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Iterative Algorithms for a System of Random Nonlinear Equations in Hilbert Spaces with Fuzzy Mappings

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Abstract

The purpose of this paper, by using the resolvent operator technique associated with randomly (A, η, m)-monotone operator, is to establish an existence and convergence theorem for a class of system of random nonlinear equations with fuzzy mappings in Hilbert spaces. Our works are improvements and generalizations of the corresponding well-known results.

Keywords: system of random nonlinear equations, relaxed cocoercive operators, randomly (A, η, m)-proximal operator equations, fuzzy mappings, Hilbert spaces.

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1 Introduction

The fuzzy sets theory is an extension of a crisp set by enlarging the truth valued set $\{0, 1\}$ to the real unit interval $[0, 1]$ ([27]). A fuzzy set is characterized and identified by a mapping called a membership grade function from the whole set into $[0, 1]$. Heilpern [13] introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy contraction mapping which is a fuzzy analogue of Nadler's fixed point theorem for multi-valued mappings.

In 1989, Chang and Zhu [6] first introduced and studied a class of variational inequalities for fuzzy mappings. Since then several classes of variational inequalities, quasi variational inequalities and complementarity problems with fuzzy mappings were considered by Agarwal *et al.* [1], Chang and Huang [8], Ding *et al.* [9], Huang [10], Lee *et al.* [21], Salahuddin [25] in the setting of Hilbert spaces and Banach spaces.

Lan [20] introduced a new concepts of (A,η)-monotone operator which generalizes the (H,η)-monotonicity and A-monotonicity in Hilbert spaces and studied some properties of (A,η)-monotone operators and applied resolvent operators associated with (A,η)-monotone operators to approximate the solution of a new class of nonlinear (A,η)-monotone operator inclusion problems with relaxed cocoercive operators in Hilbert spaces. Recently Kim *et al.* [16] introduced the (A,η,m)-proximal operator to study the system of equations in Hilbert spaces.

Recently some systems of variational inequalities, variational inclusions, complementarity problems and equilibrium problems have been studied by some authors in recent years because of their close relation to Nash equilibrium problems. Huang and Fang [11] introduced a system of order complementarity problems and established some existence results for these using fixed point theory. Kim and Kim [18] introduced and studied some system of variational inequalities and developed some iterative algorithms for approximately the solutions of system of variational inequalities.

On the other hand, random variational inequality problems, random quasi variational inequality problems and random variational inclusions and complementarity problems have been studied by Chang [5], Chang and Huang [7], Huang [10], Khan and Salahuddin [15] and Bharucha-Red [3], *etc.*

The concepts of random fuzzy mapping was first introduced by Huang [10]. Subsequently the random variational inclusion problems for random fuzzy mappings is studied by Anastassiou *et al.* [2], Salahuddin [25], Zhang and Bi [28].

Inspired and motivated by the works [2, 12, 14, 17, 23, 26], we establish the existence and convergence theorem for system of random nonlinear equations with fuzzy mapping in Hilbert spaces by using random (A,η,m)-proximal operator equations

2 Preliminaries

Throughout this paper, (Ω,Σ) is a measurable space with a set Ω and a σ-algebra Σ of a subset of Ω, H is a real separable Hilbert space endowed with a norm $\|\cdot\|$ and inner product

$\langle\cdot,\cdot\rangle$. Notations $\mathcal{B}(H)$, 2^H and $CB(H)$ denote the class of Borel σ-fields in H, the family of all nonempty subsets of H and the family of all nonempty closed bounded subset of H, respectively.

Definition 2.1. A mapping $u:\Omega\longrightarrow H$ is said to be measurable if for any $B\in\mathcal{B}(H), u^{-1}=\{t\in\Omega, u(t)\in B\in\Sigma\}$.

Definition 2.2. A mapping $f:\Omega\times H\longrightarrow H$ is called a random mapping if for each fixed $u\in H$, a mapping $f(\cdot,u):\Omega\longrightarrow H$ is measurable. A random mapping f is said to be continuous if for each fixed $t\in\Omega$, a mapping $f(t,\cdot):H\longrightarrow H$ is continuous.

