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EQUIVARIANT LOCAL INDEX AND SYMPLECTIC CUT

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1. Introduction

This is a survey article of [7]. In the joint work [1, 2, 3] with Fujita and Furuta we developed an index theory for Dirac-type operators on possibly non-compact Riemannian manifolds. We call the index in our theory the *local index* and also call its equivariant version the *equivariant local index*. As an application to Hamiltonian S^1-actions on prequantizable closed symplectic manifolds we can show that the equivariant Riemann-Roch index is obtained as the sum of the equivariant local indices for the inverse images of the integer lattice points by the moment map. When the lattice point is a regular value of the moment map we can compute its equivariant local index, see [3, 6]. So the problem is how to compute the equivariant local index when the lattice point is a critical value. The purpose of this paper is to give a formula for the equivariant local index for the reduced space in a symplectic cut space which is a special case of critical lattice points (Theorem 3.4).

This paper is organized as follows. In Section 2 we briefly recall the equivariant local index in the case of the Hamiltonian S^1-actions. After that we give the setting and the main theorem in Section 3.

Notation. In this paper we use the notation $\mathbb{C}_{(n)}$ for the irreducible representation of S^1 with weight n.

2. Equivariant local index

Let (M, ω) be a prequantizable Hamiltonian S^1-manifold and (L, ∇^L) an S^1-equivariant prequantum line bundle on (M, ω). We do not assume M is compact. Since all orbits are isotropic the restriction of (L, ∇^L) to each orbit is flat.

Definition 2.1. An orbit $\mathcal{O}$ is said to be *L-acyclic* if $H^0\left(\mathcal{O}; (L, \nabla^L)|_{\mathcal{O}}\right) = 0$.

Let V be an S^1-invariant open set whose complement is compact and which contains only L-acyclic orbits. For these data we give the following theorem.

Theorem 2.2 ([1, 2, 3]). *There exists an element* $\mathrm{ind}_{S^1}(M, V; L) \in R(S^1)$ *of the representation ring such that* $\mathrm{ind}_{S^1}(M, V; L)$ *satisfies the following properties:*

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(1) $\mathrm{ind}_{S^1}(M,V;L)$ *is invariant under continuous deformation of the data.*
(2) *If M is closed, then* $\mathrm{ind}_{S^1}(M,V;L)$ *is equal to the equivariant Riemann-Roch index* $\mathrm{ind}_{S^1}(M;L)$.
(3) *If M' is an S^1-invariant open neighborhood of $M\smallsetminus V$, then* $\mathrm{ind}_{S^1}(M,V;L)$ *satisfies the following excision property*

$$\mathrm{ind}_{S^1}(M,V;L) = \mathrm{ind}_{S^1}\left(M', M'\cap V; L|_{M'}\right).$$

(4) $\mathrm{ind}_{S^1}(M,V;L)$ *satisfies a product formula.*

We call $\mathrm{ind}_{S^1}(M,V;L)$ the *equivariant local index.*

Example 2.3. For small positive real number $\varepsilon > 0$ which is less than 1 let $D_\varepsilon\left(\mathbb{C}_{(1)}\right) = \{z \in \mathbb{C}_{(1)} \mid |z| < \varepsilon\}$ be the 2-dimensional disc of radius ε. As $(L, \nabla^L) \to (M,\omega)$ we consider

$$\left(D_\varepsilon\left(\mathbb{C}_{(1)}\right) \times \mathbb{C}_{(m)}, d + \frac{1}{2}(zd\bar{z} - \bar{z}dz)\right) \to \left(D_\varepsilon\left(\mathbb{C}_{(1)}\right), \frac{\sqrt{-1}}{2\pi} dz\wedge d\bar{z}\right).$$

First let us detect non L-acyclic orbits. Suppose the orbit $\mathcal{O}$ through $z \in D_\varepsilon\left(\mathbb{C}_{(1)}\right)$ has a non-trivial parallel section $s \in H^0\left(\mathcal{O}; (L,\nabla)|_{\mathcal{O}}\right)$. Then s satisfies the following equation

$$0 = \nabla^L_{\partial_\theta} s = \frac{\partial s}{\partial \theta} - 2\pi\sqrt{-1}r^2 s,$$

where we use the polar coordinates $z = re^{2\pi\sqrt{-1}\theta}$. Hence s is of the form $s(\theta) = s_0 e^{2\pi\sqrt{-1}r^2\theta}$ for some non-zero constant s_0. Since s is a global section on $\mathcal{O}$ s satisfies $s(0) = s(1)$. This implies $r = 0$.

