic parameter set given in Section 4.1. We also examine the effects of these two uncertainties on the leverage ratio and credit spreads using the same basic parameter set.

Figure 2A indicates that in the baseline model, an increase in σ leads to an increase in the default threshold μ_B for each debt maturity. Thus, equity holders are more likely to default earlier for each debt maturity as σ increases. This tendency is also observed in the case of constant capital stock with no investment (see Figure 2B). This implies that incorporation of the value of the option to start or terminate investment for equity holders does not modify the effect of σ on the default threshold. In addition, as argued in Corollary to Proposition 2, the case of constant capital stock with no investment when $m = \infty$ corresponds to the benchmark case of Gryglewicz (2011). Figure 2C suggests that even for the case that corresponds to Gryglewicz (2011), the comparative static result with respect to short-term

liquidity uncertainty has the same tendency as in the other cases for sufficiently large $\mu_H - \mu_L$ ($\mu_H - \mu_L \geq 0.101$), although we have the corner solution $\mu_B = \mu_L$ in this case if $\mu_H - \mu_L$ is not sufficiently large ($\mu_H - \mu_L < 0.101$).

The intuition for these results is explained as follows. Increasing volatility σ has two main effects. One effect is that a higher σ increases the magnitude of liquidity risk. The other effect is that a higher σ makes instantaneous cash flows less informative about $\overline{\mu}$. These two main effects of increasing σ cause a lower option value of the firm delaying the decision on default in order to wait for new information in both the baseline model and the constant capital stock with no investment model. Hence, in these two models, the default threshold μ_B increases for any m as σ increases.

Next, Figure 3A illustrates that in the baseline model, an increase in σ increases the investment threshold μ_I for any m. Hence, the higher σ aggravates their investment incentives and increases debt overhang for any m. Similarly, in the equity finance model, Figure 3B shows that an increase in σ increases the investment threshold μ_I^e. Note that, as has been argued, μ_I^e does not depend on m in the equity finance model. This result means that even though the value of the option to default for equity holders is incorporated, increasing σ does not modify the effect of σ on the investment threshold.

The logic behind these results is explained as follows. Less informative cash flow signals cause the firm to be less likely to be insolvent against negative cash flow shocks because such negative cash flow shocks are not quickly interpreted as a reduction in expected profitability. This direct learning effect created by an increase in volatility σ induces the firm to choose the lower investment threshold. However, given that rollover gains/losses are covered from new equity issues, the lower informativeness of cash flows raises the market value of debt per capital stock because debt is information-insensitive security. This indirect learning effect increases the transfer to debt holders and decreases equity holders' incentives to invest. Another indirect effect of increasing μ_I arises from the above finding that μ_B increases in σ. This is because the increase in μ_B triggers earlier default and induces the firm to make do with smaller cash flows as a result of the greater likelihood of failing to receive cash flows when investing or the lesser need to hedge negative liquidity shocks at each point in time. Furthermore, the larger magnitude of liquidity risk also directly increases the value of the option for equity holders to wait for investment, thereby motivating equity holders to exercise the option to start investment later, as in the standard real option model such as Dixit and Pindyck (1996). In the equity finance model, increasing volatility σ involves only the last of

these four effects because there is no debt. Thus, in the equity finance model, an increase in σ always increases the investment threshold μ_I^e. However, the baseline model also includes the remaining three effects because it considers the value of the option to default for equity holders. However, for any m, the first of these four effects is dominated by the other three effects. Consequently, consideration of the value of the option to default for equity holders does not modify the relation between μ_I and σ observed in the equity finance model.

We now consider the effect of a change in profitability uncertainty. Because we use the binomial distribution of $\overline{\mu}$, this uncertainty is measured by a mean preserving spread between the high value (μ_H) and low value (μ_L) realizations of mean earnings, as in Gryglewicz (2011). More specifically, we vary $\mu_H - \mu_L$ around the mean $\mu_0 = \frac{1}{2}(\mu_H + \mu_L) = 0.05$. Increasing $\mu_H - \mu_L$ around the mean μ_0 has two main effects. One effect is that increasing $\mu_H - \mu_L$ around the mean μ_0 directly increases the profit potential of the firm at success. The other effect is that the greater the spread of $\mu_H - \mu_L$ is, the more rapid are the learning dynamics in μ_t. The reason is that cash flow signals are then more informative about the realization of either μ_H or μ_L (see equation (4)) because μ_t is farther away from μ_L and μ_H on average. Thus, this effect increases the option value to wait for new information.

As illustrated in Figure 4A, the default threshold μ_B decreases with $\mu_H - \mu_L$ for each debt maturity in the baseline model. Thus, equity holders are more likely to default later for each debt maturity as $\mu_H - \mu_L$ increases. This is also observed in the constant capital stock with no investment model (see Figure 4B). Hence, consideration of the value of the option to start or terminate investment for equity holders does not modify the effect of $\mu_H - \mu_L$ on the default threshold. Again, Figure 2C suggests that even for the case that corresponds to Gryglewicz (2011), the comparative static result with respect to long-term solvency uncertainty has the same tendency as in the other cases for sufficiently large $\mu_H - \mu_L$ ($\mu_H - \mu_L \geq 0.101$), although we have the corner solution $\mu_B = \mu_L$ in this case if $\mu_H - \mu_L$ is not sufficiently large ($\mu_H - \mu_L < 0.101$).

The mechanism for these results is as follows. The two main effects of increasing $\mu_H - \mu_L$ discussed above create a higher option value of the firm delaying the decision on default in order to wait for new information in both the baseline model and the constant capital stock with no investment model. Hence, in these two models, the default threshold μ_B decreases for any m as $\mu_H - \mu_L$ increases.

Figure 5A shows that in the baseline model, the investment threshold μ_I decreases with $\mu_H - \mu_L$ for any m. In the equity finance model, Figure 5B also indicates that an increase in

$\mu_H - \mu_L$ decreases the investment threshold μ_I^e. Thus, this result suggests that even though the value of the option to default for equity holders is incorporated, increasing $\mu_H - \mu_L$ still induces equity holders to invest earlier.

Intuitively, the two main effects of increasing $\mu_H - \mu_L$ around μ_0 operate as follows. First, increasing $\mu_H - \mu_L$ directly increases the profit potential of the firm at success. Hence, increasing $\mu_H - \mu_L$ induces equity holders to invest earlier by decreasing the option value of delaying investment in order to wait for new information. Second, the larger degree of debt overhang arises from the direct learning effect caused by an increase in $\mu_H - \mu_L$, which raises the speed of learning from cash flow shocks about expected profitability. The reason is that the firm is expected to be more likely to be insolvent if negative cash flow shocks are quickly interpreted as a drop in expected profitability. This effect induces the firm to choose the higher investment threshold. For indirect effects, given that rollover gains/losses are covered from new equity issues and that debt is information-insensitive security, the greater informativeness of cash flows reduces the market value of debt per capital stock. This indirect learning effect decreases the transfer to debt holders and increases equity holders' incentive to invest. Furthermore, the decrease in the default threshold μ_B because of an increase in $\mu_H - \mu_L$ triggers later default and induces the firm to obtain larger cash flows as a result of the greater likelihood of being able to receive cash flows when investing or the greater need to hedge negative liquidity shocks at each point in time. Thus, this effect also motivates the firm to choose the lower investment threshold. In the equity finance model, increasing profitable uncertainty $\mu_H - \mu_L$ involves only the first profit potential effect because there is no debt. Hence, in the equity finance model, an increase in $\mu_H - \mu_L$ always decreases the investment threshold μ_I^e, thereby inducing equity holders to exercise the option to start investment earlier. On the other hand, in the baseline model, consideration of the value of the option to default for equity holders additionally involves the remaining three effects. However, as the second of these four effects is dominated by the other three effects for any m, consideration of the value of the option to default for equity holders does not modify the tendency of the equity finance model. Consequently, increasing long-term solvency uncertainty is more likely to induce equity holders to invest earlier in the baseline model.

The analysis in this section shows that the effect on the default threshold of long-term solvency uncertainty is opposite to that of short-term liquidity uncertainty for each debt maturity. Furthermore, the effect on the investment threshold (debt overhang) of long-term solvency uncertainty is also opposite to that of short-term liquidity uncertainty. Intuitively,

this is because the greater degree of uncertainty about short-term liquidity (long-term solvency) makes cash flow signals less (more) informative through the learning process of profitability uncertainty. As a result, short-term liquidity uncertainty and long-term solvency uncertainty affect the firm's demand for cash flows and the market value of debt per capital stock differently at each point in time, thereby leading to different effects on the default and investment policies of the firm. These results have not been tested empirically.

Our investigation also indicates that for each debt maturity, the effect of increasing long-term solvency uncertainty not only mitigates the incentives for equity holders to default, but also improves their incentives to invest. This result depends on the fact that increasing long-term solvency uncertainty not only raises the profit potential of the firm at success but also reduces the market value of debt per capital stock, thereby decreasing (increasing) equity holders' incentives to default (invest).

4.3. Leverage and credit spreads.—

In this subsection, we examine the effects of short-term liquidity and long-term solvency uncertainties on the leverage ratio (debt to firm value) and credit spreads in the baseline model.

Figure 6 displays the effects of the two sources of uncertainty on the leverage ratio.[16] The effect of increasing σ on the leverage ratio is positive for all m. As discussed in the preceding subsection, a higher volatility increases the magnitude of liquidity risk, makes cash flow signals less informative about $\overline{\mu}$, and causes default to be relatively early. Although the first and third effects increase the cost of debt, the second learning effect induces equity holders to issue more debt because debt is information-insensitive security. As the second learning effect dominates, the higher volatility increases the debt value and leverage ratio.

However, Figure 6 also shows that the effect of a mean preserving spread of $\mu_H - \mu_L$ around the mean is negative for all m. As argued in the preceding section, a higher spread $\mu_H - \mu_L$ increases the profit potential for the firm at success, brings out the higher informativeness of cash flows, and induces equity holders to default relatively late. In particular, because the second learning effect dominates and debt is information-insensitive security, the higher $\mu_H - \mu_L$ motivates equity holders to issue less debt and decreases the leverage ratio.

Gryglewicz (2011) suggests that the consideration of cash holdings in his model lessens

[16] In Figures 6 and 7, note that in several cases of $m < 3$, we do not obtain any computation results because the firm does not operate in this case because of $\mu_0 \leq \mu_B$.

the problem of the standard trade-off model of capital structure that the optimal leverage implied by the standard model exceeds the leverage ratio observed empirically. However, our numerical analysis also indicates that the leverage ratio is significantly lower as the debt maturity is longer. Hence, our model implies that one of the driving forces of the reduced leverage is the incorporation of debt maturity.[17]

Figure 7 illustrates the effects of short-term liquidity and long-term solvency uncertainties on credit spreads. The high volatility σ decreases credit spreads for all $m \geq 9$, but does not necessarily do so for shorter-maturity debt. The high volatility risk generates higher liquidity risk, lowers informativeness of cash flows, and results in the greater possibility of default. Indeed, as the second learning effect dominates for $m \geq 9$ (but does not necessarily dominate for $m < 9$), credit spreads decrease for $m \geq 9$ (but does not necessarily decrease for $m < 9$).

However, Figure 7 indicates that the higher spread $\mu_H - \mu_L$ increases credit spreads for all $m \geq 12$, but does not necessarily do so for shorter-maturity debt. The higher spread creates the greater profit potential of the firm at success, the greater informativeness of cash flows, and the lower default threshold. Because the second learning effect dominates for $m \geq 12$ (but does not necessarily dominate for $m < 12$), credit spreads increase for $m \geq 12$ (but does not necessarily decrease for $m < 12$).

When the maturity of debt is not sufficiently long, our results for credit spreads do not necessarily coincide with those of Gryglewicz (2011). Again, the incorporation of debt maturity affects the results significantly.

5. Equilibrium Debt Maturity and Corresponding Default and Investment Policies

5.1. Equilibrium debt maturity.—

In this subsection, we discuss the debt maturity choice of the firm. To this end, we consider the following setting. At time 0, the founder establishes the firm and maximizes its value by issuing equity and debt and selling them to outside investors. The founder also selects the maturity of debt. Because the total amount of the promised repayments of debt per capital stock, p, is taken as a fixed parameter, the valuation equations of $d(\mu)$ and $e(\mu)$ in Proposition 1 show that the choice of debt maturity determines both the initial values of

[17] Note that in the model of Gryglewicz (2011), the debt maturity is infinite.

debt and equity per capital stock, given an initial belief μ_0. Hence, the firm's determination of debt maturity is equivalent to its determination of leverage. After the founder sells equity and debt to outside investors, we assume that management acts in the interests of equity holders. Hence, it is plausible to assume that the investment and default thresholds are determined by equity holders as in the previous sections.