Definition 2.3. A multi-valued mapping $T:\Omega\longrightarrow 2^H$ is said to be measurable if for any $B\in\mathcal{B}(H), T^{-1}(B)=\{t\in\Omega: T(t)\cap B\neq\emptyset\}\in\Sigma$.

Definition 2.4. A mapping $u:\Omega\longrightarrow H$ is called a measurable selection of a measurable multi-valued mapping $T:\Omega\longrightarrow 2^H$, if u is measurable and for any $t\in\Omega$, $u(t)\in T_t(u(t))$.

Definition 2.5. A mapping $T:\Omega\times H\longrightarrow 2^H$ is called a random multi-valued mapping if for each fixed $x\in H$, $T(\cdot,x):\Omega\longrightarrow 2^H$ is a measurable multi-valued mapping. A random multi-valued mapping $T:\Omega\times H\longrightarrow CB(H)$ is said to be $\mathfrak{D}$-continuous if for each fixed $t\in\Omega$, $T(t,\cdot):\Omega\times H\longrightarrow 2^H$ is randomly continuous with respect to the Hausdorff metric $\mathfrak{D}$.

Definition 2.6. A multi-valued mapping $T:\Omega\times H\longrightarrow 2^H$ is called random if for any $x\in H, T(\cdot,x)$ is measurable (denoted by $T_{t,x}$ or T_t).

Let Ω be a set and $\mathfrak{F}(H)$ be a collection of fuzzy sets over H. A mapping $\tilde{F}:\Omega\times H\longrightarrow\mathfrak{F}(H)$ is called a fuzzy mapping on H. If $\tilde{F}$ is a fuzzy mapping on H then for any $t\in\Omega, \tilde{F}(t)$ (denote it by $\tilde{F}_t$ in the sequel) is a fuzzy mapping on H and $\tilde{F}_t(x)$ is the membership grade of x in $\tilde{F}_t$. Let $A\in\mathfrak{F}(H)$, $\alpha\in(0,1]$. Then the set

$$A_\alpha = \{x\in H:\ A(x)\geq\alpha\}$$

is called an α-cut of A.

Definition 2.7. A fuzzy mapping $\tilde{F}:\Omega\times H\longrightarrow\mathfrak{F}(H)$ is said to be measurable, if for any $\alpha\in(0,1]$, $(\tilde{F}(\cdot))_\alpha:\Omega\longrightarrow 2^H$ is a measurable multi-valued mapping.

Definition 2.8. A fuzzy mapping $\tilde{F}:\Omega\times H\longrightarrow\mathfrak{F}(H)$ is a random fuzzy mapping if for any $x\in H, \tilde{F}(\cdot,x):\Omega\times H\longrightarrow\mathfrak{F}(H)$ is a measurable fuzzy mapping (denoted by $\tilde{F}_{t,x}$ short down $\tilde{F}_t(x)$).

Let $\tilde{T}: \Omega \times H \longrightarrow \mathfrak{F}(H)$ be a random fuzzy mapping satisfying the following condition:

$(*)$: there exists a function $\alpha : H \longrightarrow (0,1]$ such that for all $(t,x) \in \Omega \times H$, we have $(\tilde{T}_{t.x(t)})_{\alpha(x(t))} \in CB(H)$, where $T_{t,x}$ denotes the value of T at (t,x). Induced multi-valued random mapping $\tilde{T}_t$ from T as follows:

$$T : \Omega \times H \longrightarrow CB(H), T_t = \tilde{T}(t,x(t))_{\alpha(x(t))}, (t,x) \longrightarrow T_{t,x_{\alpha(x)}}, \forall (t,x) \in \Omega \times H.$$

