Comment on Field Redefinitions in the AdS/CFT Correspondence

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We carry out field redefinitions in ten-dimensional Type IIB supergravity and show that they do not give rise to any physical corrections to the holographic renormalization group structure in the AdS/CFT correspondence. We in particular show that the holographic Weyl anomaly of the $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory does not change under the field redefinition of the ten-dimensional metric of the form $\boldsymbol{G}_{MN} \to \boldsymbol{G}_{MN} + \alpha \boldsymbol{R} \boldsymbol{G}_{MN} + \beta \boldsymbol{R}_{MN}$. These results are consistent with the fact that classical supergravity represents the on-shell structure of massless modes of superstrings, which should not change under redefinitions of fields.

§1. Introduction

The AdS/CFT correspondence[1)−4)] asserts that the classical theory of $(d+1)$-dimensional gravity in an AdS background is related to a d-dimensional CFT at the boundary of the AdS geometry. More precisely, we can regard an on-shell field in the gravity theory as the source coupled to a scaling operator in the CFT at the boundary. Among many applications of the AdS/CFT correspondence, the holographic renormalization group (RG)[5)−16)] is one of the most important. In the holographic RG, we regard the radial coordinate of the $(d+1)$-dimensional manifold as a scaling parameter of the corresponding boundary field theory. Using this scheme, we can describe many aspects of the RG structure of the d-dimensional boundary field theory using the $(d+1)$-dimensional classical gravity theory. For example, we can derive the Callan-Symanzik equation of the corresponding d-dimensional boundary field theory from the Hamilton-Jacobi equation of the $(d+1)$-dimensional gravity theory,[17)] which gives us a systematic formulation of the holographic RG (see also Refs. 18)–20)).

There have been numerous quantitative studies to check the validity of the AdS/CFT correspondence and the holographic renormalization group. Among such studies are calculations of the chiral anomaly[15)] and the Weyl anomaly[16)] of the four-dimensional $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory (SYM_4), which is believed to be realized on the boundary of AdS_5 after the ten-dimensional spacetime is factorized as $\text{AdS}_5 \times S^5$. Both calculations were carried out purely on the basis of the five-dimensional supergravity theory and correctly reproduce the field-theoretical results

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in the large N limit.

In this article, as another study to check the validity of the AdS/CFT correspondence, we show that the holographic RG structure does not undergo any physical corrections under field redefinitions of ten-dimensional supergravity. The AdS/CFT correspondence should have this property, since classical supergravity represents the on-shell structure of massless modes of superstrings, and the on-shell amplitudes (more precisely, the residues of one-particle poles of correlation functions) should be invariant under redefinitions of fields[21] (see also Ref. 22) for discussions in the context of string theory).*)

It is easy to demonstrate the invariance of the holographic RG structure for point-transformations of scalar fields in supergravity,

$$\phi^I \to \phi'^I = f^I(\phi), \tag{1.1}$$

because the superpotential $W(\phi)$ transforms as a scalar over the space parametrized by ϕ^I: $W(\phi) \to W'(\phi) = W(f(\phi))$, so that the beta function of the boundary field theory transforms as a vector field over such space:[17),18),**)]

$$\beta^I(\phi)\Big(= -\frac{2(d-1)}{W(\phi)}\, L^{IJ}(\phi)\, \frac{\partial}{\partial \phi^J} W(\phi)\Big) \to \beta'^I(\phi) = \frac{\partial \phi^I}{\partial f^J}\, \beta^J(f(\phi)). \tag{1.2}$$

Similar arguments can be applied to field redefinitions that include derivatives of fields, such as the redefinition of the ten-dimensional metric of the form $\boldsymbol{G}_{MN} \to \boldsymbol{G}_{MN} + \alpha \boldsymbol{R} \boldsymbol{G}_{MN} + \beta \boldsymbol{R}_{MN}$. In this case, however, the resulting gravity action obtained after such redefinitions possesses higher-order derivative terms. Thus, after the compactification on S^5, one needs to treat the five-dimensional gravity theory with curvature squared terms.