Figure 8 illustrates that the initial firm value per capital stock is increasing in debt maturity.[18] Hence, we introduce the cost of varying debt maturity in the subsequent analysis. To this end, we consider a constraint in which the firm must at least raise a certain amount $\underline{d}$ through debt financing at $t = 0$. This is equivalent to assuming that the initial market value of debt per capital stock (firm's leverage), $d(m, \mu_0)$, must be larger than or equal to $\underline{d}$, that is, $d(m, \mu_0) \geq \underline{d}$. As in Diamond and He (2014) and DeMarzo and He (2016), within the model we do not specify any particular reasons why the firm's initial leverage must be at least set equal to $\underline{d}$.[19] Holding the initial market value of debt constant by varying the promised payments, Diamond and He (2014) isolate the ex ante effect of maturity on debt overhang from the pure leverage effect on debt overhang. In contrast, as in Gertner and Scharfstein (1991) and Titman and Tsyplakov (2007), our paper adjusts $d(m, \mu_0)$ for each maturity of debt while making the fixed promised payment per capital stock p. However, our procedure also implies that the maturity of debt is adjusted so as to make $d(m, \mu_0)$ fixed at $\underline{d}$, as long as $d(m, \mu_0) \geq \underline{d}$ is binding with equality. As $d(m, \mu_0) \geq \underline{d}$ is always binding with equality, the equilibrium debt maturity is determined by $d(m, \mu_0) = \underline{d}$. In Section 5.2, we always set $\mu_0 = 0.5$ and use the set of basic parameters. In Section 5.3, we vary μ_0 relative

[18] He and Milbradt (2013) suggest that the optimal debt maturity is infinite (that is, an infinitely lived consol bond is optimal) in a Leland-style model such as Leland (1996) and He and Xiong (2012a). This point also depends on the assumption of the Leland-style model, which assumes that the firm commits to a future leverage policy with fixed debt face value. Relaxing the assumption that the firm commits to a future leverage policy with fixed debt face value, DeMarzo and He (2016) suggest that ultra-short-term debt maximizes both the firm value and the debt capacity. In our model, raising m reduces equity holders' rollover losses per capital stock, $\frac{1}{m}[p - d(m)]$, because the effect of the lower rollover frequency $\frac{1}{m}$ brought about by raising m dominates the effect of the smaller market value of debt per capital stock caused by raising m. Hence, equity holders are more likely to default later if the firm chooses the longer debt maturity structure. In addition, raising m also decreases the investment threshold. Combining these effects results in an increase in the initial firm value per capital stock in our model.

[19] Generally, the financing constraint can be justified by asymmetric information between the founder and outside investors at $t = 0$. Because the founder knows more about the firm's information, she may find it difficult to issue a sufficient amount of equity or may face a sufficiently high issuance cost of equity at $t = 0$. Hence, the founder at least needs to issue a certain amount of debt in order to raise the start-up cost of the firm. This financing is possible because debt is information-insensitive security. Note that our main results are not modified even though we instead impose a constraint on the market value of equity per capital stock such that $e(m, \mu_0) \leq \underline{e}$.

to the set of basic parameters.

5.2. The effects of short-term liquidity and long-term solvency uncertainties on equilibrium debt maturity and the corresponding default and investment policies.—

This subsection investigates how the two sources of uncertainty affect equilibrium debt maturity and change optimal default and investment policies corresponding to the equilibrium debt maturity. Figure 9 shows that an increase in σ increases m^* for any $\underline{d} \leq 0.48$, whereas an increase in $\mu_H - \mu_L$ decreases m^* for any $\underline{d} \leq 0.44$.[20] Here, m^* is the equilibrium debt maturity. However, Figure 9 indicates that an increase in σ does not necessarily increase m^* for any $\underline{d} > 0.48$, while an increase in $\mu_H - \mu_L$ does not necessarily decrease m^* for any $\underline{d} > 0.44$. Tables 1A and 1B also report that an increase in σ ($\mu_H - \mu_L$) increases (decreases) both μ_B^* and μ_I^*, where μ_B^* and μ_I^* are the default and investment thresholds that correspond to m^*. Thus, equilibrium debt maturity increases (decreases) with short-term liquidity (long-term solvency) uncertainty if $\underline{d}$ is not sufficiently large. However, the corresponding default threshold and the corresponding investment threshold—the extent of debt overhang—increase (decrease) with short-term liquidity (long-term solvency) uncertainty. This implies that the effect of long-term solvency uncertainty on equilibrium debt maturity is opposite to that of short-term liquidity uncertainty if $\underline{d}$ is not sufficiently large, and the effects of long-term solvency uncertainty on the corresponding default and investment thresholds are opposite to those of short-term liquidity uncertainty.

The intuition behind these results is explained as follows. Because the firm needs to raise debt to satisfy the constraint $d(m, \mu_0) \geq \underline{d}$, its choice of m must take into account the impact of σ on $d(m, \mu_0)$. An increase in σ magnifies liquidity shocks and makes cash flow signals less informative about profitability. Suppose that $\underline{d}$ is not sufficiently large so that the chosen m becomes sufficiently long. Then, as discussed in Section 4.2, the above two effects increase the initial market value of debt per capital stock in our parameter set (see Figure 9A). As argued in Section 4.1, $d(m, \mu_0)$ is decreasing in m for any m and σ except for extremely small m (see Figure 9A). Thus, the upward shift in $d(m, \mu_0)$ in response to the increase in σ implies that the firm must raise m^* to attain $d(m, \mu_0) = \underline{d}$ as σ increases. In Section 4.2, we also show that the direct effect of a change in σ increases both μ_B and μ_I for any m.

[20] In Figure 9A (9B), note that in several cases of $m < 2$ ($m < 3$), we do not obtain any computation results because the firm does not operate in this case because of $\mu_0 \leq \mu_B$.

However, with regard to the choices of μ_B^* and μ_I^*, we further need to consider the indirect effects of a change in σ on μ_B and μ_I through a change in m in addition to such direct effects of a change in σ on μ_B and μ_I for fixed m. The indirect effects are expressed as the effect of a change in σ on m multiplied by the effects of a change in m on μ_B and μ_I for fixed σ, that is, $\frac{\partial \mu_B}{\partial m}\frac{\partial m}{\partial \sigma}$ and $\frac{\partial \mu_I}{\partial m}\frac{\partial m}{\partial \sigma}$. As discussed above, $\frac{\partial m}{\partial \sigma}$ is positive under the restriction of $\underline{d}$ imposed here. However, in Section 4.1, we show that the effects of a change in m on μ_B and μ_I for fixed σ, $\frac{\partial \mu_B}{\partial m}$ and $\frac{\partial \mu_I}{\partial m}$, are interpreted as effects occurring along the μ_B and μ_I curves. Combining these three effects, we see that an increase in σ increases both μ_B^* and μ_I^*.

However, if $\underline{d}$ is sufficiently large so as to make the choice set of m sufficiently short, the discussion in Section 4.3 shows that credit spreads do not necessarily decrease with σ. This implies that the higher volatility risk may increase the cost of debt and reduce the initial market value of debt per capital stock if $\underline{d}$ is sufficiently large. Consequently, if $\underline{d}$ is sufficiently large, the firm does not necessarily raise m^* as σ increases. By contrast, even though m^* decreases with σ, this effect makes the indirect effects of a change in σ on μ_B^* and μ_I^* become positive, and strengthens the tendency that μ_B^* and μ_I^* increase with σ.

Higher values of $\mu_H - \mu_L$ make cash flow signals more informative about profitability, thus reducing the initial market value of debt per capital stock further if $\underline{d}$ is not sufficiently large so that the chosen m becomes sufficiently long (see the discussion in Section 4.2 and Figure 9B). In addition, $d(m, \mu_0)$ is decreasing in m for any m and $\mu_H - \mu_L$ except for extremely small m (see the discussion in Section 4.1 and Figure 9B). Hence, if $\underline{d}$ is not sufficiently large, the firm decreases m^* to attain $d(m, \mu_0) = \underline{d}$ when $\mu_H - \mu_L$ increases. The discussion of the effects of a change in $\mu_H - \mu_L$ on μ_B^* and μ_I^* is similar to that of a change in σ on μ_B^* and μ_I^*. Thus, the effects of a change in $\mu_H - \mu_L$ on μ_B^* and μ_I^* consist of the direct effects of a change in $\mu_H - \mu_L$ on μ_B and μ_I for fixed m, and the indirect effects of a change in $\mu_H - \mu_L$ on μ_B and μ_I through a change in m. Combining these effects, we find that an increase in $\mu_H - \mu_L$ decreases both μ_B^* and μ_I^* if $\underline{d}$ is not sufficiently large.

However, if $\underline{d}$ is sufficiently large so as to make the choice set of m sufficiently short, the discussion of Section 4.3 indicates that credit spreads do not necessarily increase with $\mu_H - \mu_L$. Hence, the higher spread $\mu_H - \mu_L$ may decrease the cost of debt and raise the initial market value of debt per capital stock if $\underline{d}$ is sufficiently large. Then, the firm does not necessarily reduce m^* as $\mu_H - \mu_L$ increases. By contrast, even though m^* increases with $\mu_H - \mu_L$, this effect makes the indirect effects of a change in $\mu_H - \mu_L$ on μ_B^* and μ_I^* become negative, and enhances the tendency that μ_B^* and μ_I^* decrease with $\mu_H - \mu_L$.

Gryglewicz (2011) indicates that initial cash holdings and the default threshold increase (decrease) with short-term liquidity (long-term solvency) uncertainty. Thus, he suggests that the effects of long-term solvency uncertainty on initial cash holdings and the default threshold are opposite to those of short-term liquidity uncertainty because the volatility risk and profitability uncertainty are related differently to solvency concerns. However, as he assumes perpetual debt, debt maturity is fixed in his model although he considers the initial cash holdings of the firm. By endogenizing debt maturity and incorporating investment incentives for equity holders, we newly show that the equilibrium debt maturity increases (decreases) with short-term liquidity (long-term solvency) uncertainty if the firm does not depend on debt financing substantially, whereas the corresponding investment threshold increases (decreases) with short-term liquidity (long-term solvency) uncertainty. Hence, even when endogenizing debt maturity and investment incentives for equity holders, we find that long-term solvency uncertainty and short-term liquidity uncertainty have opposite effects on key financial variables such as debt maturity and the corresponding investment threshold (debt overhang) if the firm does not depend on debt financing substantially so that the chosen m becomes sufficiently long. As discussed in the final paragraph of Section 4.2, these findings are created by the different effects of short-term liquidity and long-term solvency uncertainty on the informativeness of cash flow signals. However, if the firm depends on debt financing substantially so as to make the choice set of m sufficiently short, we do not necessarily indicate that long-term solvency uncertainty and short-term liquidity uncertainty have opposite effects on debt maturity. However, even in this case, we report that long-term solvency uncertainty and short-term liquidity uncertainty still have opposite effects on the corresponding investment threshold (debt overhang) because the indirect effect of short-term liquidity (long-term solvency) uncertainty on the investment threshold through a change in debt maturity only strengthens this tendency.[21]

Our theoretical predictions have not been tested empirically. As suggested in Gryglewicz (2011), empirical studies should pay attention to the effects of profitability uncertainty in addition to the effects of volatility risk.

[21] Cheng and Milbradt (2012) examine the effects of changes in the drift and the volatility of a Brownian motion under debt rollover and the risk-shifting incentive of the manager. However, because neither the investment decision nor the learning process is considered in their model, the implications of the effects of changes in the drift and the volatility in their model are different from those in σ and $\mu_H - \mu_L$ in our model. Consequently, unlike our model, Cheng and Milbradt (2012) suggest that both the optimal maturity and the corresponding default threshold decrease with the drift and the volatility when debt maturity is determined to maximize the total value of the firm.

5.3. The effects of the initial belief in long-term solvency uncertainty on equilibrium debt maturity and the corresponding default and investment policies.—

We discuss how the initial belief in profitability uncertainty affects equilibrium debt maturity and changes optimal default and investment policies corresponding to the endogenized debt maturity. As $\varphi_0\mu_H + (1-\varphi_0)\mu_L = \mu_0$, where φ_0 is the prior probability of μ_H at the initial time 0, an increase in μ_0 increases φ_0 because $\varphi_0 = \frac{\mu_0-\mu_L}{\mu_H-\mu_L}$. Hence, we can capture the effect of an initial optimistic or pessimistic belief in profitability uncertainty by varying μ_0.

Figure 10 and Table 1C show that for any $\underline{d}$, an increase in μ_0 increases m^* and decreases μ_B^* and μ_I^*.[22] Thus, the more optimistic the initial belief in long-term solvency uncertainty is, the longer the equilibrium debt maturity and the lower the corresponding default and investment thresholds are.

Intuitively, an increase in μ_0 increases the initial market value of debt per capital stock for any $m \geq 0$. $d(m, \mu_0)$ is also decreasing in m for any $m \geq 0$ and μ_0 (see the discussion in Section 4.1 and Figure 10). Thus, the upward shift in $d(m, \mu_0)$ for each m in response to the increase in μ_0 implies that the firm must raise m^* to attain $d(m, \mu_0) = \underline{d}$ as μ_0 increases. For the choices of μ_B^* and μ_I^*, we again need to examine both the direct effects of a change in μ_0 on μ_B and μ_I for fixed m and the indirect effects of a change in μ_0 on μ_B and μ_I through a change in m. As an increase in μ_0 implies that the firm's business condition is expected to improve, it is straightforward to see that μ_B^* and μ_I^* decrease with μ_0.