In this paper we consider the following random (A_t, η_t, m_t)-proximal operator equation system with fuzzy mappings, we consider for each fixed $t \in \Omega$ finding $(x(t), y(t)), (z(t), w(t)) \in H_1 \times H_2, u(t) \in T_t(x(t))$ and

$$\begin{aligned} &E_t(x(t), y(t)) + \rho^{-1} R^{M_t(\cdot,x(t))}_{\rho,A_{1,t}}(z(t)) = 0, \\ &G_t(u(t), y(t)) + \varrho^{-1} R^{N_t(\cdot,y(t))}_{\varrho,A_{2,t}}(w(t)) = 0 \end{aligned} \tag{2.1}$$

where $T : H_1 \times \Omega \longrightarrow \mathfrak{F}(H_1)$ is a fuzzy mapping, $E : H_1 \times H_2 \times \Omega \longrightarrow H_1, G : H_1 \times H_2 \times \Omega \longrightarrow H_2, g : H_1 \times \Omega \longrightarrow H_1, h : H_2 \times \Omega \longrightarrow H_2, \eta_1 : H_1 \times H_2 \times \Omega \longrightarrow H_1$ and $\eta_2 : H_2 \times H_2 \times \Omega \longrightarrow H_2$ are nonlinear random single-valued mappings, $A_1 : H_1 \times \Omega \longrightarrow H_1, A_2 : H_2 \times \Omega \longrightarrow H_2, M : H_1 \times H_1 \times \Omega \longrightarrow 2^{H_1}$ and $N : H_2 \times H_2 \times \Omega \longrightarrow 2^{H_2}$ are any nonlinear operators such that for all $(z(t), t) \in H_1 \times \Omega, M_t(\cdot, z_t) : H_1 \longrightarrow 2^{H_1}$ is a randomly $(A_{1,t}, \eta_{1,t}, m_{1,t})$-monotone operator with $f_t(H_1) \cap dom(M_t(\cdot, z(t))) \neq \emptyset$ and for all $(w,t) \in H_2 \times \Omega, N_t(\cdot, w(t)) : H_2 \longrightarrow 2^{H_2}$ is a randomly $(A_{2,t}, \eta_{2,t}, m_{2,t})$-monotone operator with $g_t(H_2) \cap dom(N_t(\cdot, w(t)) \neq \emptyset$, $R^{M_t(\cdot,x(t))}_{\rho_t,A_{1,t}} = I - A_{1,t}\left(J^{M_t(\cdot,x(t))}_{\rho_t,A_{1,t}}\right)$, $R^{N_t(\cdot,y(t))}_{\varrho_t,A_{2,t}} = I - A_{2,t}\left(J^{N_t(\cdot,y(t))}_{\varrho_t,A_{2,t}}\right)$, I is the identity mapping, $A_{1,t}\left(J^{M_t(\cdot,x(t))}_{\rho_t,A_{1,t}}(z(t))\right) = A_{1,t}\left(J^{M_t(\cdot,x(t))}_{\rho_t,A_{1,t}}\right)(z(t))$, $A_{2,t}\left(J^{N_t(\cdot,y(t))}_{\varrho_t,A_{2,t}}(w(t))\right) = A_{2,t}\left(J^{N_t(\cdot,y(t))}_{\varrho_t,A_{2,t}}\right)(w(t))$, $J^{M_t(\cdot,x(t))}_{\rho_t,A_{1,t}} = (A_{1,t}+\rho_t M_t(\cdot, x(t)))^{-1}$ and $J^{N_t(\cdot,y(t))}_{\varrho_t,A_{2,t}} = (A_{2,t}+\varrho_t N_t(\cdot, y(t)))^{-1}$, for all $(x(t), z(t)) \in H_1, (y(t), w(t)) \in H_2$ and $\rho, \varrho : \Omega \longrightarrow (0,1)$ are measurable mappings.

For appropriate and suitable choice of $T, E, G, M, N, f, g, A_i, \eta_i$ and H_i for $i = 1,2$ we see that (2.1) is generalized version of some problems which include the system (random) variational inclusions, (random) generalized quasi variational inequalities and (random) implicit quasi variational inequalities for fuzzy mappings, see [17, 18].