The structure of the holographic RG for higher-derivative gravity was investigated generally in Refs. 25)–28), where it is shown that if the five-dimensional gravity action is given by***)

$$\boldsymbol{S}_5 = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-\widehat{g}} \left[\frac{12}{L^2} - \frac{80a + 16b + 8c}{L^4} + \widehat{R} + a\widehat{R}^2 + b\widehat{R}_{\mu\nu}^2 + c\widehat{R}_{\mu\nu\rho\sigma}^2 \right], \tag{1.3}$$

then the Weyl anomaly of the corresponding boundary CFT is

$$\langle T_i^i \rangle = \frac{2L^3}{2\kappa_5^2} \left[\left(1 + \frac{8(5a+b+c)}{L^2} \right) \left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2 \right) + \frac{c}{2L^2}R_{ijkl}^2 \right]. \tag{1.4}$$

From this, it is seen that if c vanishes, then it may be possible to absorb the change $(1 + 8(5a+b)/L^2)$ into the five-dimensional Newton constant $2\kappa_5^2$. In fact, for field

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*) See also Ref. 23) for recent discussion about scheme independence in the renormalization group structure.

**) $L_{IJ}(\phi)$ is the metric on the space $\{\phi^I\}$, and $c(\phi) = \big(-W(\phi)\big)^{-(d-1)}$ can be identified with the c-function.

***) The cosmological constant is parametrized in such a way that the classical solution can have an AdS spacetime with radius L.

redefinitions of the form $\boldsymbol{G}_{MN} \to \boldsymbol{G}_{MN} + \alpha \boldsymbol{R} \boldsymbol{G}_{MN} + \beta \boldsymbol{R}_{MN}$, no terms including the Riemann tensor $\boldsymbol{R}_{KLMN}$ are induced, so that we only have to consider the case where $c = 0$. Furthermore, as we show in the following sections, the field equation in ten dimensions changes the radius of S^5 exactly in such a way that the change of the five-dimensional Newton constant, $2\kappa_5^2 = 2\kappa_{10}^2 / \text{volume}(S^5)$, cancels the factor $(1 + 8(5a + b)/L^2)$, together with the contribution from the Ramond-Ramond terms.

In §2, we derive the ten-dimensional Type IIB supergravity action that is obtained through the field redefinition, and then we discuss its $AdS_5 \times S^5$ solution. In §3, after explaining how to determine the five-dimensional gravity action when the geometry is compactified on S^5, we calculate the holographic Weyl anomaly for $\mathcal{N} = 4$ $SU(N)$ SYM_4 and show that the result is exactly the same as that for the original anomaly before the field redefinition. Section 4 is devoted to conclusions.

§2. Field redefinition of type IIB supergravity and the $\mathbf{AdS_5 \times S^5}$ solutions

In this section, we consider a field redefinition in the ten-dimensional type IIB supergravity theory. We first give the usual IIB supergravity action and its $AdS_5 \times S^5$ solution. We then carry out a field redefinition of the ten-dimensional metric and derive the corresponding action with its $AdS_5 \times S^5$ solution.

We start with the bosonic part of the ten-dimensional Type IIB supergravity action given by[*]

$$\boldsymbol{S}_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-\boldsymbol{G}} \left[e^{-2\phi} \left(\boldsymbol{R} + 4 |d\phi|^2 \right) - \frac{1}{4} |F_5|^2 \right]. \tag{2·1}$$

Here ϕ and F_5 are the dilaton and the self-dual Ramond-Ramond 5-form field strength, respectively, and we have set other fields of Type IIB supergravity to zero. In this equation, we have used the definitions

$$|d\phi|^2 \equiv \boldsymbol{G}^{MN} \partial_M \phi \, \partial_N \phi, \quad |F_5|^2 \equiv \frac{1}{5!} \boldsymbol{G}^{M_1 N_1} \cdots \boldsymbol{G}^{M_5 N_5} (F_5)_{M_1 \cdots M_5} (F_5)_{N_1 \cdots N_5}. \tag{2·2}$$

The self-duality of F_5 is imposed on the field equations (not in the action) as a constraint.

