Multi-objective optimization of a tower-type truss using order statics

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1. Introduction

There have been many methods developed for robust design and optimization of structures considering uncertainty in parameter values representing, e.g., material and geometrical properties [1]. Various approaches exist for uncertainty model including probabilistic and non-probabilistic models. Simplest approach is the worst-case design under constraints on worst values of responses defining the uncertain parameters using interval variables, where the upper and lower bounds are given. Robust optimization [2] for uncertain parameters in bounded convex regions is also extensively studied. However, even for this simple approach, it is difficult to obtain the worst value. Yamakawa and Ohsaki [3] showed that an approximate worst value can be obtained using the order statistics [4] within a small computational cost.

In this study, we propose a multi-objective optimization approach to robust optimization of structures. Order statistics is used for representing the robustness. In the numerical example of a tower-type truss, Pareto optimal solutions are found considering the structural weight and robustness of a tower-type truss.

2. Approximate worst response based on order statistics

Let $\theta \in \Omega$ denote an uncertain parameter representing, e.g., structural geometry and material constants, in the range Ω . The representative response quantity such as maximum stress or displacement of a structure is denoted by $g(\theta)$ with its upper bound $\bar{g}$. We use a probabilistic approach to the worst-case design, and assign a constraint so that the following inequality is satisfied with a specified probability:

$$g_{\max} := \max_{\theta \in \Omega} g(\theta) \le \bar{g} \tag{1}$$

Parameter values $\theta_1, \ldots, \theta_n$ are generated on Ω based on the prescribed probability distribution, and the values of $g(\theta_i)$ are denoted by Y_i as $\{Y_1 = g(\theta_1), \ldots, Y_n = g(\theta_n)\}$. Then the samples $Y_1, \ldots, Y_n$ obey the same probability distribution, which are sorted as

$$Y_{1,n} \geq Y_{2,n} \ldots Y_{n-1,n} \geq Y_{n,n} \tag{2}$$

The *k*th value $Y_{k,n}$ is a probabilistic value that is called *k*th order statistic. For specified values of α, γ, and *n*, we can compute the value of *k* so that satisfying the inequality

$$Y_{k,n} \leq \overline{g} \tag{3}$$

is equivalent to the following property:

- 100γ% of the samples $Y_1, Y_n, \ldots$ are less than $\overline{g}$ with the probability 100α%

See Refs. [3,4] for details.

3. Multi-objective optimization based on order statistics

A multi-objective optimization problem is formulated considering structural weight and robustness based on the order statics. Note that *k* has the property opposite to γ. If we decrease *k* keeping *n* constant, the ratio of feasible solutions for the upper-bound response ratio corresponding to *k* increases; accordingly, robust solutions with small uncertainty are to be obtained. This way, a robust optimization problem can be formulated using the order statics denoting *k* as *robustness parameter*. However, it is difficult to assign a specific robustness level as a design requirement. Therefore, a multi-objective optimization problem is formulated to maximize robustness and minimize the cost represented by structural weight.

Consider a structure subjected to specified static loads. The maximum ratio of member stresses and nodal displacements to their upper bounds, which is denoted by *R*, is used as the representative response. If we have enough number of solutions satisfying (1) with equality in good accuracy, then we can obtain Pareto optimal solutions between structural weight *C* and robustness parameter *k*. To compare the performance of the structures with different values of *R*, the product of weight *C* and response ratio *R*, denoted as $F = CR$, is considered as the performance measure.

Furthermore, to obtain the good candidate solutions for the Pareto set, we first optimize the structure under constraint on the response ratio corresponding to the nominal values of the uncertain parameters. A heuristic method called simulated annealing (SA) is used for optimization. Then, the solutions obtained in the process of optimization by SA are regarded as the candidate solutions of the Pareto set. The process is summarized as follows:

Step 1: Obtain a set of optimal cross-sectional areas for minimizing the structural weight *C* under constraints on the response ratio under static loads. Optimal solution is found using SA, and no uncertainty is considered at this step.

Step 2: Collect feasible solutions generated in the process of SA in Step 1.

Step 3: Assign uncertainty in Young's modulus of each feasible solution in Step 2 to generate ordered set of *n* samples of maximum representative response ratios $Y_1 \geq Y_2 \ldots Y_{n-1} \geq Y_n$.

Step 4: Plot the product of weight and response ratio $F = CR$ with respect to the order *k* for all solutions.

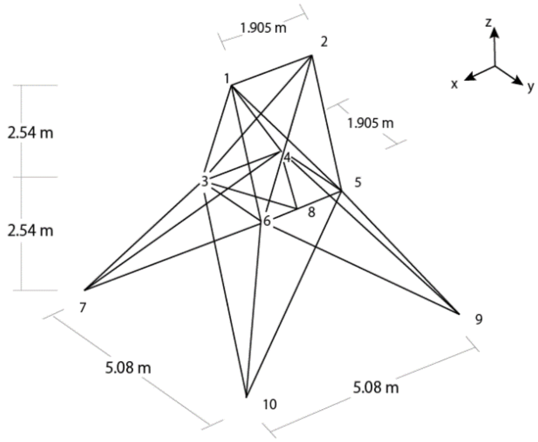

Fig. 1: A small tower model [5].

4. Optimization of a small tower model

Consider a small tower model as shown in Fig. 1, which is investigated as a test problem of structural optimization in many papers. The details are described in, e.g., Ref. [5]. The members are classified into eight groups, and their cross-sectional areas are selected from the list {0.0, 0.1, 0.2,…2.6, 2.8, 3.0, 3.2, 3.4} (×645.16 mm^2). The (x, y, z) coordinates (m) of nodes 1, 3, and 7 are (−0.9525, 0, 5.08), (−0.9525, 0.9525, 2.54), and (−2.54, 2.54, 0), respectively, and those for other nodes are defined based on symmetry of the truss.

