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1+1 Large N_c QCD and its Holographic Dual ∼ Soliton Picture of Baryons in Single-Flavor World

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We study baryons in holographic QCD corresponding to 1+1 dimensional single-flavor (N_f=1) QCD for the first time. We formulate 1+1 QCD using an S^1-compactified D2/D8/$\overline{\text{D8}}$ branes in the superstring theory, and describe the baryon as a topological configuration in 1+1 N_f=1 QCD, corresponding to $\Pi_1(\text{U}(1)) = \mathbf{Z}$. Unlike 1+3 QCD with $N_f \geq 2$, however, we find that the low-dimensional baryonic soliton is generally unstable against a scale transformation/variation and swells infinitely in 1+1 N_f=1 QCD at the leading of large N_c. We thus point out a serious difficulty on the soliton picture of baryons in large N_c in the single-flavor world in both 1+1 and 1+3 QCD. We also compare the low-dimensional holographic baryon with the Abrikosov vortex, i.e., a stable topological configuration in Type-II superconductors.

KEYWORDS: holographic QCD, low-dimensional QCD, baryon, soliton

1. Introduction: Holographic QCD and Baryons in Large N_c

Since 1973, quantum chromodynamics (QCD) has been established as the fundamental theory of the strong interaction. Nevertheless, it is very difficult to solve QCD directly in an analytical manner, and many effective models of QCD have been used instead of QCD, but most models cannot be derived from QCD and its connection to QCD is unclear. To analyze nonperturbative QCD, the lattice QCD Monte Carlo simulation has been also used as a first-principle calculation of the strong interaction. However, it has several weak points. For example, the state information (e.g. the wave function) is severely limited, because lattice QCD is based on the path-integral formalism. Also, it is difficult to take the chiral limit, because zero-mass pions require infinite volume lattices. There appears a notorious "sign problem" at finite density.

On the other hand, holographic QCD [1–3] has a direct connection to QCD, and can be derived from QCD in some limit. In fact, holographic QCD is equivalent to infrared QCD in large N_c and strong 't Hooft coupling λ, via gauge/gravity correspondence. Remarkably, holographic QCD is successful to reproduce many hadron phenomenology such as vector meson dominance, the KSRF relation, hidden local symmetry, the GSW model and the Skyrme soliton picture [2]. Unlike lattice QCD simulations, holographic QCD is usually formulated in the chiral limit, and does not have the sign problem at finite density [3].

In general, when large N_c is taken, QCD reduces a weakly interacting theory of mesons (and glueballs), and the baryon is described as a Skyrmion, i.e., a topological chiral soliton of mesons (mainly Nambu-Goldstone bosons) [4] Actually, in holographic QCD with large N_c, the theory is described by pseudoscalar, vector and axial-vector mesons [2], and baryons do not appear as explicit degrees of freedom but appear as spatially-extended topological solitons composed of mesons [3].

2. Puzzle in Single Flavor (N_f=1) World in Large N_c

In large N_c QCD, the effective theory includes only meson degrees of freedom, and therefore one has to take the soliton picture for the description of baryons [4]. In our real world with $N_f \geq 2$, there occurs spontaneous breaking of the chiral symmetry, i.e., $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$, and baryons can be described as topological chiral solitons, according to the nontrivial homotopy group $\Pi_3(\mathrm{SU}(N_f)_A) = \mathbf{Z}$ [4]. Actually, this topological chiral-soliton picture of baryons is successful for the semi-quantitative description of baryons [5].

In the single-flavor (N_f=1) world, however, baryons cannot be described as topological objects, because of absence of the topological charge, i.e., $\Pi_3(\mathrm{U}(1)_A) = 1$. Of course, QCD with single flavor (N_f=1) is a possible quantum field theory. Actually, if the Higgs coupling to d, s-quarks were large enough, the single-flavor world would be realized. In N_f=1 QCD, there appear only a massive pseudoscalar meson $\eta'(\bar{u}\gamma_5 u)$, a vector meson $\omega(\bar{u}\gamma^\mu u)$, and a baryon $\Delta^{++}(uuu)$, as low-lying hadrons. Unlike the $N_f \geq 2$ case, the single-flavor QCD does not have the topological charge because of $\Pi_3(\mathrm{U}(1)_A)$=1, and therefore baryons cannot be described as topological objects. Thus, in the single-flavor world, it is difficult to describe baryons with mesons in large N_c, where baryons do not appear explicitly. This is an open problem still now.