In the context of the AdS_5/CFT_4 correspondence, we are interested in an $AdS_5 \times S^5$ solution that is realized as the near horizon limit of the black 3-brane solution:[29]

$$ds^2 = \frac{l_0^2}{r^2} \, dr^2 + \frac{r^2}{l_0^2} \, \eta_{ij} \, dx^i dx^j + l_0^2 \, d\Omega_5^2,$$

$$(F_5)_{r0123} = -\frac{4}{g_s} \frac{r^3}{l_0^4}, \quad (F_5)_{y^1 \cdots y^5} = \frac{4}{g_s} \, l_0^4,$$

$$e^\phi = g_s. \tag{2·3}$$

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[*] The coefficient of $|F_5|^2$ is chosen to be $(-1/4)$, which is one half of the canonical value $(-1/2)$. This is necessary for the action to be invariant under T-duality transformations (see, e.g., Ref. 24)).

Here, $d\Omega_5^2 = (\delta_{ab} + y_a y_b/(1-y^2))dy^a dy^b$ $(-1 \le y^a \le 1, \ a, b = 1, \cdots, 5)$ is the metric of the unit five-sphere and $i, j \in \{0, 1, 2, 3\}$. In this case, the AdS$_5$ and S^5 have the same radius, l_0, whose value is determined by the D3-brane charge as

$$l_0 = (4\pi g_s N)^{1/4}, \tag{2.4}$$

where N is the number of the coincident D3-branes, and we have set the string length $\sqrt{\alpha'}$ to 1.

As discussed in the Introduction, we can make an arbitrary field redefinition $\delta \boldsymbol{G}_{MN} = X_{MN}$ without changing the content of the Type IIB supergravity theory.[22] Now we make an infinitesimal change of the metric as

$$\boldsymbol{G}_{MN} \to \boldsymbol{G}'_{MN} \equiv \boldsymbol{G}_{MN} + \alpha \boldsymbol{R} \boldsymbol{G}_{MN} + \beta \boldsymbol{R}_{MN}. \quad \left(F_5' \equiv F_5, \quad \phi' \equiv \phi \right) \tag{2.5}$$

Then, the new gravity action is obtained as

$$\begin{aligned}
\widetilde{\boldsymbol{S}}_{10}[\boldsymbol{G}_{MN}] &\equiv \boldsymbol{S}_{10}[\boldsymbol{G}'_{MN}] \\
&= \boldsymbol{S}_{10}[\boldsymbol{G}_{MN} + \alpha \boldsymbol{R} \boldsymbol{G}_{MN} + \beta \boldsymbol{R}_{MN}],
\end{aligned} \tag{2.6}$$

which is expressed explicitly as

$$\begin{aligned}
\widetilde{\boldsymbol{S}}_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-\boldsymbol{G}} \Bigg\{ & e^{-2\phi} \Big[\boldsymbol{R} + 4\,|d\phi|^2 + a\boldsymbol{R}^2 + b\boldsymbol{R}_{MN}^2 \\
& + a\boldsymbol{R}\,|d\phi|^2 + b\,\boldsymbol{R}^{MN}\partial_M\phi\,\partial_N\phi \Big] \\
& - \frac{1}{4}\,|F_5|^2 + \frac{b}{8}\,\boldsymbol{R}\,|F_5|^2 - \frac{b}{4}\frac{1}{4!}\,\boldsymbol{R}_{MN}(F_5)^{MPQRS}(F_5)^N{}_{PQRS} \Bigg\}.
\end{aligned} \tag{2.7}$$

Here a and b are defined as

$$a = 4\alpha + \frac{1}{2}\beta, \quad b = -\beta. \tag{2.8}$$

Since $\boldsymbol{G}'_{MN}$ and F_5 can be expressed as in Eq. (2.3),

$$\begin{aligned}
ds'^2 &= \boldsymbol{G}'_{MN}\,dX^M dX^N = \frac{l_0^2}{r^2}\,dr^2 + \frac{r^2}{l_0^2}\,\eta_{ij}\,dx^i dx^j + l_0^2\,d\Omega_5^2, \\
(F_5)_{r0123} &= -\frac{4}{g_s}\frac{r^3}{l_0^4}, \quad (F_5)_{y^1\cdots y^5} = \frac{4}{g_s}\,l_0^4, \quad e^\phi = g_s,
\end{aligned} \tag{2.9}$$

we can easily construct an AdS$_5 \times S^5$ solution for the action (2.7):