5.4. Empirical implications.—

Badoer and James (2016) report that firms with a long-term credit rating by S&P of A or higher are more likely to issue long-term debt. As the higher long-term credit rating can be regarded as reflecting less long-term solvency uncertainty, their result is consistent with our prediction of the effect of long-term solvency uncertainty on debt maturity, particularly if these firms do not need to depend on debt financing considerably.

In our model, the more pessimistic initial belief in long-term solvency uncertainty decreases debt maturity and increases the investment threshold. This implies that debt maturity is more likely to be shorter and investment is more likely to be reduced when pessimistic beliefs in long-term solvency dominate. If deteriorating economic conditions are more likely

[22] In Figure 10, for any value of m where we do not obtain any computation results, the firm does not operate because of $\mu_0 \leq \mu_B$.

to generate pessimistic beliefs in the long-term solvency of each firm, this prediction is consistent with several empirical findings. For example, Krishnamurthy (2010) indicates that financial firms were shortening their debt maturity just before the financial crisis of 2007–2008. Xu (2014) reports that debt maturity is longer in good times in speculative-grade firms. Almeida, Campello, Laranjeira, and Weisbenner (2011) also find that firms whose long-term debt was largely maturing just after the 2007 credit crisis cut their investment substantially.

If a growth firm is defined as one that has substantial uncertainty about its expected investment performance but achieves higher earnings when its investment succeeds, our result has important empirical implications. Because a growth firm can then be regarded as one with a larger mean preserving spread $\mu_H - \mu_L$, our result shows that such a growth firm should have shorter-term debt, particularly if it does not need to depend on debt financing substantially. By contrast, if uncertainty about the expected investment performance of a firm is not large and lower earnings are only attained when the investment of the firm succeeds, our result implies that such a firm should have longer-term debt, particularly if it does not need to depend on debt financing substantially. Although Diamond and He (2014) suggest a similar result for debt maturity, they define a growth firm as one in which there is substantial uncertainty about its investment opportunities and the value of its new investment projects is highly correlated with the value of its existing investment projects. In this sense, our result provides more straightforward empirical implications regarding debt maturity in a growth firm.

The above empirical implication of our model can also be applied to the analysis of the effect of deregulation on the debt maturity structure of regulated firms. Indeed, if deregulation causes substantial uncertainty about the expected investment performance of firms but brings about higher earnings when the firms' investment succeeds, our model predicts that the firms should have shorter-term debt after deregulation than before deregulation, particularly if they do not need to depend on debt financing substantially. This empirical prediction of our model is consistent with the finding of Ovtchinnikov (2016) regarding the effect of deregulation on regulated firms' debt decisions: regulated firms depend more on long-term debt but reduce their dependence substantially during deregulation.

We show that short-term liquidity uncertainty and long-term solvency uncertainty have opposite effects on several key financial variables such as debt maturity and investment policies under certain conditions. These findings depend on the different effects of short-

term liquidity uncertainty and long-term solvency uncertainty on the informativeness of cash flow signals, and have not been tested empirically. Hence, our findings suggest that long-term solvency uncertainty should be distinguished from short-term liquidity uncertainty in the empirical literature on debt rollover and investment when an endogenous interaction between solvency and liquidity concerns, generated by a learning process, is regarded as important.

6. Conclusion

We explore the effects of the interaction between rollover risk and solvency concern on the debt maturity, default, investment, and leverage policy decisions of a firm under debt rollover and the learning process of the firm's solvency risk. We distinguish between short-term liquidity uncertainty (cash flow shock) and long-term solvency uncertainty (profitability uncertainty) and incorporate an assessment of the firm's solvency risk via the learning process over time. Under the learning model framework, the effects on the decisions of the firm about debt maturity, default, investment, and leverage policies resulting from long-term solvency uncertainty are separated from those resulting from short-term liquidity uncertainty. We consider how the two sources of uncertainty affect such decisions of the firm under the endogenous interaction between rollover risk and solvency concern, generated by both debt rollover and the learning process, when newly issued equity covers losses under debt rollover.

If debt maturity is determined exogenously, our results indicate that default is more likely to occur if debt maturity is shorter. Our results also show that as debt maturity becomes longer, less debt overhang occurs subsequently. Our findings further show that an increase in short-term liquidity (long-term solvency) uncertainty raises (reduces) the firm's incentives to default and the leverage ratio for any debt maturity, whereas an increase in short-term liquidity (long-term solvency) uncertainty reduces (raises) the firm's incentives to invest for any debt maturity. The final result regarding the firm's investment incentives implies that the effect of long-term solvency uncertainty on the investment policy—debt overhang—is opposite to that of short-term liquidity uncertainty for any debt maturity. In addition, an increase in short-term liquidity (long-term solvency) uncertainty reduces (raises) credit spreads if debt maturity is sufficiently long.

If debt maturity is determined endogenously, we show that the equilibrium debt maturity increases (decreases) with short-term liquidity (long-term solvency) uncertainty if the

firm does not depend on debt financing substantially so that the chosen debt maturity becomes sufficiently long; and that for any debt maturity, the firm's incentives to default increase (decrease) with short-term liquidity (long-term solvency) uncertainty whereas the firm's incentives to invest decrease (increase) with short-term liquidity (long-term solvency) uncertainty. Thus, the effect of long-term solvency uncertainty on debt maturity and the firm's incentives to default and invest (debt overhang) are opposite to those of short-term liquidity uncertainty under certain conditions. We also show that the more pessimistic the initial belief regarding long-term solvency uncertainty is, the shorter the equilibrium debt maturity, the higher the firm's incentive to default, and the lower the firm's incentive to invest are.

Appendix

Proof of Proposition 1: We begin by solving for the debt value function per capital stock. Initially, we assume that equity holders use the investment threshold policy. Later, we will prove that the investment threshold policy is optimal for equity holders. Then, if $\mu \geq \mu_I$, ordinary differential equation (8) has a solution of the following general form:

$$d(\mu) = \frac{m}{2[1+m(r-i+\delta)]\sigma^2}\beta_1(\beta_1-1)(\mu_H-\mu_L)^2\left[A_1(\mu-\mu_L)^{1-\beta_1}(\mu_H-\mu)^{\beta_1}\right.$$
$$\left.+A_2(\mu-\mu_L)^{\beta_1}(\mu_H-\mu)^{1-\beta_1}\right]+\frac{p}{1+m(r-i+\delta)},$$

where $\beta_1 > 1$ is the positive root of

$$\beta_1^2-\beta_1-\frac{2[1+m(r-i+\delta)]\sigma^2}{m(\mu_H-\mu_L)^2}=0. \tag{A1}$$

Because $A_2(\mu-\mu_L)^{\beta_1}(\mu_H-\mu)^{1-\beta_1} \to \pm\infty$ as $\mu \to \mu_H$, boundary condition (12) implies that $A_2 = 0$. Thus, using (A1), we obtain

$$d(\mu) = A_1(\mu-\mu_L)^{1-\beta_1}(\mu_H-\mu)^{\beta_1}+\frac{p}{1+m(r-i+\delta)}, \qquad \text{if } \mu \geq \mu_I. \tag{A2}$$

If $\mu_I > \mu \geq \mu_B$, ordinary differential equation (8) still has a solution of the following general form:

$$d(\mu) = \frac{m}{2[1+m(r+\delta)]\sigma^2}\beta_2(\beta_2-1)(\mu_H-\mu_L)^2\left[A_3(\mu-\mu_L)^{1-\beta_2}(\mu_H-\mu)^{\beta_2}\right.$$
$$\left.+A_4(\mu-\mu_L)^{\beta_2}(\mu_H-\mu)^{1-\beta_2}\right]+\frac{p}{1+m(r+\delta)},$$

where $\beta_2 > 1$ is determined by

$$\beta_2^2-\beta_2-\frac{2[1+m(r+\delta)]\sigma^2}{m(\mu_H-\mu_L)^2}=0. \tag{A3}$$

Thus, it follows from (A3) that the above solution can be reduced to

$$d(\mu) = A_3(\mu-\mu_L)^{1-\beta_2}(\mu_H-\mu)^{\beta_2}+A_4(\mu-\mu_L)^{\beta_2}(\mu_H-\mu)^{1-\beta_2}+\frac{p}{1+m(r+\delta)}, \qquad \text{if } \mu_I > \mu \geq \mu_B. \tag{A4}$$

Solving the constants A_1, A_3, and A_4 using (9)–(11), (A2), and (A4), we obtain

$$\begin{aligned} A_1 = &\left[\frac{\mu_B}{r+\delta} - \frac{p}{1+m(r+\delta)}\right]\frac{1}{\mu_B-\mu_L}\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{\beta_2}\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2-\beta_1} \\ &+\left[\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\beta_1+\beta_2-1} - \left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\beta_2-1}\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2-\beta_1}\right]A_4 \\ &-\frac{\frac{pmi}{\mu_H-\mu_I}\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\beta_1-1}}{[1+m(r+\delta)]\,[1+m(r-i+\delta)]}, \end{aligned} \tag{A5}$$

$$A_3 = \left[\frac{\mu_B}{r+\delta} - \frac{p}{1+m(r+\delta)}\right]\frac{1}{\mu_B-\mu_L}\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{\beta_2} - \left(\frac{\mu_H-\mu_B}{\mu_B-\mu_L}\right)^{1-2\beta_2}A_4, \tag{A6}$$

$$\begin{aligned} A_4 = &\left\{\Psi_3 + \left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\beta_2-1}\Psi_2\right. \\ &\left.+\left[\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\beta_1+\beta_2-1} - \left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\beta_2-1}\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2-\beta_1}\right]\Psi_1\right\}^{-1} \\ &\times\left\{\left[\frac{\mu_B}{r+\delta} - \frac{p}{1+m(r+\delta)}\right]\frac{1}{\mu_B-\mu_L}\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{\beta_2}\left[\Psi_2 - \left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2-\beta_1}\Psi_1\right]\right. \\ &\left.+\frac{pmi\left(\frac{\beta_1}{\mu_H-\mu_I}+\frac{\beta_1-1}{\mu_I-\mu_L}\right)}{[1+m(r+\delta)]\,[1+m(r-i+\delta)]}\right\}, \end{aligned} \tag{A7}$$

where

$$\Psi_1 = \beta_1\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_1-1} + (\beta_1-1)\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_1}, \tag{A8}$$

$$\Psi_2 = \beta_2\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2-1} + (\beta_2-1)\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\beta_2}, \tag{A9}$$

$$\Psi_3 = \beta_2\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\beta_2-1} + (\beta_2-1)\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\beta_2}. \tag{A10}$$

Now, we move on to equity. We first show that the optimal investment policy for fixed

debt maturity is given by the investment threshold policy. Note that $\frac{\mu_H-\lambda(r+\delta)}{r-i+\delta} > \frac{p}{1+m(r-i+\delta)}$ is assumed,[23] which ensures that $e(\mu_H) > \lambda$. Hence, investment is optimal and default does not occur at $\mu = \mu_H$. Then, it follows from (12) and (14) that $e(\mu) \to \frac{\mu_H-\lambda i}{r-i+\delta} - \frac{p}{1+m(r-i+\delta)} > \lambda$ as $\mu \to \mu_H$. Given $e(\mu_B) = 0$ from (17), there must exist a solution $\mu_I \in (\mu_B, \mu_H)$ that satisfies $e(\mu_I) = \lambda$. Suppose that we have multiple solutions to $e(\mu_I) = \lambda$, and take the smallest one as μ_{I0}. To prove that the threshold strategy is optimal, we need to show that $e(\mu) > \lambda$ for any $\mu \in (\mu_{I0}, \mu_H]$, where $e(\mu)$ solves the following ordinary differential equation for any $\mu \in (\mu_{I0}, \mu_H]$:

$$(r-i+\delta)e(\mu) = \mu - \lambda i + \frac{1}{2\sigma^2}(\mu-\mu_L)^2(\mu_H-\mu)^2 e''(\mu) - \frac{1}{m}[p-d(\mu)]. \tag{A11}$$

Suppose that there are at least two other solutions $\mu_{I1}, \mu_{I2} \in (\mu_{I0}, \mu_H]$ that satisfy $\mu_{I1} < \mu_{I2}$, $e(\mu_{Ii}) = \lambda$ for $i = 1, 2$, and $e'(\mu_{I1}) < 0 < e'(\mu_{I2})$. Then, there are intermediate points $\mu_I^\circ \in (\mu_{I0}, \mu_{I1})$ and $\mu_I^{\circ\circ} \in (\mu_{I1}, \mu_{I2})$ so that $e(\mu_I^\circ) > \lambda > e(\mu_I^{\circ\circ})$, $e'(\mu_I^\circ) = e'(\mu_I^{\circ\circ}) = 0$, $e''(\mu_I^\circ) < 0$, and $e''(\mu_I^{\circ\circ}) > 0$.[24,25] Now, evaluating (A11) at μ_I° and $\mu_I^{\circ\circ}$ and using $e(\mu_I^\circ) > \lambda > e(\mu_I^{\circ\circ})$, we obtain

$$\begin{aligned}&\mu_I^\circ - \lambda i + \frac{1}{2\sigma^2}(\mu_I^\circ-\mu_L)^2(\mu_H-\mu_I^\circ)^2 e''(\mu_I^\circ) - \frac{1}{m}[p-d(\mu_I^\circ)]\\ &> \frac{\lambda}{r-i+\delta} > \mu_I^{\circ\circ} - \lambda i + \frac{1}{2\sigma^2}(\mu_I^{\circ\circ}-\mu_L)^2(\mu_H-\mu_I^{\circ\circ})^2 e''(\mu_I^{\circ\circ}) - \frac{1}{m}[p-d(\mu_I^{\circ\circ})].\end{aligned}$$