Lemma 2.9. [4] Let $M : \Omega \times H \longrightarrow CB(H)$ be a $\mathfrak{D}$-continuous random multi-valued mapping. Then for a measurable mapping $x : \Omega \longrightarrow H$, the mapping $M(\cdot, x(\cdot)) : \Omega \longrightarrow CB(H)$ is measurable.

Lemma 2.10. [4] Let $M, V : \Omega \longrightarrow CB(H)$ be two measurable multi-valued mappings and $\epsilon > 0$ be a constant and $x : \Omega \longrightarrow H$ be a measurable selection of M. Then there exists a measurable selection $y : \Omega \longrightarrow H$ of V such that for all $t \in \Omega$

$$\|x(t) - y(t)\| \ \leq \ (1+\epsilon)\,\mathfrak{D}(M(t), V(t)).$$

Lemma 2.11. [22] Let (H, d) be a complete metric space. Suppose that $G : H \longrightarrow CB(H)$ satisfies

$$\mathfrak{D}(G(x), G(y)) \leq \omega d(x, y), \forall x, y \in H,$$

where $\omega \in (0, 1)$ is a constant. Then the mapping G has a fixed point in H.

Definition 2.12. Let $x, y, w : \Omega \longrightarrow H$ be random mappings and $t \in \Omega$. A random mapping $T : \Omega \times H \times H \longrightarrow H$ is said to be:

(i) randomly monotone in the first argument if

$$\langle T_t(x(t), w(t)) - T_t(y(t), w(t)), x(t) - y(t)\rangle \geq 0,$$

for all $x(t), y(t) \in H$.

(ii) randomly strictly monotone in the first argument if T_t is monotone and

$$\langle T_t(x(t), w(t)) - T_t(y(t), w(t)), x(t) - y(t)\rangle = 0$$

if and only if $x(t) = y(t)$;

(iii) randomly r_t-strongly monotone in the first argument if there exists a measurable function $r_t : \Omega \longrightarrow (0, \infty)$ such that

$$\langle T_t(x(t), w(t)) - T_t(y(t), w(t)), x(t) - y(t)\rangle \geq r_t\|x(t) - y(t)\|^2,$$

for all $x(t), y(t) \in H$.

(iv) randomly m_t-relaxed monotone in the first argument if there exists a measurable function $m_t : \Omega \longrightarrow (0, \infty)$ such that

$$\langle T_t(x(t), w(t)) - T_t(y(t), w(t)), x(t) - y(t)\rangle \geq -m_t\|x(t) - y(t)\|^2,$$

for all $x(t), y(t) \in H$.

(v) randomly s_t-cocoercive in the first argument if there exists a measurable function $s_t : \Omega \longrightarrow (0, \infty)$ such that

$$\langle T_t(x(t), w(t)) - T_t(y(t), w(t)), x(t) - y(t)\rangle \geq s_t\|T_t(x(t), w(t)) - T_t(y(t), w(t))\|^2,$$

for all $x(t), y(t), w(t) \in H \times H \times H$.

(vi) randomly γ_t-relaxed cocoercive with respect to A_t in the first argument if there exists a measurable function $\gamma_t \longrightarrow (0,\infty)$ such that

$$\langle T_t(x(t),w(t))-T_t(y(t),w(t)), A_t(x(t))-A_t(y(t))\rangle \geq -\gamma_t\|T_t(x(t),w(t))-T_t(y(t),w(t))\|^2,$$

for all $x(t), y(t), w(t) \in H \times H \times H$.

(vii) randomly (γ_t, α_t)-relaxed cocoercive with respect to A_t in the first argument if there exist measurable functions $\gamma_t, \alpha_t : \Omega \longrightarrow (0,\infty)$ such that

$$\begin{aligned}&\langle T_t(x(t),w(t)) - T_t(y(t),w(t)), A_t(x(t)) - A_t(y(t))\rangle \\ &\geq -\gamma_t\|T_t(x(t),w(t)) - T_t(y(t),w(t))\|^2 + \alpha_t\|x(t) - y(t)\|^2,\end{aligned}$$

for all $x(t), y(t), w(t) \in H \times H \times H$.