In this context, we notice that 1+1 single-flavor QCD has the topological object, corresponding to the nontrivial homotopy group $\Pi_1(\mathrm{U}(1)) = \mathbf{Z}$. In fact, as a natural possibility, it is expected that the baryon can be described as the topological object in 1+1 single-flavor QCD, like 1+3 QCD with $N_f \geq 2$. This is a motivation to investigate baryons in 1+1 single-flavor QCD, especially in large N_c, where baryons do not appear explicitly.

3. Holographic QCD corresponding to 1+1 Single-Flavor QCD and Baryons

The baryons in 1+1 QCD have been usually studied with the bosonization technique [6]. However, to investigate the topological aspect relating to $\Pi_1(\mathrm{U}(1)) = \mathbf{Z}$, one has to describe baryons as topological objects. For this purpose, holographic QCD is suitable, because baryons appear as topological objects in this framework [7].

Similarly to the Sakai-Sugimoto model, massless 1+1 QCD can be constructed with an S^1-compactified D2/D8/$\overline{\mathrm{D8}}$-brane system [8], as shown in Fig. 1. Here, N_c D2-branes give color degrees of freedom, and N_f=1 D8-brane gives flavor degrees of freedom. The gluons appear as 2-2 string modes on N_c D2-branes, and the left/right quarks appear as 2-8 string modes at the cross point between D2 and D8/$\overline{\mathrm{D8}}$ branes. In this paper, we use the M_{KK} = 1 unit [2].

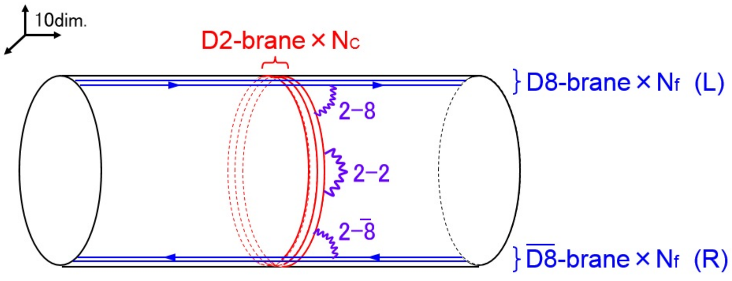

Fig. 1. The S^1-compactified D2/D8/$\overline{\mathrm{D8}}$-brane system which gives 1+1 QCD with N_c color and N_f flavor.

In large N_c, N_c D2-branes are extremely massive and can be replaced by a gravitational background, via the gauge/gravity correspondence. From this D2/D8/$\overline{\mathrm{D8}}$-brane system, the effective theory of massless 1+1 QCD is derived as a 1+2 dimensional U(N_f=1) gauge theory in the flavor space,

$$S = \frac{N_c}{8\pi}\int dtdxdz\left[f(z)F_{\mu z}F^{\mu z} - g(z)\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \epsilon^{LMN}A_L F_{MN}\right], \tag{1}$$

at the leading of $1/N_c$ and $1/\lambda$ expansion [8]. Remarkably, this theory can be treated at the classical level. Here, z is the extra spatial dimension appearing in holographic QCD, and $f(z) \equiv (1+z^2)^{1/2}$ and $g(z) \equiv (1+z^2)^{-11/10}$ appear as the gravitational effect from N_c D2-branes [8].

The action (1) has two parts, i.e., the Dirac-Born-Infeld (DBI) action (the 1st and 2nd terms) and the Chern-Simons (CS) three-form (the last term). [The Greek index runs over (0,1)= (t, x) and the capital index runs over (0,1,2)= (t, x, z).] In 1+1 QCD, the CS term is the leading order as well as the DBI terms [8], while the CS term is subleading of $1/\lambda$ expansion compared with the leading DBI term in 1+3 QCD [3].

In this two-dimensional spatial system on (x, z), we note a topological charge, called the Pontryagin index,

$$Q \equiv \int dxdz\frac{1}{4\pi}\epsilon_{ij}F_{ij} = \frac{1}{2\pi}\int dxdzF_{xz} = \frac{1}{2\pi}\int dxdzH \in \mathbf{Z}, \tag{2}$$

which is an integer, according to Π_1(U(N_f=1))=**Z**. Here, $H \equiv F_{xz}$ is the magnetic field in the U(N_f=1) gauge theory, and Q is the total magnetic flux divided by 2π. From the holographic viewpoint, this topological charge corresponds to the baryon number, similarly in the holographic dual of 1+3 QCD [7]. On the topological object in the two-dimensional space (x, z), we note that its direct analogue is the Abrikosov vortex with "quantization of the magnetic flux" in Type-II superconductors.