Thus,

$$\mu_I^\circ - \mu_I^{\circ\circ} > \frac{1}{m}[d(\mu_I^{\circ\circ}) - d(\mu_I^\circ)] + \frac{1}{2\sigma^2}(\mu_I^{\circ\circ}-\mu_L)^2(\mu_H-\mu_I^{\circ\circ})^2 e''(\mu_I^{\circ\circ}) - \frac{1}{2\sigma^2}(\mu_I^\circ-\mu_L)^2(\mu_H-\mu_I^\circ)^2 e''(\mu_I^\circ). \tag{A12}$$

Given $\mu_I^\circ \in (\mu_{I0}, \mu_{I1})$, $\mu_I^{\circ\circ} \in (\mu_{I1}, \mu_{I2})$, $e''(\mu_I^\circ) < 0$, and $e''(\mu_I^{\circ\circ}) > 0$, it follows from (A12) that $d(\mu_I^{\circ\circ}) < d(\mu_I^\circ)$. Because of $e(\mu_I^\circ) > \lambda > e(\mu_I^{\circ\circ})$, this implies that $v(\mu_I^\circ) > v(\mu_I^{\circ\circ})$. However, this contradicts the assumption that $v(\mu)$ is increasing in μ. Hence, the solution to $e(\mu_I) = \lambda$ is uniquely determined. Consequently, we verify that $e(\mu) > \lambda$ for any $\mu \in (\mu_I, \mu_H]$.

We next proceed to characterize the equity value function per capital stock. As discussed

[23] Note that our parameter set satisfies this assumption.

[24] Even though $\mu_{I0} = \mu_{I1}$ ($\mu_{I1} = \mu_{I2}$) so that $e(\mu_{I1}) = \lambda$ and $e'(\mu_{I1}) = 0$ ($e(\mu_{I2}) = \lambda$ and $e'(\mu_{I2}) = 0$), we can set $\mu_I^\circ = \mu_{I1}$ ($\mu_I^{\circ\circ} = \mu_{I2}$). Then, the following argument still holds.

[25] μ_I° is a local maximum point of $e(\mu)$. Thus, $e(\mu)$ is flat and concave at μ_I°. In contrast, $\mu_I^{\circ\circ}$ is a local minimum point of $e(\mu)$. Thus, $e(\mu)$ is flat and convex at $\mu_I^{\circ\circ}$.

in Diamond and He (2014), the equity value per capital stock can be indirectly derived as the difference between the total firm value per capital stock and the debt value per capital stock: $e(\mu) = v(\mu) - d(\mu)$. The total firm value per capital stock $v(\mu)$ satisfies

$$v(\mu) = \begin{cases} \frac{\mu-\lambda i}{r-i+\delta} + B_1(\mu-\mu_L)^{1-\gamma_1}(\mu_H-\mu)^{\gamma_1}, & \text{if } \mu \geq \mu_I, \\ \frac{\mu}{r+\delta} + B_2(\mu-\mu_L)^{1-\gamma_2}(\mu_H-\mu)^{\gamma_2} + B_3(\mu-\mu_L)^{\gamma_2}(\mu_H-\mu)^{1-\gamma_2}, & \text{if } \mu_I > \mu \geq \mu_B. \end{cases} \tag{A13}$$

The function $v(\mu)$ can be interpreted as follows. If $\mu \geq \mu_I$, the first term is equal to the firm value per capital stock that would be realized if the firm always invested and did not default. The second term indicates the adjustment for stopping investment at least temporarily. If $\mu_I > \mu \geq \mu_B$, the first term is the firm value per capital stock without investment in the case of no default. The remaining terms reflect the adjustment for entering the investment region again in the future.

Thus, it follows from (A2), (A13), and $e(\mu) = v(\mu) - d(\mu)$ that if $\mu \geq \mu_I$, the equity value per capital stock is given by

$$e(\mu) = \frac{\mu-\lambda i}{r-i+\delta} + B_1(\mu-\mu_L)^{1-\gamma_1}(\mu_H-\mu)^{\gamma_1} - A_1(\mu-\mu_L)^{1-\beta_1}(\mu_H-\mu)^{\beta_1} - \frac{p}{1+m(r-i+\delta)}. \tag{A14}$$

Then,

$$\begin{aligned} e'(\mu) &= v'(\mu) - d'(\mu) \\ &= \frac{1}{r-i+\delta} + (1-\gamma_1)B_1\left(\frac{\mu_H-\mu}{\mu-\mu_L}\right)^{\gamma_1} - \gamma_1 B_1\left(\frac{\mu-\mu_L}{\mu_H-\mu}\right)^{1-\gamma_1} \\ &\quad - \left[(1-\beta_1)A_1\left(\frac{\mu_H-\mu}{\mu-\mu_L}\right)^{\beta_1} - \beta_1 A_1\left(\frac{\mu-\mu_L}{\mu_H-\mu}\right)^{1-\beta_1}\right], \end{aligned} \tag{A15}$$

$$e''(\mu) = \frac{(\mu_H-\mu_L)^2}{(\mu-\mu_L)(\mu_H-\mu)^2}\left[\gamma_1(\gamma_1-1)B_1\left(\frac{\mu_H-\mu}{\mu-\mu_L}\right)^{\gamma_1} - \beta_1(\beta_1-1)A_1\left(\frac{\mu_H-\mu}{\mu-\mu_L}\right)^{\beta_1}\right]. \tag{A16}$$

Using (14) and $v(\mu) = d(\mu) + e(\mu)$, it is found from (A1), (A2), (A13), and (A16) that $\gamma_1 > 1$ is the positive root of

$$\gamma_1^2 - \gamma_1 - \frac{2\sigma^2(r-i+\delta)}{(\mu_H-\mu_L)^2} = 0. \tag{A17}$$

If $\mu_I > \mu \geq \mu_B$, we repeat the above argument with (14), (A3), (A4), (A13), and $e(\mu) = v(\mu) - d(\mu)$. Then, we can show that the equity value per capital stock is given by

$$\begin{aligned} e(\mu) = \frac{\mu}{r+\delta} &+ B_2(\mu-\mu_L)^{1-\gamma_2}(\mu_H-\mu)^{\gamma_2} + B_3(\mu-\mu_L)^{\gamma_2}(\mu_H-\mu)^{1-\gamma_2} \\ &- A_3(\mu-\mu_L)^{1-\beta_2}(\mu_H-\mu)^{\beta_2} - A_4(\mu-\mu_L)^{\beta_2}(\mu_H-\mu)^{1-\beta_2} - \frac{p}{1+m(r+\delta)}, \end{aligned} \tag{A18}$$

where $\gamma_2 > 1$ is given by the positive root of

$$\gamma_2^2 - \gamma_2 - \frac{2\sigma^2(r+\delta)}{(\mu_H-\mu_L)^2} = 0. \tag{A19}$$

We now solve the constants B_1, B_2, and B_3. Combining (9)–(11), (17), (19), and (20) under $v(\mu) = e(\mu) + d(\mu)$, we have

$$v(\mu_B) = \frac{\mu_B}{r+\delta}, \tag{A20}$$

$$\lim_{\mu\uparrow\mu_I} v(\mu) = \lim_{\mu\downarrow\mu_I} v(\mu), \tag{A21}$$

$$\lim_{\mu\uparrow\mu_I} v'(\mu) = \lim_{\mu\downarrow\mu_I} v'(\mu). \tag{A22}$$

Then, it follows from (A13) and (A20)–(A22) that

$$B_1 = \frac{1}{\Psi_4}\left\{\frac{i}{(r-i+\delta)(r+\delta)} - \left[\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\gamma_2-1}\Psi_5 + \Psi_6\right]B_3\right\}, \tag{A23}$$

$$B_2 = -\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\gamma_2-1} B_3, \tag{A24}$$

$$\begin{aligned} B_3 = &\left\{(\mu_I-\mu_L)\left(\frac{\mu_B-\mu_L}{\mu_H-\mu_B}\right)^{2\gamma_2-1}\left[\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\gamma_1}\frac{\Psi_5}{\Psi_4} - \left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\gamma_2}\right]\right. \\ &\left. +(\mu_H-\mu_I)\left(\frac{\mu_I-\mu_L}{\mu_H-\mu_I}\right)^{\gamma_2} + (\mu_I-\mu_L)\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\gamma_1}\frac{\Psi_6}{\Psi_4}\right\}^{-1} \\ &\times\left\{\frac{i}{(r-i+\delta)(r+\delta)}\left[\mu_I + \frac{(\mu_I-\mu_L)\left(\frac{\mu_H-\mu_I}{\mu_I-\mu_L}\right)^{\gamma_1}}{\Psi_4}\right] - \frac{\lambda i}{r-i+\delta}\right\}, \end{aligned} \tag{A25}$$

where

$$\Psi_4 = \gamma_1 \left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\gamma_1 - 1} + (\gamma_1 - 1)\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\gamma_1}, \tag{A26}$$

$$\Psi_5 = \gamma_2 \left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\gamma_2 - 1} + (\gamma_2 - 1)\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\gamma_2}, \tag{A27}$$

$$\Psi_6 = \gamma_2 \left(\frac{\mu_I - \mu_L}{\mu_H - \mu_I}\right)^{\gamma_2 - 1} + (\gamma_2 - 1)\left(\frac{\mu_I - \mu_L}{\mu_H - \mu_I}\right)^{\gamma_2}. \tag{A28}$$

Now, it follows from (A14) and (A18) that the investment and default thresholds μ_I and μ_B are simultaneously determined by (15) and (18), that is,

$$\begin{aligned}
&\frac{\mu_I - \lambda i}{r - i + \delta} - \frac{p}{1 + m(r - i + \delta)} - \lambda \\
&+ (\mu_I - \mu_L)\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\gamma_1} \frac{1}{\Psi_4}\left\{\frac{i}{(r - i + \delta)(r + \delta)} - \left[\left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{2\gamma_2 - 1} \Psi_5 + \Psi_6\right] B_3\right\} \\
&- (\mu_I - \mu_L)\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\beta_1} \left\{\left[\frac{\mu_B}{r + \delta} - \frac{p}{1 + m(r + \delta)}\right] \frac{1}{\mu_B - \mu_L}\left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{\beta_2}\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\beta_2 - \beta_1}\right. \\
&+ \left[\left(\frac{\mu_I - \mu_L}{\mu_H - \mu_I}\right)^{\beta_1 + \beta_2 - 1} - \left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{2\beta_2 - 1}\left(\frac{\mu_H - \mu_I}{\mu_I - \mu_L}\right)^{\beta_2 - \beta_1}\right] A_4 \\
&\left. - \frac{\frac{pmi}{\mu_H - \mu_I}\left(\frac{\mu_I - \mu_L}{\mu_H - \mu_I}\right)^{\beta_1 - 1}}{[1 + m(r + \delta)]\,[1 + m(r - i + \delta)]}\right\} \\
&= 0,
\end{aligned} \tag{A29}$$

$$\begin{aligned}
&\frac{1}{r + \delta} + \left[\gamma_2 \left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{\gamma_2 - 1} \frac{\mu_H - \mu_L}{\mu_H - \mu_B} + (\gamma_2 - 1)\left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{\gamma_2 - 1} \frac{\mu_H - \mu_L}{\mu_H - \mu_B}\right] B_3 \\
&+ \left[\beta_2 \left(\frac{\mu_H - \mu_B}{\mu_B - \mu_L}\right)^{\beta_2 - 1} + (\beta_2 - 1)\left(\frac{\mu_H - \mu_B}{\mu_B - \mu_L}\right)^{\beta_2}\right] A_3 \\
&- \left[\beta_2 \left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{\beta_2 - 1} + (\beta_2 - 1)\left(\frac{\mu_B - \mu_L}{\mu_H - \mu_B}\right)^{\beta_2}\right] A_4 = 0.
\end{aligned} \tag{A30}$$

Finally, to prove $\frac{\mu_B}{r+\delta} < \frac{p}{1+m(r+\delta)}$, we assume that $\frac{\mu_B}{r+\delta} \geq \frac{p}{1+m(r+\delta)}$. Then, it follows from (9) that $d(\mu_B) \geq \frac{p}{1+m(r+\delta)}$, which means that the debt is riskless. Thus, with the option to default, equity holders must incur strictly negative expected cash flows at default, as

discussed in Diamond and He (2014). However, because equity holders set $i_t = 0$ at μ_B, it follows from (5) that the expected net cash flow per capital stock for equity at μ_B is at least equal to $\{\mu_B + \frac{1}{m}[d(\mu_B) - p]\}dt > \left(\mu_B - \frac{p(r+\delta)}{1+m(r+\delta)}\right) dt \geq 0$, which is a contradiction. ■