(viii) randomly μ_t-Lipschitz continuous in the first argument if there exists a measurable function $\mu_t : \Omega \longrightarrow (0,\infty)$ such that

$$\|T_t(x(t),w(t)) - T_t(y(t),w(t))\| \leq \mu_t\|x(t) - y(t)\|,$$

for all $x(t), y(t), w(t) \in H \times H \times H$.

In a similar way, we can define a randomly Lipschitz continuity of the operator $T(\cdot,\cdot,\cdot)$ in the second argument.

Definition 2.13. Let $T : H \times \Omega \longrightarrow 2^H$ be a random multi-valued mapping. Then T is said to be randomly τ_t-$\widetilde{\mathcal{D}}$-Lipschitz continuous in the first argument if there exists a measurable mapping $\tau : \Omega \longrightarrow (0,1)$ such that

$$\widetilde{\mathcal{D}}(T_t(x(t)), T_t(y(t))) \leq \tau_t\|x(t) - y(t)\|,$$

for all $x(t), y(t) \in H, t \in \Omega$, where $\widetilde{\mathcal{D}} : 2^H \times 2^H \longrightarrow (-\infty, +\infty) \cup \{+\infty\}$ is the Hausdorff metric *i.e.,*

$$\widetilde{\mathcal{D}}(A,B) = \max\left\{\sup_{x(t)\in A}\inf_{y(t)\in B}\|x(t) - y(t)\|, \sup_{x(t)\in B}\inf_{y(t)\in A}\|x(t) - y(t)\|\right\}, \forall A, B \in 2^H.$$

In a similar way we can define randomly $\widetilde{\mathcal{D}}$-Lipschitz continuity of the $T(\cdot,\cdot)$ in the second argument.

Lemma 2.14. Let (H,d) be a complete metric space and $T_1, T_2 : H \longrightarrow CB(H)$ be two set-valued contractive mappings with same contractive constant $t \in (0,1)$ *i.e.*,

$$\widetilde{\mathcal{D}}(T_i(x), T_i(y)) \leq td(x,y), \forall x, y \in H, i = 1,2.$$

Then

$$\widetilde{\mathcal{D}}(F(T_i), F(T_i)) \leq \frac{1}{1-t} \sup_{x \in H} \widetilde{\mathcal{D}}(T_1(x), T_2(x)),$$

where $F(T_1)$ and $F(T_2)$ are the sets of fixed points of T_1 and T_2, respectively.

Definition 2.15. Let $A : H \times \Omega \longrightarrow H, \eta : H \times H \times \Omega \longrightarrow H$ be two random single-valued mappings. The set-valued mapping $M : H \times H \times \Omega \longrightarrow 2^H$ is said to be randomly (A_t, η_t, m_t)-monotone if

(1) M is randomly m_t-relaxed η_t-monotone mapping;

(2) $(A_t + \rho_t M_t)(H) = H$, where $\rho : \Omega \longrightarrow (0,1)$ is a measurable mapping.

Definition 2.16. Let $A : \Omega \times H \longrightarrow H$ be a randomly r_t-strongly η_t-monotone mapping and $M : \Omega \times H \longrightarrow 2^H$ be a randomly (A_t, η_t)-monotone operator. Then random operator $(A_t + \rho_t M_t)^{-1}$ is a single-valued random mapping for any measurable function $\rho : H \longrightarrow (0, \infty)$ and $t \in \Omega$.

Definition 2.17. Let $A : \Omega \times H \longrightarrow H$ be a randomly strictly η_t-monotone mapping and $M : \Omega \times H \longrightarrow 2^H$ be a randomly (A_t, η_t, m_t)-monotone mapping. Then for any given measurable mapping $\rho : \Omega \longrightarrow (0,1)$, the random resolvent operator $J^{\eta_t, M_t}_{\rho_t, A_t} : H \longrightarrow H$ is defined by

$$J^{\eta_t, M_t}_{\rho_t, A_t}(x(t)) = (A_t + \rho_t M_t)^{-1}(x(t)), \forall t \in \Omega, x(t) \in H.$$