Next, we put $V = D_\varepsilon\left(\mathbb{C}_{(1)}\right) \smallsetminus \{0\}$ and let us compute $\mathrm{ind}_{S^1}(M,V;L)$. We recall the definition of $\mathrm{ind}_{S^1}(N,V;L)$. For $t \geq 0$ consider the following perturbation of the Spin^c Dirac operator $D\colon \Gamma\left(\wedge^{0,*}T^*M \otimes L\right) \to \Gamma\left(\wedge^{0,*}T^*M \otimes L\right)$ associated with the standard Hermitian structure on $M = D_\varepsilon\left(\mathbb{C}_{(1)}\right)$

$$D_t = D + t\rho D_{S^1},$$

where ρ is a cut-off function of V and D_{S^1} is a first order formally self-adjoint differential operator of degree-one

$$D_{S^1} : \Gamma\left((\wedge^* T^*M^{0,1} \otimes L)|_V\right) \to \Gamma\left((\wedge^* T^*M^{0,1} \otimes L)|_V\right)$$

that satisfies the following conditions:

(1) D_{S^1} contains only derivatives along orbits.
(2) The restriction $D_{S^1}|_{\mathcal{O}}$ to an orbit $\mathcal{O}$ is the de Rham operator with coefficients in $L|_{\mathcal{O}}$.
(3) For any S^1-equivariant section u of the normal bundle $\nu_{\mathcal{O}}$ of $\mathcal{O}$ in M, D_{S^1} anti-commutes with the Clifford multiplication of u.

See [1, 2, 3] for more details. From the second condition and $\{0\}$ is the unique non L-acyclic orbit we can see $\ker\left(D_{S^1}|_{\mathcal{O}}\right) = 0$ for all orbits $\mathcal{O} \neq \{0\}$. Extend the complement of a neighborhood of 0 in $D_\varepsilon\left(\mathbb{C}_{(1)}\right)$ cylindrically so that all the data are translationally invariant. Then we showed in [1, 2] that for a sufficiently large t D_t is Fredholm, namely, $\ker D_t \cap L^2$ is finite dimensional and its super-dimension is independent of a sufficiently large t. So we define

$$\mathrm{ind}_{S^1}(M,V;L) = \ker D_t^0 \cap L^2 - \ker D_t^1 \cap L^2$$

for a sufficiently large t. In this case, by the direct computation using the Fourier expansion of s with respect to θ, we can show that

$$\ker D_t^0 \cap L^2 \cong \mathbb{C}, \quad \ker D_t^1 \cap L^2 = 0,$$

and $\ker D_t^0 \cap L^2$ is spanned by a certain L^2-function $a_0(r)$ on $D_\varepsilon(\mathbb{C}_{(1)})$ which depends only on $r = |z|$. Since the S^1-action on $\ker D_t^0 \cap L^2$ is given by pull-back and the S^1-action on the fiber is given by $\mathbb{C}_{(m)}$ we obtain

$$\begin{aligned}\mathrm{ind}_{S^1}(M,V;L) &= \mathrm{ind}_{S^1}\left(D_\varepsilon(\mathbb{C}_{(1)}), D_\varepsilon(\mathbb{C}_{(1)}) \smallsetminus \{0\}; D_\varepsilon(\mathbb{C}_{(1)}) \times \mathbb{C}_{(m)}\right) \\ &= \mathbb{C}_{(-m)}.\end{aligned}$$

For more details see [1, Remark 6.10], or [5, Section 5.3.2].

It is well-known that the lift of S^1-action on M to L defines the moment map $\mu\colon M \to \mathbb{R}$ by the Kostant formula

$$\mathcal{L}_X s = \nabla_X^L s + 2\pi\sqrt{-1}\mu s, \tag{2.1}$$

where s is a section of L, X is the vector field which generates the S^1-action on (M,ω), and $\mathcal{L}_X s$ is the Lie derivative which is defined by

$$\mathcal{L}_X s(x) = \left.\frac{d}{d\theta}\right|_{\theta=0} e^{-2\pi\sqrt{-1}\theta} s(e^{2\pi\sqrt{-1}\theta}x).$$

Lemma 2.4. *If an orbit $\mathcal{O}$ is not L-acyclic, namely, $H^0\left(\mathcal{O};(L,\nabla^L)|_{\mathcal{O}}\right) \neq 0$, then, $\mu(\mathcal{O}) \in \mathbb{Z}$.*

If M is closed, then we have the following localization formula for the equivariant Riemann-Roch index.