Proof of Proposition 2: Suppose that $i = \delta = 0$. Given that the investment threshold μ_I does not need to be considered, we can derive (23)–(25) by applying the same procedure as that of the proof of Proposition 1 with $i = \delta = 0$. Given boundary conditions (12) and (16), note that (23) and (24) are determined so that neither $d(\mu)$ nor $e(\mu)$ becomes infinite as $\mu \to \mu_H$. Again, note that μ_B^c must satisfy $\frac{\mu_B^c}{r} < \frac{p}{1+mr}$. ■

Proof of Proposition 3: In this case, the firm does not issue any debt. Then, the equity value per capital stock is obtained as

$$e^e(\mu) = \begin{cases} \frac{\mu - \lambda i}{r-i+\delta} + B_1^e (\mu - \mu_L)^{1-\gamma_1^e} (\mu_H - \mu)^{\gamma_1^e}, & \text{if} \quad \mu \geq \mu_I^e, \\ \frac{\mu}{r+\delta} + B_3^e (\mu - \mu_L)^{\gamma_2^e} (\mu_H - \mu)^{1-\gamma_2^e}, & \text{if} \quad \mu_I^e > \mu, \end{cases} \tag{A31}$$

where $\gamma_1^e > 1$ and $\gamma_2^e > 1$ are the positive roots of $(\gamma_1^e)^2 - \gamma_1^e - \frac{2\sigma^2(r-i+\delta)}{(\mu_H-\mu_L)^2} = 0$ and $(\gamma_2^e)^2 - \gamma_2^e - \frac{2\sigma^2(r+\delta)}{(\mu_H-\mu_L)^2} = 0$, respectively. Note that the equity value per capital stock is determined so that it does not become infinite as $\mu \to \mu_H$ or $\mu \to \mu_L$. It follows from (15) and (A31) that

$$B_1^e = \left[\frac{\lambda(r+\delta) - \mu_I^e}{r-i+\delta}\right] (\mu_I^e - \mu_L)^{-1+\gamma_1^e} (\mu_H - \mu_I^e)^{-\gamma_1^e}. \tag{A32}$$

It is also found from (15), (19), and (A31) that

$$B_3^e = \left[\frac{\lambda(r+\delta) - \mu_I^e}{r+\delta}\right] (\mu_I^e - \mu_L)^{-\gamma_2^e} (\mu_H - \mu_I^e)^{-1+\gamma_2^e}. \tag{A33}$$

Substituting (A32) and (A33) into (A31), we obtain (26a) and (26b). It also follows from (26a), (26b), and (20) that (27) is derived.

The effect of the promised debt payment per capital stock p: To confirm that our choice of the value of the promised debt payment per capital stock p does not lead to extraordinary results, we examine the effect of p on the default and investment thresholds, the debt value per capital stock, and the firm value per capital stock. We report the results in Figure A1. These results suggest that our choice of p does not cause any trouble. ■

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	μ_B				
$\underline{d}/\sigma$	0.20	0.25	0.30	0.35	0.40
0.40	0.0123	0.0141	0.0156	0.0168	0.0178
0.42	0.0137	0.0156	0.0172	0.0185	0.0196
0.44	0.0154	0.0173	0.0191	0.0205	0.0216
0.46	0.0170	0.0193	0.0213	0.0229	0.0243
0.48	0.0191	0.0217	0.0240	0.0262	0.0285

	μ_I				
$\underline{d}/\sigma$	0.20	0.25	0.30	0.35	0.40
0.40	0.0344	0.0358	0.0369	0.0377	0.0384
0.42	0.0359	0.0373	0.0384	0.0393	0.0400
0.44	0.0377	0.0390	0.0401	0.0410	0.0418
0.46	0.0395	0.0409	0.0421	0.0431	0.0440
0.48	0.0416	0.0433	0.0447	0.0460	0.0475

Table 1A: The effects of a change in σ on μ_B and μ_I when debt maturity is endogenously determined for different $\underline{d}$. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $p = 0.8$, and $\mu_0 = 0.5$.

	μ_B				
$\underline{d}/\mu_H - \mu_L$	0.06	0.08	0.10	0.12	0.14
0.40	0.0241	0.0208	0.0156	0.0097	0.0034
0.42	0.0259	0.0224	0.0172	0.0113	0.0051
0.44	0.0284	0.0244	0.0191	0.0132	0.0070
0.46	0.0360	0.0272	0.0213	0.0153	0.0092
0.48	0.0375	0.0330	0.0240	0.0177	0.0116

	μ_I				
$\underline{d}/\mu_H - \mu_L$	0.06	0.08	0.10	0.12	0.14
0.40	0.0415	0.0398	0.0369	0.0333	0.0295
0.42	0.0430	0.0413	0.0384	0.0349	0.0312
0.44	0.0450	0.0431	0.0401	0.0368	0.0332
0.46	0.0511	0.0454	0.0421	0.0388	0.0353
0.48	0.0522	0.0504	0.0447	0.0411	0.0376

Table 1B: The effects of a change in $\mu_H - \mu_L$ on μ_B and μ_I when debt maturity is endogenously determined for different $\underline{d}$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $p = 0.8$, and $\mu_0 = 0.5$.

	μ_B				
$\underline{d}/\mu_0$	0.01	0.03	0.05	0.07	0.09
0.40			0.0156	0.0118	0.0109
0.42			0.0172	0.0128	0.0117
0.44			0.0191	0.0138	0.0127
0.46			0.0213	0.0149	0.0136
0.48			0.0240	0.0160	0.0147

	μ_I				
$\underline{d}/\mu_0$	0.01	0.03	0.05	0.07	0.09
0.40			0.0369	0.0332	0.0323
0.42			0.0384	0.0341	0.0332
0.44			0.0401	0.0351	0.0340
0.46			0.0421	0.0361	0.0350
0.48			0.0447	0.0372	0.0359

Table 1C: The effects of a change in μ_0 on μ_B and μ_I when debt maturity is endogenously determined for different $\underline{d}$. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

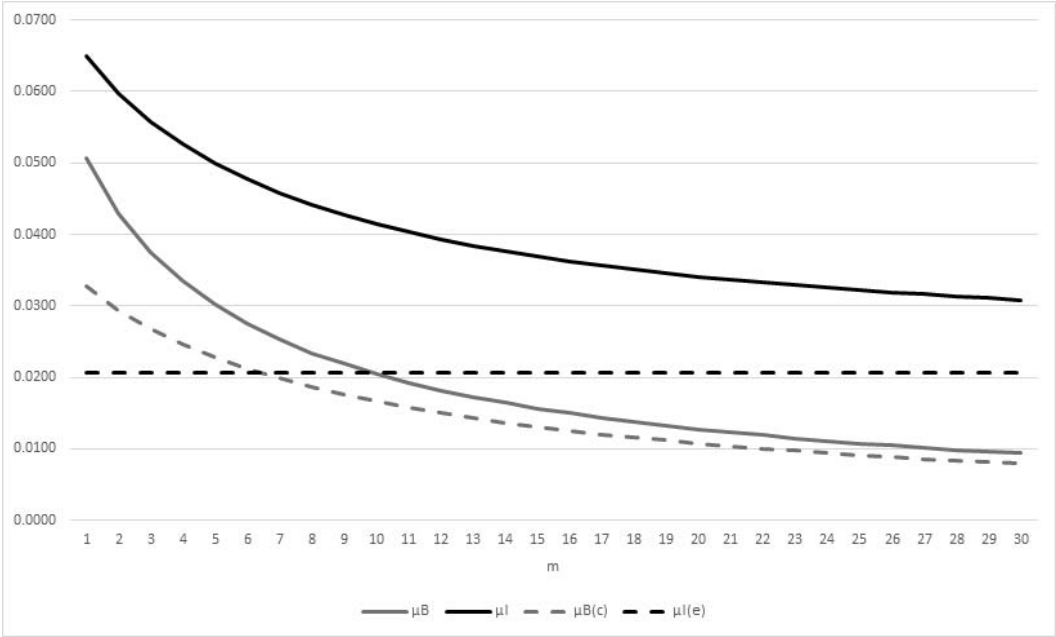

Figure 1. The optimal default and investment policies for different debt maturities. The thin (thick) solid line expresses the optimal default (investment) threshold μ_B (μ_I) in the baseline model. The thin (thick) dashed line represents the optimal default (investment) threshold μ_B^c (μ_I^e) in the constant capital stock with no investment (equity finance) model. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

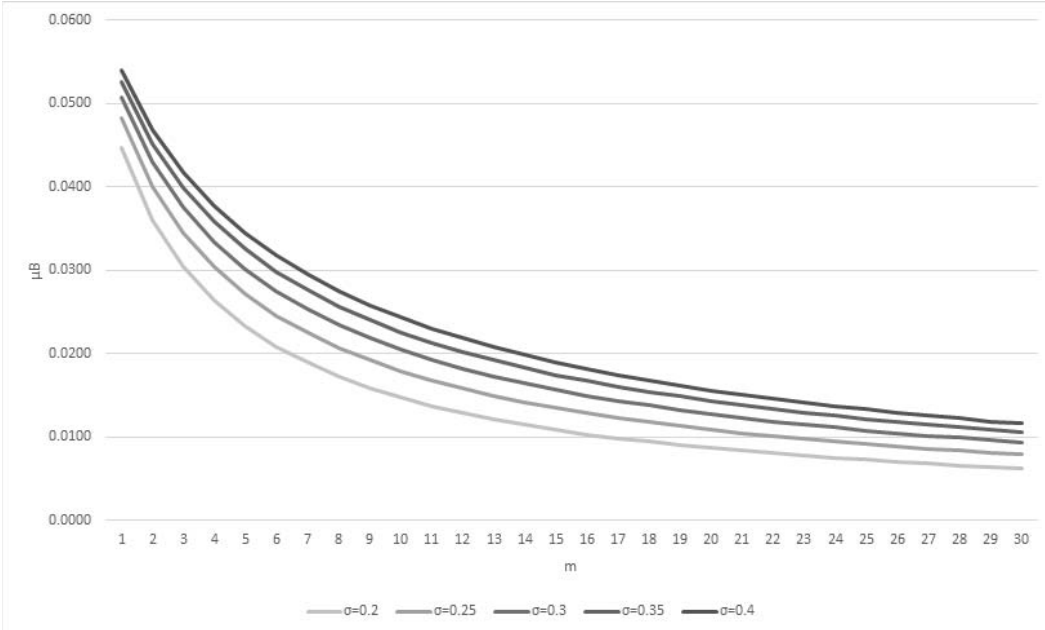

Figure 2A. The effect of a change in σ on μ_B for different debt maturities in the baseline model. Each solid line expresses the optimal default threshold relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

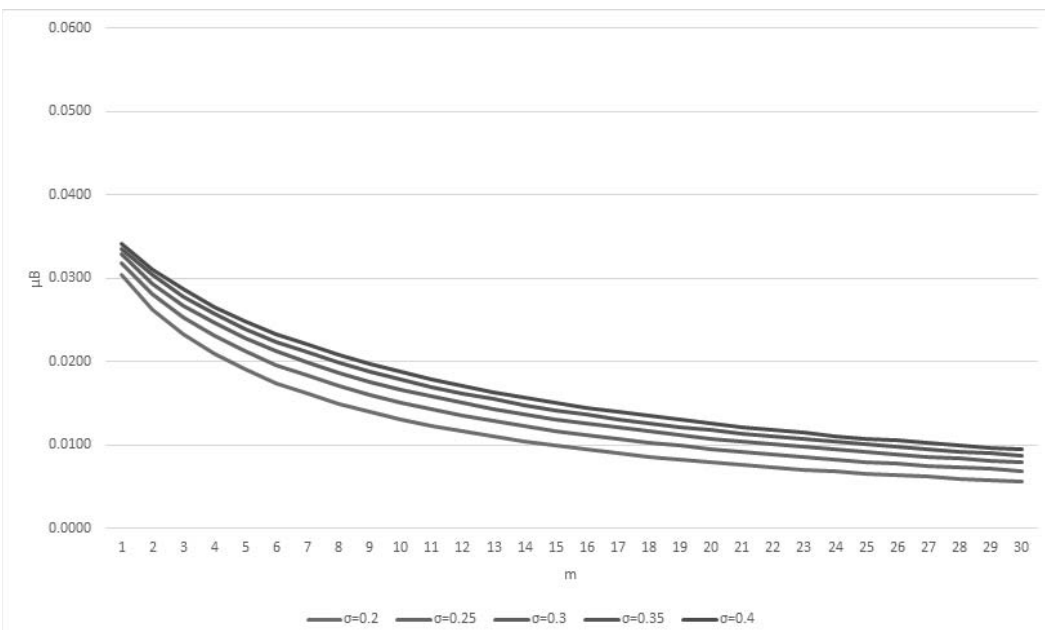

Figure 2B. The effect of a change in σ on μ_B^c for different debt maturities in the constant capital stock with no investment model. Each solid line expresses the optimal default threshold relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