Next, we consider the topological soliton solution corresponding to the (multi)baryon with the baryon number $B = Q(\in \mathbf{Z})$. For the calculation, we take the temporal gauge $A_0 = 0$, which leads to the ordinary canonical formalism. From action (1), the Lagrangian density reads

$$\mathcal{L} = \frac{N_c}{8\pi}\left[f(z)\{\dot{A}_z^2 - (\partial_x A_z - \partial_z A_x)^2\} + g(z)\dot{A}_x^2 + 2(A_z\dot{A}_x - A_x\dot{A}_z)\right], \tag{3}$$

and the Hamiltonian density $\mathcal{H} \equiv \Pi_{A_x}\dot{A}_x + \Pi_{A_z}\dot{A}_z - \mathcal{L}$ with $\Pi_{A_i} \equiv \delta L/\delta\dot{A}_i$ is written as

$$\mathcal{H} = \frac{N_c}{8\pi}\left\{g(z)\dot{A}_x^2 + f(z)\dot{A}_z^2 + f(z)(\partial_x A_z - \partial_z A_x)^2\right\} = \frac{N_c}{8\pi}\left\{g(z)E_x^2 + f(z)E_z^2 + f(z)H^2\right\}, \tag{4}$$

where the CS term disappears and $E_i = \dot{A}_i$ in the temporal gauge. Here, $\mathcal{H}$ is non-negative because of $f(z) > 0$ and $g(z) > 0$. Then, the total energy $M[\vec{A}]$ of the configuration $\vec{A} = (A_x, A_z)$ is given by

$$M[\vec{A}] = \frac{N_c}{8\pi}\int dxdz\left\{g(z)E_x^2 + f(z)E_z^2 + f(z)H^2\right\}. \tag{5}$$

We now consider the ground-state soliton, i.e., the lowest (multi)baryon state, under the topological constraint of $B = Q \equiv \frac{1}{2\pi}\int dxdzH(\in \mathbf{Z})$. Since this topological condition does not act on the electric field $\vec{E}$, we can take $E_x = E_z = 0$ for the ground-state soliton without change of the topological charge Q, so that $\vec{A}$ is t-independent. Thus, the ground-state soliton mass is described as

$$M[\vec{A}(x,z)] = \frac{N_c}{8\pi}\int dxdzf(z)H^2 = \frac{N_c}{8\pi}\int dxdzf(z)\left[\partial_x A_z(x,z) - \partial_z A_x(x,z)\right]^2. \tag{6}$$

4. A Scale Instability of Holographic Baryons in 1+1 Single-Flavor QCD

From Eq.(6), one can investigate the (multi)baryonic solutions in holographic QCD corresponding to 1+1 QCD with $N_f = 1$, at the leading of $1/N_c$ and $1/\lambda$ expansions. As a remarkable conclusion, we find that all the (multi)baryonic solutions are generally *unstable* against some scale transformation (scale variation) and *swell infinitely*. We show this scale instability of baryonic solitons below.

Suppose we obtain a topological (baryonic) solution $\vec{A}^{\rm sol}(x,z) = (A_x^{\rm sol}(x,z), A_z^{\rm sol}(x,z))$, which minimizes the mass $M[\vec{A}(x,z)]$ and satisfies the topological condition of $B = Q[\vec{A}(x,z)](\in \mathbf{Z})$. As a general property of the solution, its total energy M must be a minimum, i.e.,

$$M[\vec{A}^{\rm sol}] \le M[\vec{A}^{\rm sol} + \delta\vec{A}], \tag{7}$$

against any small variations $\delta\vec{A}(x,z)$ consistent with the topological condition $B = Q[\vec{A}^{\rm sol} + \delta\vec{A}](\in \mathbf{Z})$.

As a simple variation, we consider a "scaled configuration" of

$$\vec{A}_\lambda(x,z) \equiv (\lambda A_x^{\rm sol}(\lambda x,z), A_z^{\rm sol}(\lambda x,z)), \tag{8}$$

which includes the original solution $\vec{A}^{\rm sol}(x,z)$ at $\lambda = 1$, i.e., $\vec{A}_{\lambda=1}(x,z) = \vec{A}^{\rm sol}(x,z)$. The scaled configuration $\vec{A}_\lambda(x,z)$ has the same topological charge as

$$\begin{aligned} Q[\vec{A}_\lambda] &= \frac{1}{2\pi}\int dxdz\left[\partial_x A_z^{\rm sol}(\lambda x,z) - \partial_z A_x^{\rm sol}(\lambda x,z)\right] \\ &= \frac{1}{2\pi}\int d\bar{x}dz\left[\partial_{\bar{x}} A_z^{\rm sol}(\bar{x},z) - \partial_z A_x^{\rm sol}(\bar{x},z)\right] = Q[\vec{A}^{\rm sol}], \end{aligned} \tag{9}$$