Corollary 2.5. *Suppose M is closed. For $i \in \mu(M) \cap \mathbb{Z}$ let V_i be an S^1-invariant open neighborhood of $\mu^{-1}(i)$ such that they are mutually disjoint, namely, $V_i \cap V_j \neq \emptyset$ for all $i \neq j$. Then,*

$$\mathrm{ind}_{S^1}(M;L) = \bigoplus_{i \in \mu(M)\cap\mathbb{Z}} \mathrm{ind}_{S^1}(V_i, V_i \cap V; L|_{V_i}). \tag{2.2}$$

3. The setting and the main theorem

Let (M,ω) be a Hamiltonian S^1-space with moment map $\mu\colon M \to \mathbb{R}$. For a real number n the cut space $\overline{M}_{\mu\leq n}$ of (M,ω) by the symplectic cutting [4] is the reduced space of the diagonal S^1-action on $(M,\omega) \times \left(\mathbb{C}_{(1)}, \frac{\sqrt{-1}}{2\pi} dz \wedge d\bar{z}\right)$, namely,

$$\overline{M}_{\mu\leq n} = \left\{ (x,z) \in (M,\omega) \times \left(\mathbb{C}_{(1)}, \frac{\sqrt{-1}}{2\pi} dz \wedge d\bar{z}\right) \middle| \mu(x) + |z|^2 = n \right\} / S^1.$$

We denote the reduced space $\mu^{-1}(n)/S^1$ by M_n.

Proposition 3.1. (1) *If S^1 acts on $\mu^{-1}(n)$ freely, then, $\overline{M}_{\mu\leq n}$ is a smooth Hamiltonian S^1-space. The S^1-action is given as*

$$t[x,z]=[tx,z] \tag{3.1}$$

for $t\in S^1$ and $[x,z]\in\overline{M}_{\mu\leq n}$.
(2) *Under the assumption in* (1), *the reduced space M_n and $\{x\in M \mid \mu(x)\leq n\}$ are symplectically embedded into $\overline{M}_{\mu\leq n}$ by $M_n\ni[x]\mapsto[x,0]\in\overline{M}_{\mu\leq n}$ and $\{x\in M\mid\mu(x)\leq n\}\ni x\mapsto\left[x,\sqrt{n-\mu(x)}\right]\in\overline{M}_{\mu\leq n}$, respectively. In particular, $\overline{M}_{\mu\leq n}$ can be identified with the disjoint union $\{x\in M\mid\mu(x)\leq n\}\coprod M_n$ and with this identification M_n is fixed by the S^1-action* (3.1).

Suppose that (M,ω) is equipped with a prequantum line bundle $(L,\nabla^L)\to(M,\omega)$ and the S^1-action lifts to (L,∇^L) in such a way that μ satisfies the Kostant formula (2.1).

Proposition 3.2. *If n is an integer and the S^1-action on $\mu^{-1}(n)$ is free, then $\overline{M}_{\mu\leq n}$ is prequantizable. In this case a prequantum line bundle $(\overline{L},\nabla^{\overline{L}})$ on $\overline{M}_{\mu\leq n}$ is given by*

$$(\overline{L},\nabla^{\overline{L}})=\left((L,\nabla^L)\otimes\mathbb{C}_{(n)}\right)\boxtimes\left(\mathbb{C}_{(1)}\times\mathbb{C}_{(0)},d+\frac{1}{2}(zd\bar{z}-\bar{z}dz)\right)\Big|_{\Phi^{-1}(0)}/S^1,$$

where Φ is the moment map $\Phi\colon M\times\mathbb{C}_{(1)}\to\mathbb{R}$ associated to the lift of the diagonal S^1-action which is written as $\Phi(x,z)=\mu(x)+|z|^2-n$, and the lift of the S^1-action (3.1) *on $\overline{M}_{\mu\leq n}$ to $(\overline{L},\nabla^{\overline{L}})$ is given by*

$$t[u\otimes v\boxtimes(z,w)]=[(tu)\otimes v\boxtimes(z,w)] \tag{3.2}$$

for $t\in S^1$ and $[u\otimes v\boxtimes(z,w)]\in\overline{L}$. The moment map $\overline{\mu}\colon\overline{M}_{\mu\leq n}\to\mathbb{R}$ associated with the lift (3.2) *is written as $\overline{\mu}([x,z])=\mu(x)=n-|z|^2$.*

See [4] for more details.