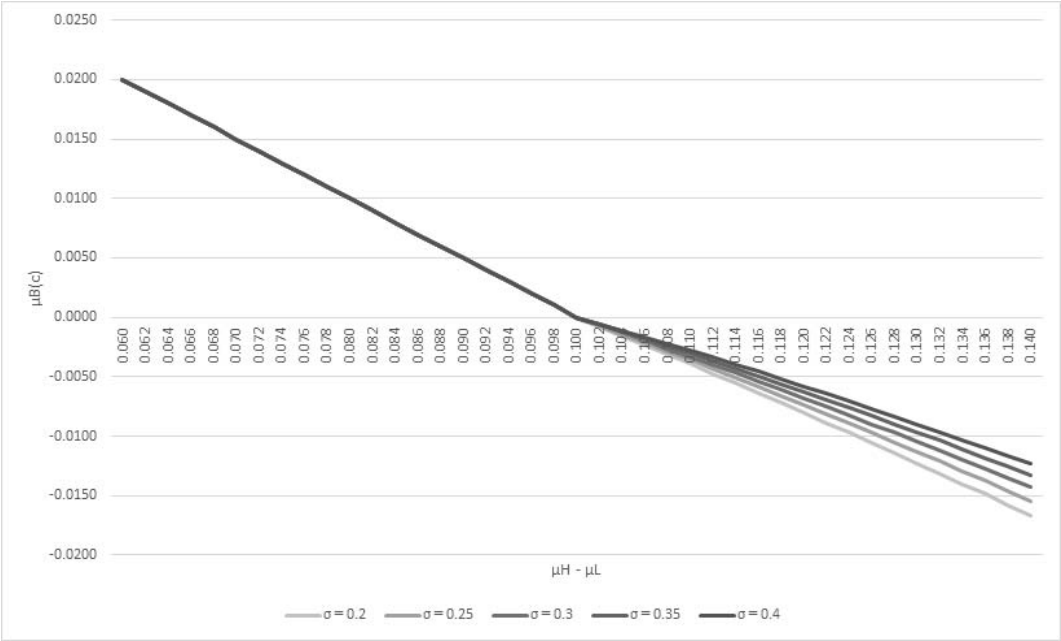

Figure 2C. The effects of changes in σ and $\mu_H - \mu_L$ on μ_B^c for different debt maturities when m is infinite. Each solid line expresses the optimal default threshold as a function of $\mu_H - \mu_L$ relative to σ. Parameters are $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

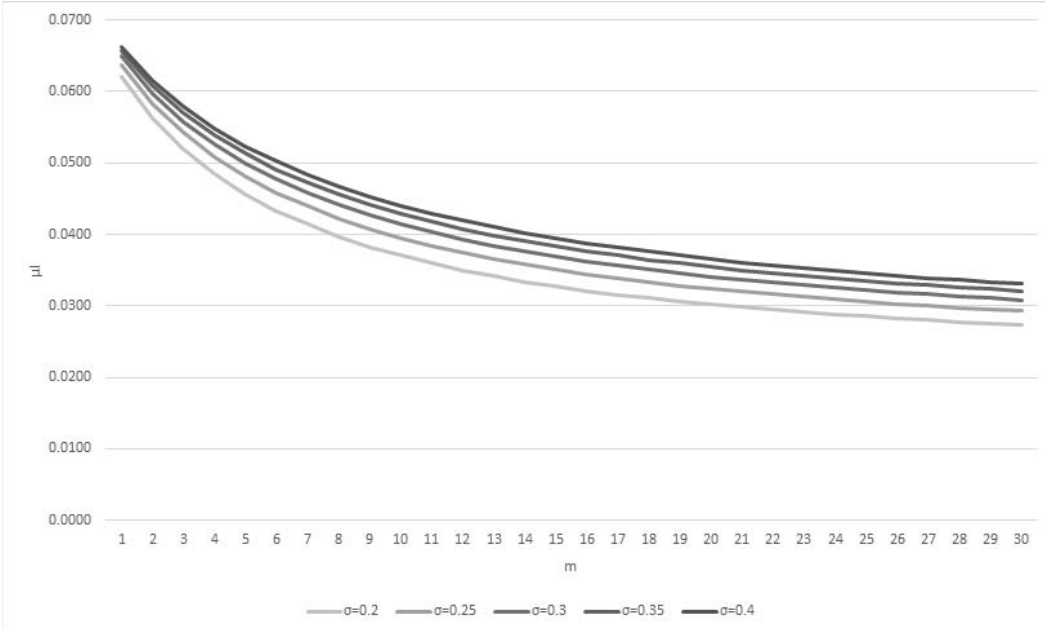

Figure 3A. The effect of a change in σ on μ_I for different debt maturities in the baseline model. Each solid line expresses the optimal investment threshold relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

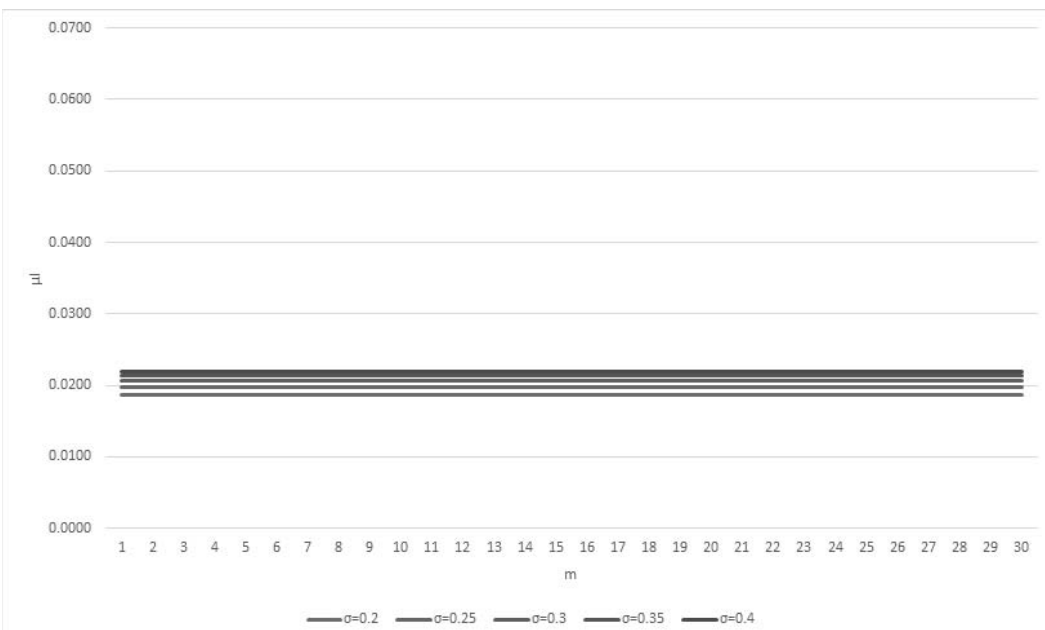

Figure 3B. The effect of a change in σ on μ_I^e for different debt maturities in the equity finance model. Each solid line expresses the optimal investment threshold relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

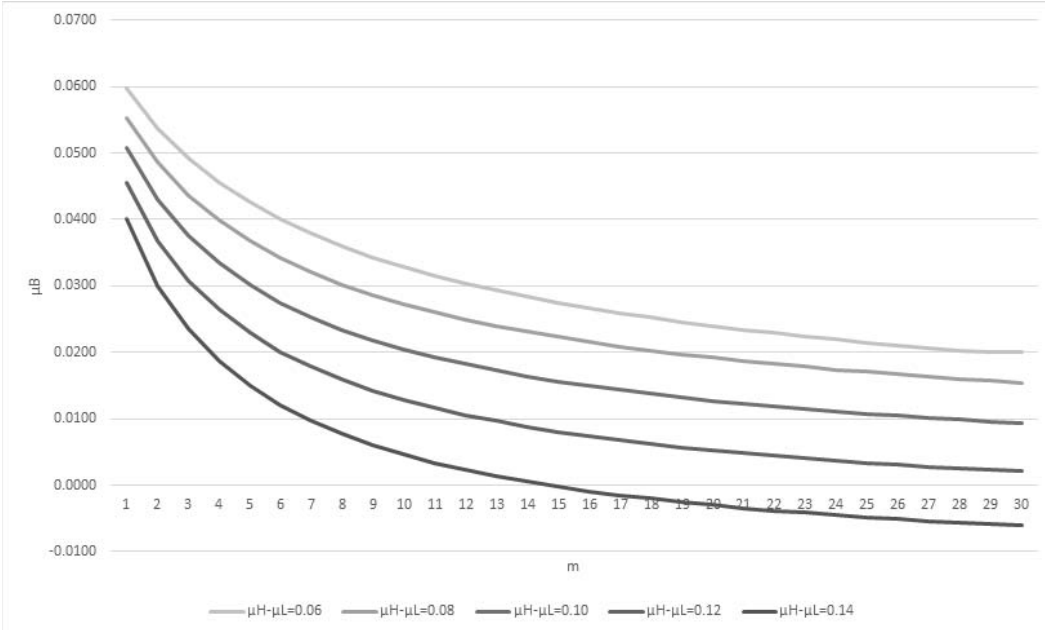

Figure 4A. The effect of a change in $\mu_H - \mu_L$ on μ_B for different debt maturities in the baseline model. Each solid line expresses the optimal default threshold relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

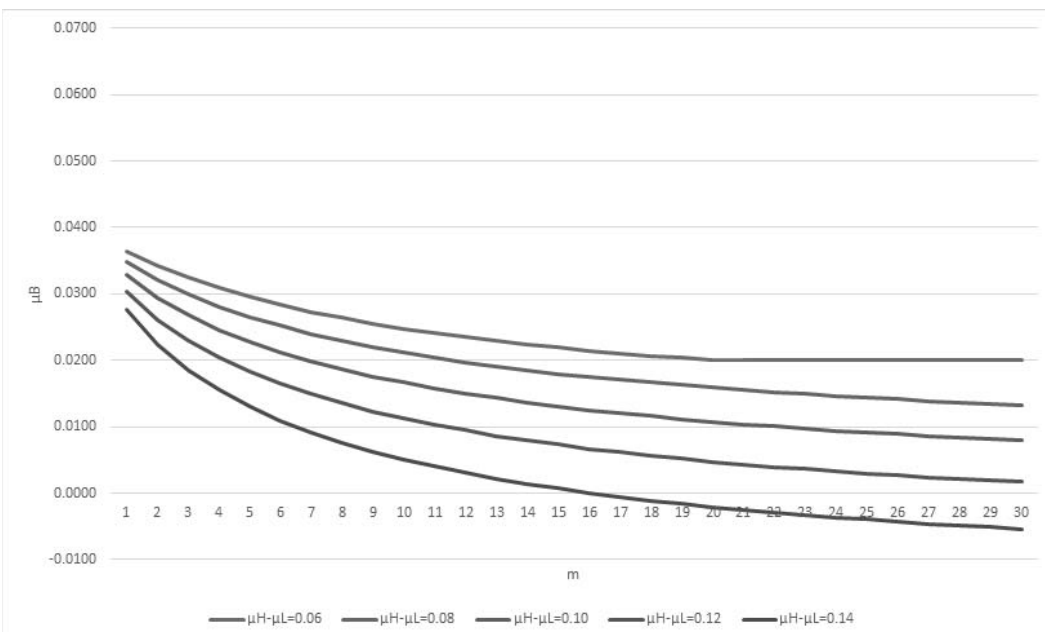

Figure 4B. The effect of a change in $\mu_H - \mu_L$ on μ_B^c for different debt maturities in the constant capital stock with no investment model. Each solid line expresses the optimal default threshold relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

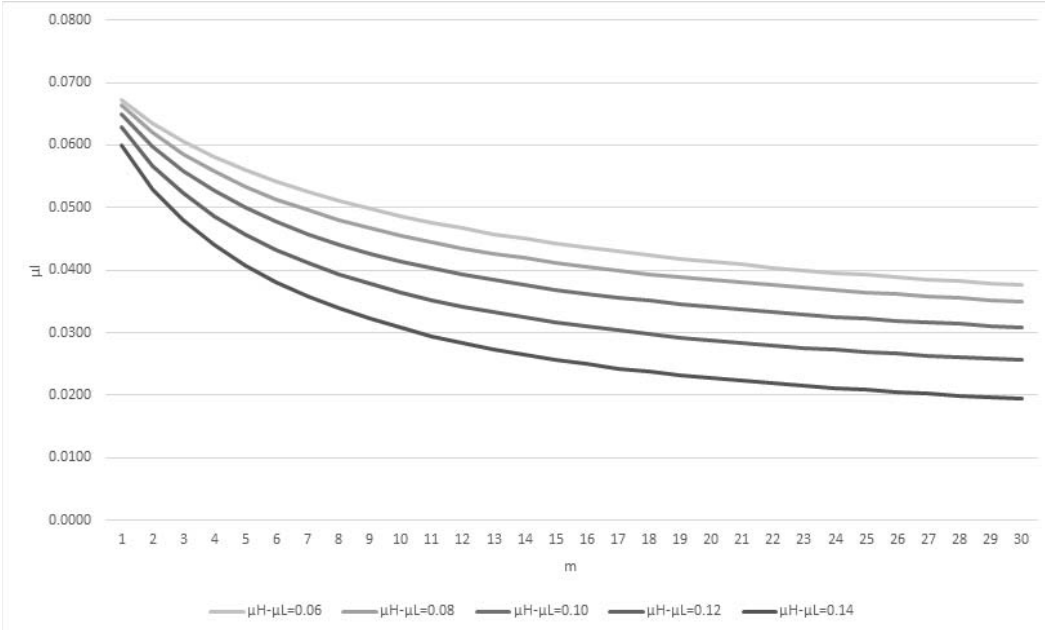

Figure 5A. The effect of a change in $\mu_H - \mu_L$ on μ_I for different debt maturities in the baseline model. Each solid line expresses the optimal investment threshold relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