Proposition 2.18. [19] Let H be a Hilbert space and $\eta : \Omega \times H \times H \longrightarrow H$ be a randomly τ_t-Lipschitz continuous mapping, $A : \Omega \times H \longrightarrow H$ be a randomly r_t-strongly η_t-monotone mapping and $M : \Omega \times H \longrightarrow 2^H$ be a randomly (A_t, η_t, m_t)-monotone mapping. Then the random resolvent operator $J^{\eta_t, M_t}_{\rho_t, A_t} : H \longrightarrow H$ is a randomly $(\frac{\tau_t}{r_t - \rho_t m_t})$-Lipschitz continuous mapping *i.e.*,

$$\|J^{\eta_t, M_t}_{\rho_t, A_t} x(t) - J^{\rho_t, A_t}_{\eta_t, M_t} y(t)\| \leq \frac{\tau_t}{r_t - \rho_t m_t} \|x(t) - y(t)\|,$$

where $\rho_t \in (0, \frac{r_t}{m_t})$ is a real-valued random variable for all $t \in \Omega$.

In connection with a randomly (A_t, η_t, m_t)-proximal operator equation system (2.1), we consider the system of random nonlinear equation with fuzzy mappings for finding measurable mappings $x, u : \Omega :\longrightarrow H_1, y : \Omega \longrightarrow H_2$ such that for all $t \in \Omega$ and each fixed $\tilde{T}_{t,x(t)}(u(t)) \geq \alpha(x(t))$ and

$$\begin{aligned} 0 &\in E_t(x(t), y(t)) + M_t(x(t), x(t)), \\ 0 &\in G_t(u(t), y(t)) + N_t(y(t), y(t)). \end{aligned} \tag{2.2}$$

Lemma 2.19. For $t \in \Omega$, $x, u : Q \longrightarrow H_1$ and $y : \Omega \longrightarrow H_2$, $(x(t), y(t), u(t))$ is a solution of problem (2.2) if and only if $(x(t), u(t)) \in H_1, y(t) \in H_2$ such that

$$\begin{aligned} x(t) &= J^{M_t(\cdot,x(t))}_{A_{1,t},\rho_t}[A_{1,t}(x(t)) - \rho_t E_t(x(t), y(t))] \\ y(t) &= J^{N_t(\cdot,y(t))}_{A_{2,t},\varrho_t}[A_{2,t}(y(t)) - \varrho_t G_t(u(t), y(t))] \end{aligned} \tag{2.3}$$

where $J^{M_t(\cdot,x(t))}_{A_{1,t},\rho_t} = (A_{1,t} + \rho_t M_t(\cdot, x(t)))^{-1}$ and $J^{N_t(\cdot,y(t))}_{A_{2,t},\varrho_t} = (A_{2,t} + \varrho_t N_t(\cdot, y(t)))^{-1}$ are corresponding random resolvent operator in the first argument of a random $(A_{1,t}, \eta_{1,t})$-monotone operator $M_t(\cdot, \cdot)$, random $(A_{2,t}, \eta_{2,t})$-monotone operator $N_t(\cdot, \cdot)$, respectively, $A_{i,t}$ is a randomly $r_{i,t}$-strongly monotone operator for $i = 1, 2$ and $\rho, \varrho : \Omega \longrightarrow (0, 1)$ are measurable mappings.

Now we prove that problem (2.1) is equivalent to problem (2.3).

Lemma 2.20. For $t \in \Omega$ the problem (2.1) has a solution $(x(t), y(t), u(t))$ with $u(t) \in \tilde{T}_t(x(t))$ if and only if the problem (2.3) has a solution $(x(t), y(t), u(t))$ with $u(t) \in \tilde{T}_t(x(t))$, where

$$x(t) = J^{M_t(\cdot,x(t))}_{A_{1,t},\rho_t}(z(t)), \quad y(t) = J^{N_t(\cdot,y(t))}_{A_{2,t},\varrho_t}(w(t)) \tag{2.4}$$

and

$$\begin{aligned} z(t) &= A_{1,t}(x(t)) - \rho_t E_t(x(t), y(t)), \\ w(t) &= A_{2,t}(y(t)) - \varrho_t G(u(t), y(t)), \end{aligned}$$

where $\rho, \varrho : \Omega \longrightarrow (0, 1)$ are measurable mappings.