Remark 3.3. We denote the restriction of $(\overline{L},\nabla^{\overline{L}})$ to M_n by (L_n,∇^{L_n}). (L_n,∇^{L_n}) is a prequantum line bundle on M_n. The S^1-action (3.2) on L_n is given by the fiberwise multiplication with weight n. Recall that M_n is fixed by the S^1-action (3.1). See Proposition 3.1.

Suppose that n be an integer and the S^1-action on $\mu^{-1}(n)$ is free. Then, the cut space $\overline{M}_{\mu\leq n}$ becomes a prequantizable Hamiltonian S^1-manifold and the S^1-equivariant prequantum line bundle $(\overline{L},\nabla^{\overline{L}})$ is given by Proposition 3.1 and Proposition 3.2.

Suppose also that $\mu^{-1}(n)$ is compact. We take a sufficiently small S^1-invariant open neighborhood O of M_n in $\overline{M}_{\mu\leq n}$ so that the intersection $\overline{\mu}(O)\cap\mathbb{Z}$ consists of the unique point n. Then we can define the equivariant local index $\mathrm{ind}_{S^1}\left(O,O\smallsetminus M_n;\overline{L}|_O\right)$ of M_n in $\overline{M}_{\mu\leq n}$. We give the following formula for $\mathrm{ind}_{S^1}\left(O,O\smallsetminus M_n;\overline{L}|_O\right)$.

Theorem 3.4. *Let (M,ω), (L,∇^L), and μ be as above. Let n be an integer. Suppose S^1 acts on $\mu^{-1}(n)$ freely and $\mu^{-1}(n)$ is compact. Let O be a sufficiently small S^1-invariant open neighborhood of M_n in $\overline{M}_{\mu\leq n}$ which satisfies $\overline{\mu}(O)\cap\mathbb{Z}=\{n\}$. Then, the equivariant local index is given as*

$$\mathrm{ind}_{S^1}\left(O,O\smallsetminus M_n;\overline{L}|_O\right)=\mathrm{ind}(M_n;L_n)\mathbb{C}_{(n)},$$

where $\operatorname{ind}(M_n; L_n)$ *is the Riemann-Roch number of* M_n.

Remark 3.5. By replacing $\mathbb{C}_{(1)}$ with $\mathbb{C}_{(-1)}$ in the above construction we obtain the other cut space $\overline{M}_{\mu\geq n} = \{(x,z) \in M \times \mathbb{C}_{-1} \colon \mu(x) - |z|^2 = n\}/S^1$. Theorem 3.4 also holds for $\overline{M}_{\mu\geq n}$.

The outline of the proof of Theorem 3.4. By the definition of the symplectic cutting, the normal bundle ν of M_n in $\overline{M}_{\mu\leq n}$ is given by

$$\nu = \mu^{-1}(N) \times_{S^1} \mathbb{C}_{(1)}.$$

For a sufficiently small $\varepsilon > 0$ let $D_\varepsilon(\mathbb{C}_{(1)}) = \{z \in \mathbb{C}_{(1)} \colon |z| < \varepsilon\}$ be the open disc of radius ε. We put $D_\varepsilon(\nu) = \mu^{-1}(n) \times_{S^1} D_\varepsilon(\mathbb{C}_{(1)})$, and define an S^1-action on $D_\varepsilon(\nu)$ by

$$t[x,z] = [tx, z]. \tag{3.3}$$

Let $p \colon D_\varepsilon(\nu) \to M_n$ be the natural projection. We define a complex line bundle $L_{D_\varepsilon(\nu)}$ on $D_\varepsilon(\nu)$ by

$$L_{D_\varepsilon(\nu)} = p^* L_n \otimes \left(\mu^{-1}(n) \times_{S^1} (D_\varepsilon(\mathbb{C}_{(1)}) \times \mathbb{C}_{(0)})\right),$$

and define an lift of the S^1-action (3.3) to $L_{D_\varepsilon(\nu)}$ by

$$t\left(([x,z],[u\otimes v]) \otimes [x',z',w]\right) = \left(([tx,z],[(tu)\otimes v]) \otimes [tx',z',w]\right). \tag{3.4}$$