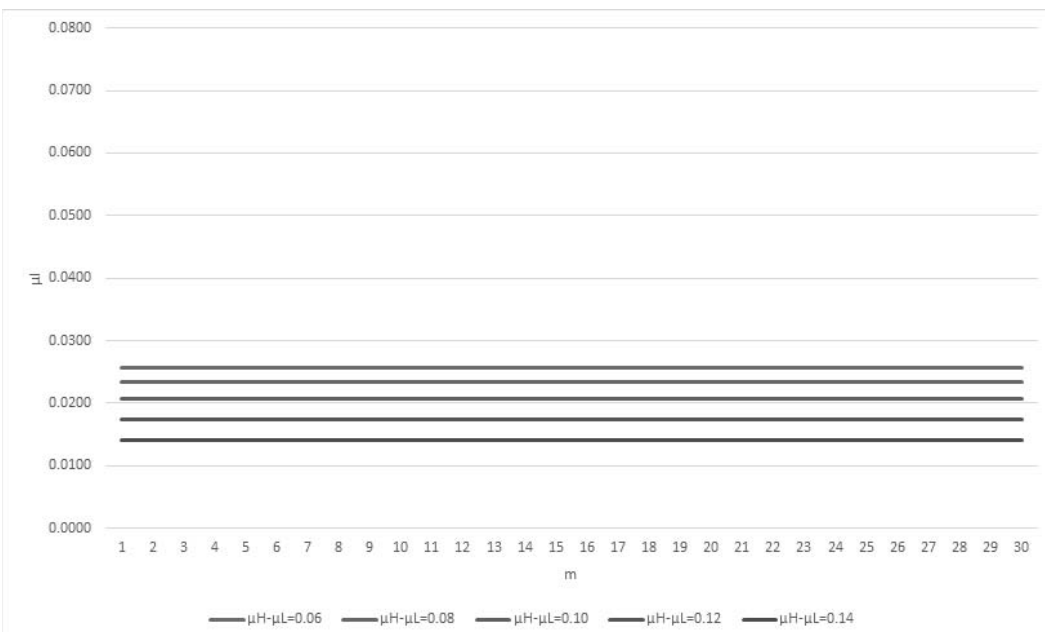

Figure 5B. The effect of a change in $\mu_H - \mu_L$ on μ_I^e for different debt maturities in the equity finance model. Each solid line expresses the optimal investment threshold relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

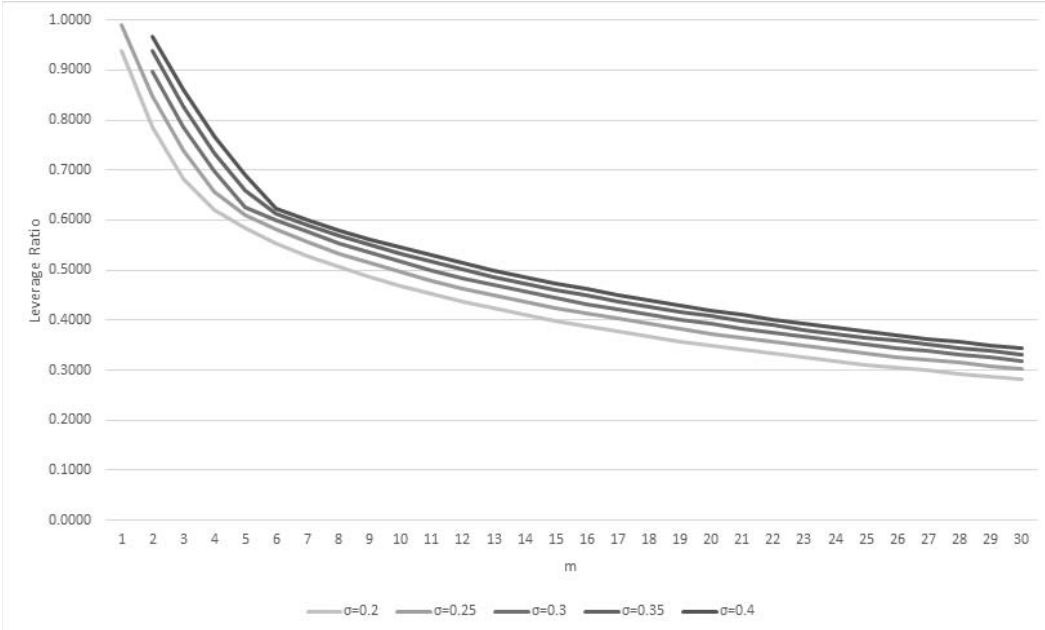

Figure 6A. The effect of a change in σ on the leverage ratio for different debt maturities. Each solid line expresses the leverage ratio relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

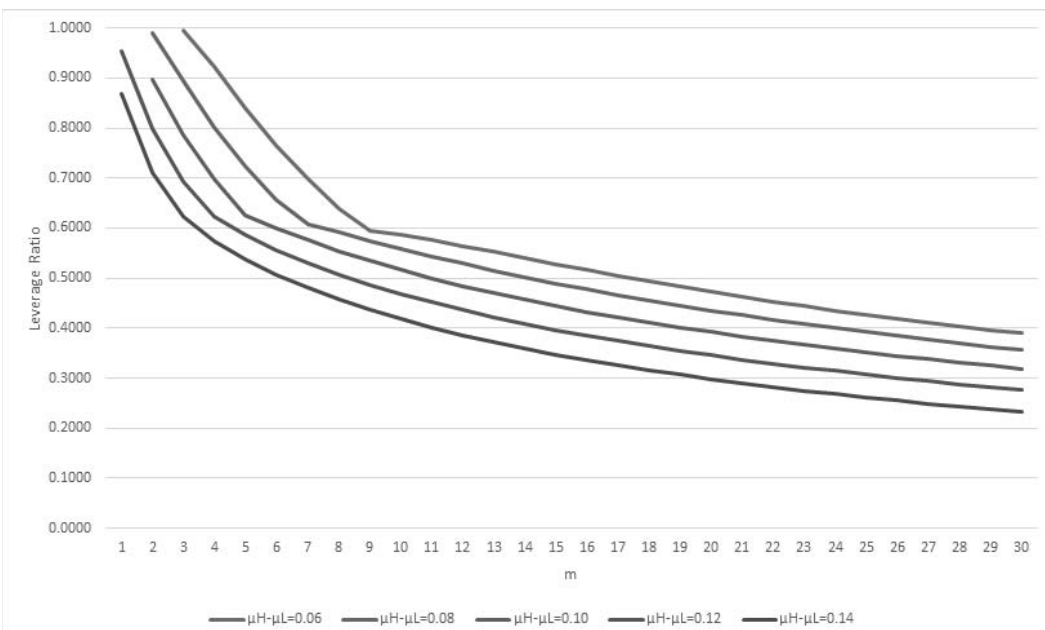

Figure 6B. The effect of a change in $\mu_H - \mu_L$ on the leverage ratio for different debt maturities. Each solid line expresses the leverage ratio relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

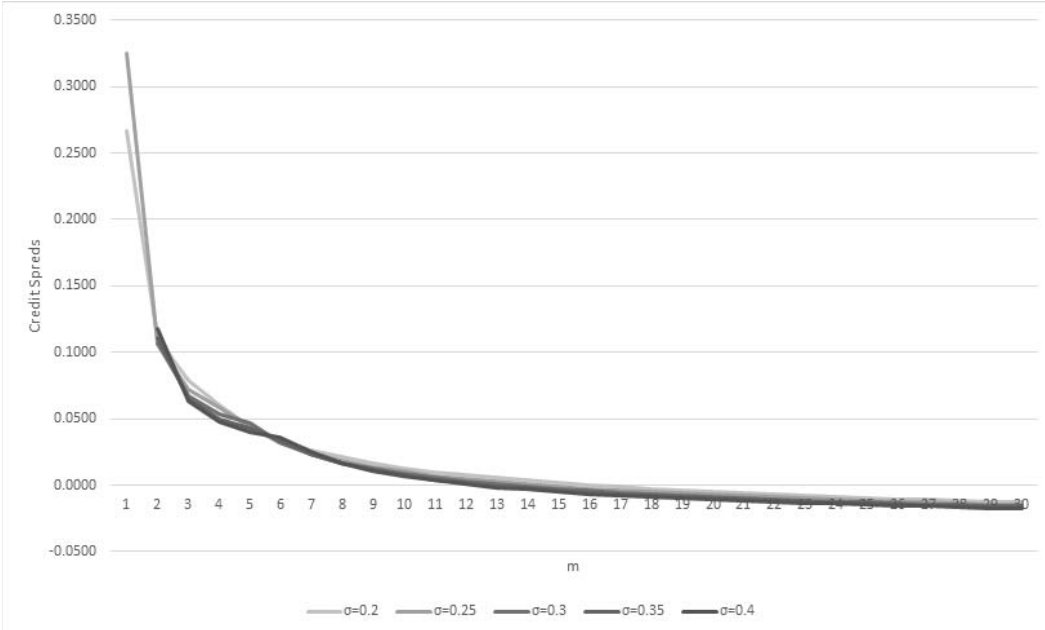

Figure 7A. The effect of a change in σ on credit spreads for different debt maturities. Each solid line expresses credit spreads relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

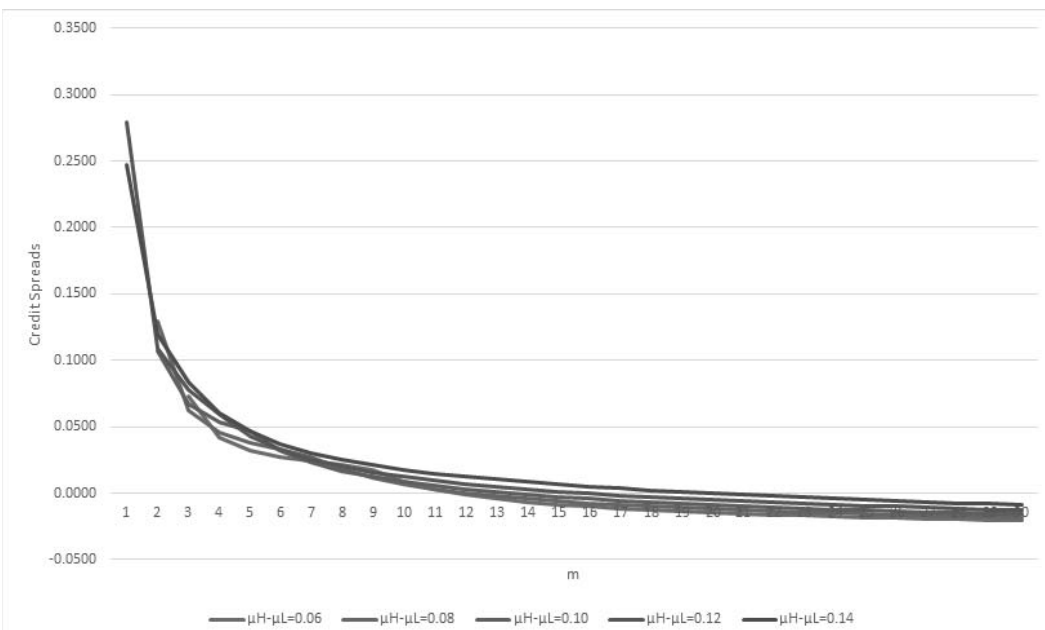

Figure 7B. The effect of a change in $\mu_H - \mu_L$ on credit spreads for different debt maturities. Each solid line expresses credit spreads relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

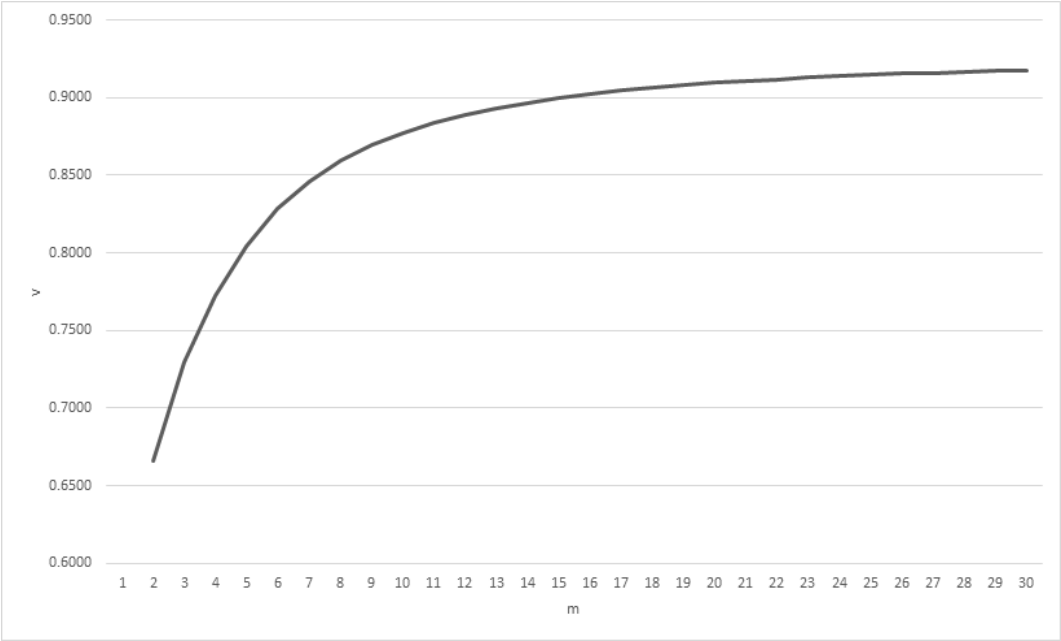

Figure 8. The initial firm value per capital stock. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $p = 0.8$, and $\mu_0 = 0.5$.