3 Main Results

In this section, we first discuss the existence thorem. And then we developed an algorithm for the problem and proved the convergence of the random sequence generated by given algorithm.

Theorem 3.1. Let (Ω, σ) be a measurable space. Let $A_i : H_i \times \Omega \longrightarrow H_i$ be a randomly $r_{i,t}$-strongly monotone and randomly $s_{i,t}$-Lipschitz continuous mapping for each $i = 1, 2, T : H_1 \times \Omega \longrightarrow \mathfrak{F}(H_1)$ be a fuzzy mapping induced by a set-valued mapping $\tilde{T} : H_1 \times \Omega \longrightarrow H_1$, and

$\alpha : H_1 \longrightarrow (0,1)$ and $\tilde{T}_{t,x(t)}(x(t)) \geq \alpha(x(t))$ satisfying the condition $(*)$. Let $\tilde{T} : H_1 \times \Omega \longrightarrow H_1$ be the randomly $\kappa_t - \widetilde{\mathcal{D}}$-Lipschitz continuous mapping induced by T, where $\widetilde{\mathcal{D}}$ is the Hausdorff pseudo metric on 2^{H_i}, for each $i = 1, 2$. Let $M : H_1 \times H_1 \times \Omega \longrightarrow 2^{H_1}$ be a randomly $(A_{1,t}, \eta_{1,t})$-monotone mapping with measurable mapping $m_1 : \Omega \longrightarrow (0,1)$ in the first variable and $N : H_2 \times H_2 \times \Omega \longrightarrow 2^{H_2}$ be a randomly $(A_{2,t}, \eta_{2,t})$-monotone mapping with measurable mapping $m_2 : \Omega \longrightarrow (0,1)$ in the first variable. Let $\eta_1 : H_1 \times H_1 \Omega \longrightarrow H_1$ be a randomly $\tau_{2,t}$-Lipschitz continuous mapping, $\eta_2 : H_2 \times H_2 \times \Omega \longrightarrow H_2$ be randomly $\tau_{2,t}$-Lipschitz continuous mapping, $E : H_1 \times H_2 \times \Omega \longrightarrow H_1$ be the randomly Lipschitz continuous mapping with respect to first variable with measurable mapping $\beta : \Omega \longrightarrow (0,1)$, and second argument with respect to the measurable mapping $\xi : \Omega \longrightarrow (0,1)$ and randomly $(\gamma_{1,t}, \alpha_{1,t})$-relaxed cocoercive with respect to $A_{1,t}$ and first variable of E_t with measurable mappings $\gamma, \alpha : \Omega \longrightarrow (0,1)$. Let $G : H_1 \times H_2 \times \Omega \longrightarrow H_2$ be the randomly Lipschitz continuous with respect to first and second variables with measurable mappings $\mu, \zeta : \Omega \longrightarrow (0,1)$, respectively. Let G be a randomly $(\gamma_{2,t}, \alpha_{2,t})$-relaxed cocoercive mapping with respect to $A_{2,t}$ with measurable mappings $\gamma_2, \alpha_2 : \Omega \longrightarrow (0,1)$, respectively. If in addition $\rho : \Omega \longrightarrow (0, \frac{r_{1,t}}{m_{1,t}})$ and $\varrho : \Omega \longrightarrow (0, \frac{r_{2,t}}{m_{2,t}})$ are measurable mappings and

$$\|J_{\rho_t, A_{1,t}}^{M_t(\cdot,x(t))}(z(t)) - J_{\rho_t, A_{1,t}}^{M_t(\cdot,y(t))}(z(t))\| \leq \upsilon_{1,t}\|x(t) - y(t)\|, \tag{3.1}$$