$$\begin{aligned}
ds^2 = \boldsymbol{G}_{MN}\,dX^M dX^N &= \Big(\boldsymbol{G}'_{MN} - \alpha \boldsymbol{R}'\boldsymbol{G}'_{MN} - \beta \boldsymbol{R}'_{MN} \Big) dX^M dX^N \\
&= \Big(1 - \frac{8b}{l^2} \Big) \frac{l^2}{r^2}\,dr^2 + \frac{r^2}{l^2}\,\eta_{ij}\,dx^i dx^j + l^2 d\Omega_5^2,
\end{aligned}$$

$$(F_5)_{r0123} = \frac{4}{g_s}\left(1 + \frac{8b}{l^2}\right)\frac{r^3}{l^4}, \quad (F_5)_{y^1\cdots y^5} = \frac{4}{g_s}\left(1 - \frac{8b}{l^2}\right)l^4, \quad e^\phi = g_s. \qquad (2\cdot10)$$

Here we have used the fact that with the solution (2·3), the Ricci tensor becomes

$$\boldsymbol{R}_{\mu\nu} = -\frac{4}{l_0^2}\,\boldsymbol{G}_{\mu\nu}, \quad \boldsymbol{R}_{ab} = +\frac{4}{l_0^2}\,\boldsymbol{G}_{ab} \qquad (2\cdot11)$$

for $\mu,\nu \in \{r,0,1,2,3\}$ and $a,b \in \{y^1\cdots y^5\}$, and have rewritten the expression using the radius l of the new S^5, which is calculated as

$$l = \left(1 + \frac{2b}{l_0^2}\right)l_0. \qquad (2\cdot12)$$

Note that after the field redefinition, the radius of S^5, l, differs from that of AdS$_5$, L, which is expressed as

$$L \equiv \left(1 - \frac{4b}{l^2}\right)l = \left(1 - \frac{2b}{l_0^2}\right)l_0. \qquad (2\cdot13)$$

§3. Five-dimensional effective action and the Weyl anomaly

In this section, we calculate the four-dimensional holographic Weyl anomaly from the higher-derivative gravity action (2·7) using the classical solution (2·10), and show that the resulting anomaly exactly reproduces the anomaly of the original gravity theory before making the field redefinition.

To derive the five-dimensional gravity action, we use the following strategy. First, we assume that the geometry of the ten-dimensional spacetime is a direct product of a five-dimensional Lorentzian manifold M_5 and a five-dimensional sphere S^5. Next, we decompose all terms in the action into two parts, one of which is expressed by the fields on M_5 with metric $\widehat{g}_{\mu\nu}$ and the other of which is expressed over S^5 of radius l. For example, the scalar curvature $\boldsymbol{R}$ in the ten-dimensional gravity action becomes $\widehat{R} + 20/l^2$. (Here $\widehat{R}$ is the scalar curvature of M_5.) However, there appears a problem in decomposing the kinetic part of the self-dual five-form field strength F_5. In fact, inserting the classical solution of F_5 into the action would give a trivial, vanishing result due to the self-duality of F_5 ($*F_5 = F_5$).[*] To avoid this problem, we use the ansatz that F_5 has non-zero values only for the S^5 components, and we rescale F_5 in the action by the factor $\sqrt{2}$: $F_5 \to \sqrt{2}F_5$. Finally, we integrate over S^5 in the ten-dimensional action and obtain the five-dimensional gravity action.

Following this strategy, we first calculate the Weyl anomaly of $\mathcal{N} = 4$ $SU(N)$ SYM$_4$ from the action (2·1). Since $\boldsymbol{R} = \widehat{R} + 20/l_0^2$ and $-(1/4)\left|\sqrt{2}F_5\right|^2 = -8/l_0^2$, we have the five-dimensional action

$$\boldsymbol{S}_5 = \frac{\pi^3 l_0^5}{2\kappa_{10}^2 g_s^2}\int d^5x\sqrt{-\widehat{g}}\left(\frac{12}{l_0^2} + \widehat{R}\right). \qquad (3\cdot1)$$

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[*] $\sqrt{-\boldsymbol{G}}\left|F_5\right|^2 = F_5 \wedge *F_5 = F_5 \wedge F_5 = 0.$

This action actually has an AdS$_5$ solution with radius l_0, which justifies our ansatz. Using the formula (1·4), we obtain the Weyl anomaly as

$$
\begin{aligned}
\langle T_i^i \rangle &= \frac{2\pi^3 l_0^8}{2\kappa_{10}^2 g_s^2}\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right) \\
&= \frac{N^2}{4\pi^2}\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right).
\end{aligned}
\tag{3·2}
$$

Here we have used $2\kappa_{10}^2 = (2\pi)^7$ and (2·4).

