# Inequalities of Mobility Functions between the Cycle Classes and their Intersections with Divisors

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**Introduction.** The volume of a Cartier divisor on a projective integral variety measures the asymptotic rate of growth of the dimension of the global sections of the multiples of the divisor. It provides a good way to understand the big divisor classes. As a generalization, the mobility defined on the cone of pseudo-effective cycle classes measures the asymptotic rate of growth of the dimensions of the global sections of the multiples of the cycle classes. It provides a way to understand the big cycle classes. The mobility function for divisors coincides with the volume function, and the mobility function for 0-cycles is just n! times the degree function. For cycles in other dimensions, the mobility is difficult to compute in general.

**Example.** For $\mathbf{X} = \mathbf{P}^3$ and $\mathbf{H}$, being the hyperplane in $\mathbf{X}$, we only have the estimation $1 \leq mob(\mathbf{H}^2) \leq 3.45$, and it is still a conjecture that $1 = mob(\mathbf{H}^2)$.

**Theorem.** Let α be a pseudo-effective cycle class and $\mathbf{A}$ be a nef divisor class. Then

$$mob(\alpha \cdot \mathbf{A}) \geq mob(\boldsymbol{\alpha})^{\frac{n-k}{n-k+1}} vol(\mathbf{A})^{\frac{1}{n-k+1}}.$$

**Corollary.** Let α be a pseudo-effective cycle class. Then we have estimations of $mob(\alpha)$, in terms of its intersections with divisors,

$$(1) \qquad mob(\alpha) \geq sup \prod_{j=1}^{n-k} vol(\mathbf{A_j})^{\frac{1}{n-k}},$$

where $\mathbf{A_j}$ runs over all nef divisor classes such that $\alpha - \prod_{j=1}^{n-k} \mathbf{A_j}$ is pseudo-effective, and

$$(2) \qquad mob(\alpha) \leq inf\left(\frac{n!\alpha \cdot \prod_{h=1}^{k} \mathbf{A}_{\boldsymbol{h}}}{\prod_{h=1}^{k} vol(\mathbf{A}_{\boldsymbol{h}})^{\frac{1}{n}}}\right)^{\frac{n}{n-k}},$$

where $\mathbf{A}_{\boldsymbol{h}}$ runs over all nef and big divisor classes.