for all $(x(t), y(t), z(t), t) \in H_1 \times H_1 \times H_1 \times \Omega$,

$$\|J_{\varrho_t, A_{2,t}}^{N_t(\cdot,x(t))}(z(t)) - J_{\varrho_t, A_{2,t}}^{N_t(\cdot,y(t))}(z(t))\| \leq \upsilon_{2,t}\|x(t) - y(t)\|, \tag{3.2}$$

for all $(x(t), y(t), z(t), t) \in H_2 \times H_2 \times H_2 \times \Omega$, where $x, u : \Omega \longrightarrow H_1$ and $y : \Omega \longrightarrow H_2$ are measurable mappings, then problem (2.1) has a random solution $(x^*(t), y^*(t), u^*(t))$.

4 Iterative algorithms and convergence analysis

In this section, based on Lemma 2.20 and Nadler results [23], we shall construct a new class of iterative algorithms for solving problems (2.1) and discuss the convergence analysis of the algorithms.

Algorithm 4.1. Assume that $H_i, A_i, \eta_i, M, N, E, G, T, \tilde{T}$ are same as in the problem (2.1) for each $i = 1, 2$ and $x_0 : \Omega \longrightarrow H_1, y_0 : \Omega \longrightarrow H_2$ are measurable mappings. For $a : H_2 \longrightarrow (0,1)$, $n \geq 0$ and the random element $(x(t), y(t), u(t)) \in H_1 \times H_2 \times H_1$, we define the iterative sequences $\{x_n(t)\}, \{y_n(t)\}, \{u_n(t)\}$ by

$$x_{n+1}(t) = (1 - \lambda_n(t))x_n(t) + \lambda_n(t)\left[J_{\rho_t, A_{1,t}}^{M_t(\cdot,x_n(t))}(A_{1,t}(x_n(t)) - \rho_t E_t(x_n(t), y_n(t)))\right] + p_n(t), \tag{4.1}$$

$$y_{n+1}(t) = (1-\lambda_n(t))y_n(t) + \lambda_n(t)\left[J^{N_t(\cdot,y_n(t))}_{\varrho_t,A_{2,t}}(A_{2,t}(y_n(t)) - \varrho_t G_t(u_n(t),y_n(t)))\right] + q_n(t), \quad (4.2)$$

$$\tilde{T}_{t,x(t)}(u_n(t)) \geq a(x_n(t)),\ \|u_n(t)-u(t)\| \leq (1+\iota)\widetilde{\mathcal{D}}(\tilde{T}_t(x_n(t)),\tilde{T}_t(x(t))), \quad (4.3)$$

where $\rho, \varrho : \Omega \longrightarrow (0,1)$ are measurable, $\{\lambda_n(t)\}$ is a measurable sequence in $(0,1]$, and $p_n(t), q_n(t)$ are two random error sequences satisfying the same conditions in H_1 and H_2, respectively.

Lemma 4.2. [24] Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be three sequences of nonnegative real numbers satisfying the following conditions:

(i) $0 \leq b_n < 1, n = 0,1,2,\cdots$ and $\limsup_n b_n < 1$;

(ii) $\Sigma_{n=0}^{\infty} c_n < +\infty$;

(ii) $a_{n+1} \leq b_n a_n + c_n, n = 0,1,2,\cdots$.

Then $\lim_{n\longrightarrow\infty} a_n = 0$.

Theorem 4.3. Let $H_1, H_2, T_t, \tilde{T}_t, \eta_{1,t}, \eta_{2,t}, A_{1,t}, A_{2,t}, M_t, N_t, E_t, G_t$ be the same as in Theorem 3.1. Assume that all the conditions of Theorem 3.1 hold and

$$\limsup_n \lambda_n(t) < 1, \Sigma_{n=0}^{\infty}(\|p_n(t)\| + \|q_n(t)\|) < +\infty. \quad (4.4)$$

Then the random iterative sequences $(x_n(t), y_n(t))$ with $u_n(t) \in \tilde{T}_t(x(t))$ defined by Algorithm 4.1, converges strongly to the random solution $(x^*(t), y^*(t), u^*(t))$ of (2.1).

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