Next we apply our strategy to the action (2·7). From the solution (2·10), we compactify ten-dimensional spacetime on S^5 of radius l. Then, the (dimensionally reduced) five-dimensional action is obtained as

$$
\begin{aligned}
\widetilde{\boldsymbol{S}}_5 &= \frac{\pi^3 l^5}{2\kappa_{10}^2 g_s^2}\left(1 + \frac{40a + 4b}{l^2}\right) \\
&\quad \times \int d^5x\sqrt{-\widehat{g}}\left[\left(\frac{12}{l^2} - \frac{80a - 80b}{l^4}\right) + \widehat{R} + a\widehat{R}^2 + b\widehat{R}_{\mu\nu}^2\right].
\end{aligned}
\tag{3·3}
$$

This action has an AdS$_5$ solution with radius $\left(1 - 4b/l^2\right)l$, which is consistent with the AdS$_5 \times S^5$ solution (2·10). From this solution, we can read off the parameters in Eq. (1·3),

$$
\frac{1}{2\kappa_5^2} = \frac{\pi^3 l^5}{2\kappa_{10}^2 g_s^2}\left(1 + \frac{40a + 4b}{l^2}\right), \quad L = \left(1 - \frac{4b}{l^2}\right)l, \quad c = 0.
\tag{3·4}
$$

Thus the corresponding Weyl anomaly is calculated again by using the formula (1·4) as

$$
\begin{aligned}
\langle T_i^i \rangle &= \frac{2L^3}{2\kappa_5^2}\left(1 - \frac{40a + 8b}{l^2}\right)\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right) \\
&= \frac{2\pi^3 l^8}{2\kappa_{10}^2 g_s^2}\left(1 - \frac{16b}{l^2}\right)\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right) \\
&= \frac{2\pi^3 l_0^8}{2\kappa_{10}^2 g_s^2}\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right) \\
&= \frac{N^2}{4\pi^2}\left(-\frac{1}{24}R^2 + \frac{1}{8}R_{ij}^2\right).
\end{aligned}
\tag{3·5}
$$

This is identical to the result (3·2).

§4.   Conclusion

In this paper, we quantitatively checked the validity of the AdS/CFT correspondence by showing that the holographic RG structure is invariant under field redefinitions in Type IIB supergravity. In particular, we carried out a redefinition of the ten-dimensional metric of the form $\boldsymbol{G}_{MN} \to \boldsymbol{G}_{MN} + \alpha\boldsymbol{R}\boldsymbol{G}_{MN} + \beta\boldsymbol{R}_{MN}$ (Eq. (2·5)) and calculated explicitly the modified Type IIB action. We then constructed

effective five-dimensional gravity when ten-dimensional spacetime is compactified on S^5 and calculated the holographic Weyl anomaly. We showed that the obtained anomaly is identical to that of the $\mathcal{N} = 4$ $SU(N)$ SYM$_4$ in the large N limit, even though the five-dimensional action contains higher-order derivative terms. This result is consistent with the assertion of the AdS/CFT correspondence that on-shell fields in the gravity theory are coupled to scaling operators of the corresponding CFT at the boundary of the AdS geometry. In fact, the theorem of Kamefuchi, O'Raifeartaigh and Salam guarantees that a field redefinition does not change the on-shell structure of the theory.

We finally point out that this invariance of the holographic Weyl anomaly under a redefinition of the metric holds only if there is a simultaneous change of the ten-dimensional metric given by (2·5). In fact, if we only change the five-dimensional metric in the effective five-dimensional action, $\widehat{g}_{\mu\nu} \to \widehat{g}_{\mu\nu} + \alpha \widehat{R} \widehat{g}_{\mu\nu} + \beta \widehat{R}_{\mu\nu}$, then the resulting Weyl anomaly differs from the field-theoretical anomaly in the large N limit. However, this is not a contradiction, because if field redefinitions are carried out only for five-dimensional components, generally the on-shell conditions for a ten-dimensional field theory are broken. Thus, there is no reason to expect that the AdS/CFT correspondence holds for such redefinitions.

Acknowledgements

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