ACC For Log Canonical Threshold Polytopes

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Preliminaries

Log canonical threshold polytope (LCT-polytope). Let \((X, \Delta)\) be a lc pair and \(D_1, \ldots, D_n\) be effective \(\mathbb{R}\)-Cartier divisors. The LCT-polytope

\[ P(X, \Delta, D_1, \ldots, D_n) \subset \mathbb{R}^n \]

is defined in \([LM11]\). The above result is proven under the smoothness assumption on \(X\) by generic limit method.

Example. Let \(s = 1\), \(P(X, \Delta, D)\) is just the interval \([0, \text{lct}(X, \Delta, D)]\), where \(\text{lct}(X, \Delta, D)\) is the log canonical threshold of \(D\).

DCC and ACC set. For a partially ordered set, DCC refers to descending chain condition, and ACC refers to ascending chain condition. For sequences of polytopes, we choose inclusion “⊆” as the partial order.

Main Theorem

For two divisors \((s = 2)\).

Let \(P_i := P_i(X_i, D_{i1}, D_{i2}) \subset \mathbb{R}^2\) be an increasing sequence of LCT-polytopes. Intersect each \(P_i\) by a vertical line \(\{x = p\}\). The intersection point \((p, t_i(p))\) is determined by \(t_i(p) = \sup \{t \mid (x, pD_{i1} + tD_{i2})\}\) is log canonical.

Idea of proof

Global ACC for linear coefficients

Let \(n \in \mathbb{N}\) and \(a, b \in \mathbb{R}\) be fixed numbers. Let \(\mathcal{F}\) be a set of real linear functions and \(\{(X, \Delta(t))\}\) be a set of log pairs. Suppose they satisfy the following properties:

1. \(X\) is normal with \(\dim X = n\).
2. For any \(f(t) \in \mathcal{F}, f(t) \geq 0\) on \([a, b]\), and \(\mathcal{F}_{[a, b]} = \{f(c) \mid f(t) \in \mathcal{F}\}\).
3. The coefficients of \(\Delta(t)\) are in \(\mathcal{F}\).
4. There exists \(c < x < b\), such that \((X, \Delta(t))\) is log on \([a, b]\), and \(K_X + \Delta(t) \equiv 0\) on \([a, b]\).

Then there is a finite set \(\mathcal{F}'\) such that the coefficients of \(\Delta(t)\) are in \(\mathcal{F}'\) for each \((X, \Delta(t))\).

Similarly, one can obtain the result of ACC for Fano spectrum of LCT-polytopes.

Potential applications

LCT-polytopes might be applied to the problems on the existence of Kähler-Einstein metrics. Analytically, \(\alpha\)-invariant is introduced to deal with such problem. \(\alpha\)-invariant is the log canonical thresholds of some \(\mathbb{R}\)-linear system. Log canonical threshold also appeared in the study of stabilities of varieties. It is desirable to see if LCT-polytopes could give some refined measurement for the existence of Kähler-Einstein metrics.

References