with $\bar{x} = \lambda x$. The total energy M of this scaled configuration $\vec{A}_\lambda(x,z)$ is λ times of the original mass $M[\vec{A}^{\rm sol}]$:

$$\begin{aligned} M[\vec{A}_\lambda] &= \frac{N_c}{8\pi}\int dxdzf(z)\left[\partial_x A_z^{\rm sol}(\lambda x,z) - \partial_z \lambda A_x^{\rm sol}(\lambda x,z)\right]^2 \\ &= \lambda\frac{N_c}{8\pi}\int d\bar{x}dzf(z)\left[\partial_{\bar{x}} A_z^{\rm sol}(\bar{x},z) - \partial_z A_x^{\rm sol}(\bar{x},z)\right]^2 = \lambda M[\vec{A}^{\rm sol}]. \end{aligned} \tag{10}$$

Therefore, the total energy becomes smaller continuously to zero, when λ goes to zero from unity. This leads to "swelling instability" of the topological configuration with any baryon number $B = Q$, as shown in Fig. 2. Thus, in holographic QCD of 1+1 single-flavor QCD, all the (multi)baryonic configurations are unstable against this type of scale variation, and any topological (baryonic) configuration swells infinitely, at the leading of $1/N_c$ and $1/\lambda$ expansions.

Fig. 2. The scale instability of the baryonic soliton configuration in the holographic dual of 1+1 QCD with N_f=1. All the (multi)baryonic solutions swell infinitely at the leading of $1/N_c$ and $1/\lambda$ expansions.

5. Comparison with Abrikosov Vortex in Type-II Superconductor

Finally, to understand the physical reason of the swelling instability of the topological (baryonic) configuration in holographic QCD of 1+1 single-flavor QCD, we compare it with the Abrikosov

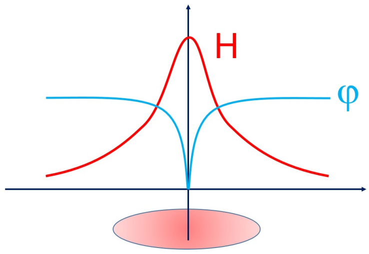

Fig. 3. The Abrikosov vortex in Type-II superconductors: a stable topological configuration composed of the magnetic field H and the Cooper-pair scalar field φ. The vortex has a topological charge on $\Pi_1(U(1)) = \mathbf{Z}$.

vortex, which is a stable topological configuration appearing in the Type-II superconductor in an external magnetic field. (See Fig. 3.)

The superconducting theory consists of the photon field A_μ and the Cooper-pair scalar field φ,

$$\mathcal{H} = \frac{1}{2}\vec{H}^2 + |(i\vec{\partial} + e\vec{A})\varphi|^2 + \lambda(|\varphi|^2 - v^2)^2, \tag{11}$$

and has a topological charge of the Pontryagin index, e.g., $Q = \frac{1}{2\pi}\int dxdyH_z \in \mathbf{Z}$, corresponding to $\Pi_1(U(1)) \in \mathbf{Z}$, and the Abrikosov vortex is a topological configuration with $Q = 1$.

On the scale transformation, the photon-field contribution is to promote "swelling" of the soliton, and the scalar-field contribution is to promote "shrinkage" of the soliton. Because of the competition between these two opposite effects, the Abrikosov vortex is stable against the scale transformation.

On the other hand, holographic QCD of 1+1 QCD has only the vector field A_M in 1+2 dimension, at the leading order of $1/N_c$ and $1/\lambda$. On the scale transformation, the vector-field contribution is to promote swelling of the soliton. Because of this one-side effect, the topological (baryonic) soliton is unstable against the scale transformation, unlike the stable Abrikosov vortex in superconductors.

6. Summary

We have studied baryons in holographic QCD corresponding to 1+1 dimensional single-flavor QCD for the first time. After formulating 1+1 QCD with an S^1-compactified D2/D8/$\overline{\text{D8}}$ branes, we have described the baryon as a topological configuration in 1+1 N_f=1 QCD, corresponding to Π_1(U(1)) = $\mathbf{Z}$. Unlike 1+3 QCD with $N_f \geq 2$, we have found that the low-dimensional (multi)baryonic soliton is generally unstable against a scale transformation/variation and swells infinitely in 1+1 N_f=1 QCD at the leading of $1/N_c$ and $1/\lambda$. In this way, there is a serious difficulty on the soliton picture of baryons in large N_c in the single-flavor world in both 1+1 QCD and 1+3 QCD.

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