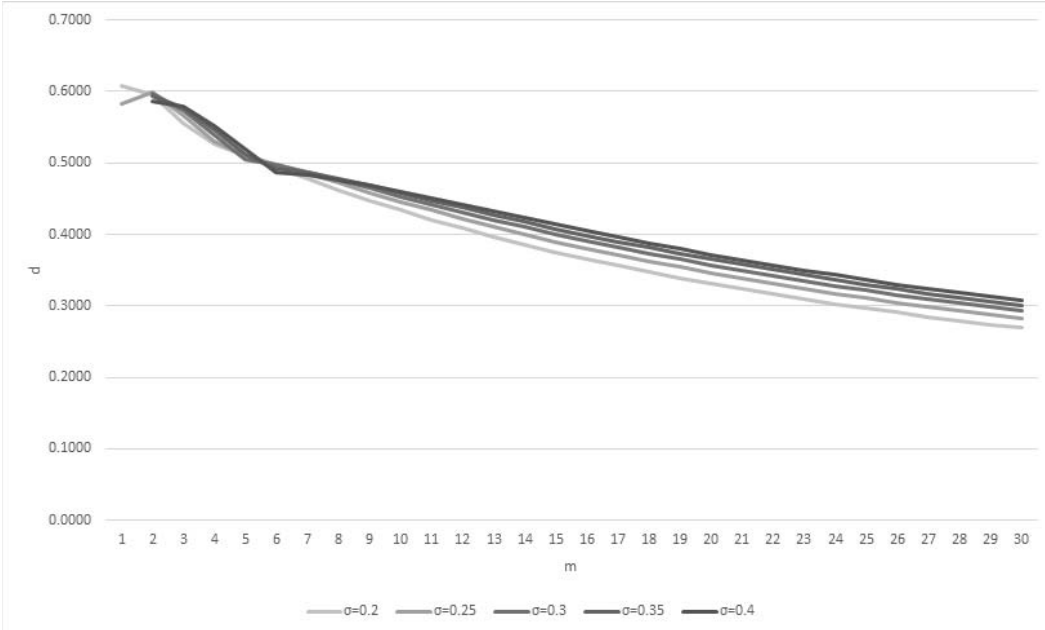

Figure 9A. The effect of a change in σ on m when m is endogenously determined for different $\underline{d}$. Each solid line expresses the initial market value of debt per capital stock relative to σ. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $p = 0.8$, and $\mu_0 = 0.5$.

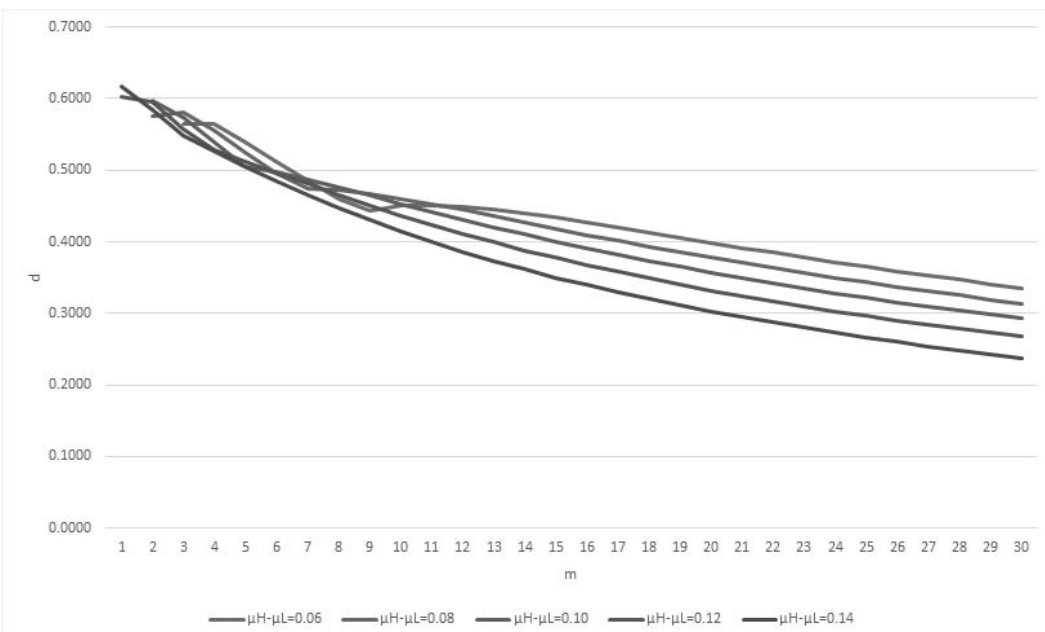

Figure 9B. The effect of a change in $\mu_H - \mu_L$ on m when m is endogenously determined for different $\underline{d}$. Each solid line expresses the initial market value of debt per capital stock relative to $\mu_H - \mu_L$. Parameters are $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $p = 0.8$, and $\mu_0 = 0.5$.

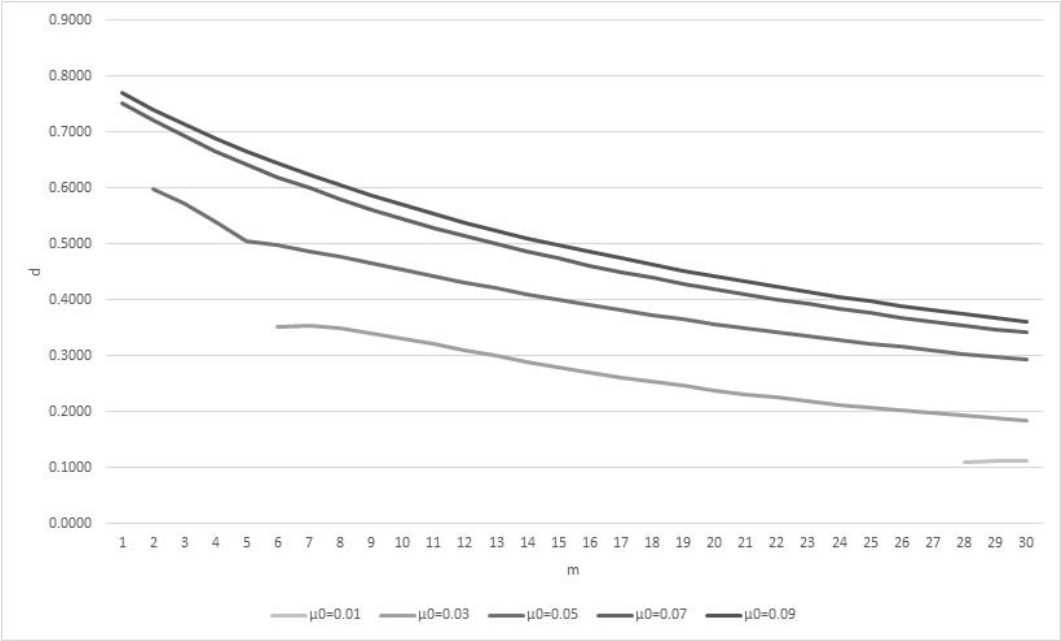

Figure 10. The effect of a change in μ_0 on m when m is endogenously determined for different $\underline{d}$. Each solid line expresses the initial market value of debt per capital stock relative to μ_0. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, and $p = 0.8$.

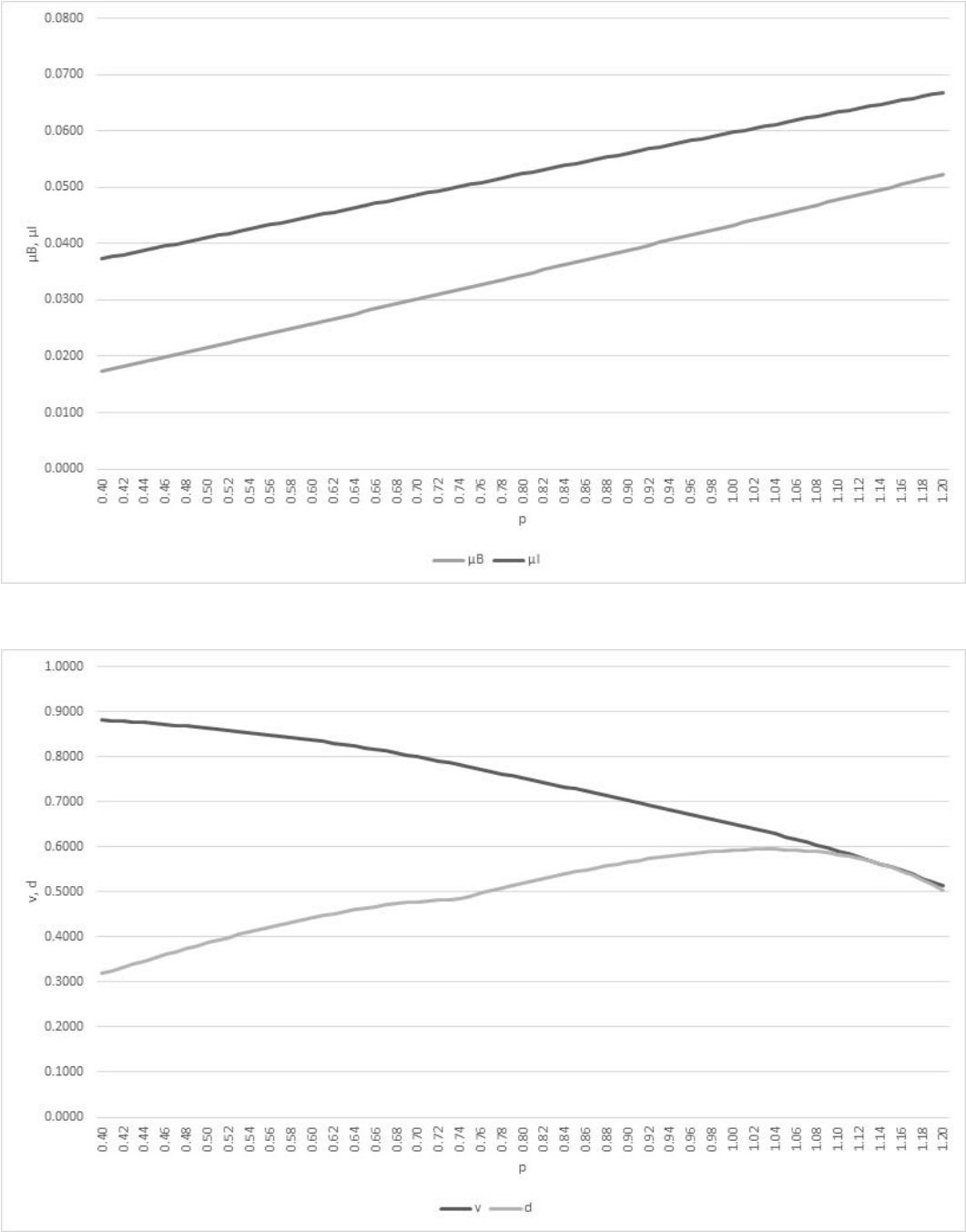

Figure A1. The effects of a change in p on μ_B, μ_I, the debt value per capital stock, d, and the firm value per capital stock, v. Parameters are $\mu_L = 0$, $\mu_H = 0.1$, $\sigma = 0.3$, $r = 0.05$, $\lambda = 0.3$, $i = 0.05$, $\delta = 0.04$, $\mu_0 = 0.5$, and $m = 5$.