Let P = 4448.2 (N), and assign (x, y, z) directional loads (P, $-10P$, $-10P$) at node 1, (0, $-10P$, $-10P$) at node 2, (0.5P, 0, 0,) at node 3, and (0.6P, 0, 0) at node 6. The upper-bound displacements in (x, y, z) directions are 8.89 mm. The nominal value of Young's modulus is 68947.5 N/mm^2, and the weight density is 27.8 kN/m^2. The upper-bound stress is 275.8 N/mm^2 for both tension and compression. The design variables are cross-sectional areas, (x, y, z) coordinates of node 4, and (x, y) coordinates of node 8. The coordinates of other nodes are defined based on symmetry. The bounds of coordinates (m) are assigned as follows:

$$\begin{aligned} &0.508 \le x_4 \le 1.524, \quad 1.016 \le y_4 \le 2.032 \\ &2.286 \le z_4 \le 3.302, \quad 1.016 \le x_8 \le 2.032 \\ &2.540 \le y_8 \le 3.556 \end{aligned} \tag{4}$$

The coordinates are discretized into equally spaced 21 values in each region.

Uncertainty is considered in the ratio of Young's modulus of each of 25 members to its nominal value. Truncated normal distribution is assumed with mean 1.0, standard deviation 0.055, lower bound 0.9, and upper bound 1.1. For the SA, the initial temperature is 1.0, the number of neighbourhood solutions is 78, the temperature reduction ratio is 0.98, and the number of steps is 10000. The scaling parameter of objective function is assigned so that the acceptance ratio is 0.5 for 10% increase of objective value at the initial step.

5. Optimization results

Pareto optimal solutions are generated as shown in Fig. 2 for n = 150 and k = 20. It is seen from the figure that Pareto set corresponds to Design A for k = 1, …, 4, Design B for k = 5, and Design C for k = 5, …, 20. The cross-sectional areas of Designs A, B, and C are shown in Fig. 3. Note that Designs A and B have the same topology.

Assigning k in the horizontal axis to solve a single-objective optimization problem for minimizing F is equivalent to solving the multi-objective optimization problem using a constraint method.

If α is fixed at 0.9, (n, k) = (150, 1) corresponds to γ = 0.985, and (n, k) = (150, 20) corresponds to γ = 0.831. This way, solutions with various values of γ, i.e., various levels of robustness, are found by generating Pareto optimal solutions for various values of k. The solutions in Fig. 2 correspond to rather high robustness level; and accordingly, the topology of Designs A and B are the same as that of the optimal solution without uncertainty.

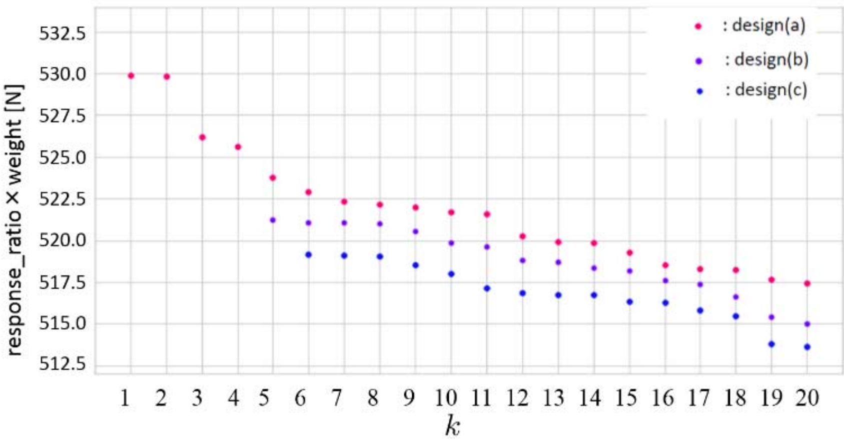

Fig. 2: Relation between k and F for solutions containing Pareto solutions.

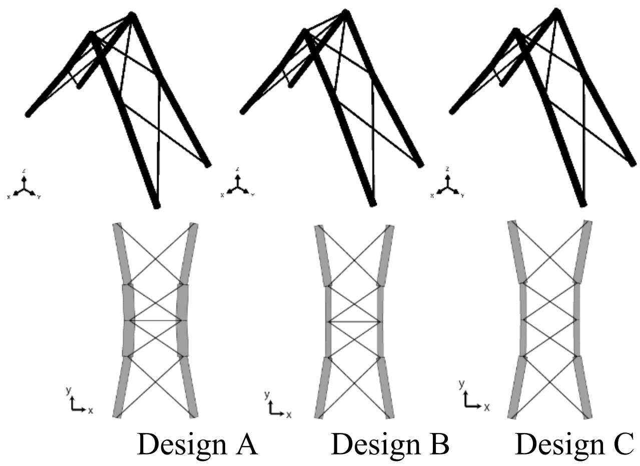

Fig. 3: Diagonal view and plan view of optimal solutions with uncertainty.

6. Conclusions

A new multi-objective optimization method has been proposed based on order statics for robust design of trusses. The objective function is the order k of the solution and the value of response ratio multiplied by the structural weight.

Pareto solutions have been generated for a tower-type truss to show that the parameter k can be used as a practical parameter representing robustness level, and various solutions with different robustness levels can be found using the proposed method with small computational cost.

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