Then we can show that for a sufficiently small $\varepsilon > 0$ $L_{D_\varepsilon(\nu)}$ on $D_\varepsilon(\nu)$ is equivariantly identified with $\overline{L}$ restricted to certain neighborhood of M_n in $\overline{M}_{\mu\leq n}$. By using this identification and the equivariant version of the product formula [2, Theorem 8.8] we obtain

$$\begin{aligned}
&\operatorname{ind}_{S^1}\left(O, O \smallsetminus M_n; \overline{L}|_O\right)\\
&= \operatorname{ind}_{S^1}\left(D_\varepsilon(\nu),\ D_\varepsilon(\nu) \smallsetminus M_n; L_{D_\varepsilon(\nu)}\right)\\
&= \operatorname{ind}_{S^1}\left(M_n; L_n \otimes \mu^{-1}(n) \times_{S^1} \operatorname{ind}_{S^1}(D_\varepsilon(\mathbb{C}_{(1)}), D_\varepsilon(\mathbb{C}_{(1)}) \smallsetminus \{0\}; D_\varepsilon(\mathbb{C}_{(1)}) \times \mathbb{C}_{(0)})\right).
\end{aligned}$$

Note that the product formula for the S^1-equivariant local index holds since the S^1-action preserves all the data. See [3, Section 6.2] for more details. From Example 2.3 the equivariant local index $\operatorname{ind}_{S^1}(D_\varepsilon(\mathbb{C}_{(1)}), D_\varepsilon(\mathbb{C}_{(1)}) \smallsetminus \{0\}; D_\varepsilon(\mathbb{C}_{(1)}) \times \mathbb{C}_{(0)})$ is equal to $\mathbb{C}_{(0)}$. By definition, L_n is naturally identified with the restriction of $\overline{L}$ to M_n. With this identification we can see that the restriction of the S^1-action (3.2) to $L_n \to M_n$ is nothing but the fiberwise multiplication of t^{-n}. Since the S^1-action on $\operatorname{ind}_{S^1}(M_n; L_n)$ is defined by the pull-back, the S^1-action on $\operatorname{ind}_{S^1}(M_n; L_n)$ is given by the multiplication of t^n as we mentioned in Remark 3.3. This proves the theorem. □

Example 3.6 (Complex projective space). As $(L, \nabla) \to (M, \omega)$ we adopt

$$\left((\mathbb{C}_{(1)})^m \times \mathbb{C}_{(0)}, d + \frac{1}{2}\sum_{i=1}^m (z_i d\bar{z}_i - \bar{z}_i dz_i)\right) \to \left((\mathbb{C}_{(1)})^m, \frac{\sqrt{-1}}{2\pi}\sum_{i=1}^m dz_i \wedge d\bar{z}_i\right).$$

For $n = 1$ the obtained $\overline{M}_{\mu\leq n}$, $\overline{L}$, and M_n are $\mathbb{C}P^m$, $\mathcal{O}(1)$, and $\mathbb{C}P^{m-1}$, respectively. The induced S^1-actions on $\mathbb{C}P^m$ and $\mathcal{O}(1)$ are given by

$$\begin{aligned}
t[z_0 : z_1 : \cdots : z_m] &= [z_0 : tz_1 : \cdots : tz_m],\\
t[z_0 : z_1 : \cdots : z_m, w] &= [z_0 : tz_1 : \cdots : tz_m, w].
\end{aligned} \tag{3.5}$$

The moment map $\overline{\mu}$ associated to the S^1-action (3.5) is given by $\overline{\mu}([z_0 : \cdots : z_m]) = \sum_{i=1}^m |z_i|^2$. For $k = 0, 1$ let O_k be a sufficient small S^1-invariant open neighborhood of $\overline{\mu}^{-1}(k)$. Then the equivariant local index $\mathrm{ind}_{S^1}\left(O_k, O_k \smallsetminus \overline{\mu}^{-1}(k); \overline{L}|_{O_k}\right)$ is defined and By Corollary 2.2 the equivariant Riemann-Roch index $\mathrm{ind}_{S^1}\left(\overline{M}_{\mu\leq n}, \overline{L}\right)$ satisfies following equality

$$\begin{aligned}\mathrm{ind}_{S^1}\left(\overline{M}_{\mu\leq n}, \overline{L}\right) =& \mathrm{ind}_{S^1}\left(O_0, O_0 \smallsetminus \overline{\mu}^{-1}(0); \overline{L}|_{O_0}\right) \\ &+ \mathrm{ind}_{S^1}\left(O_1, O_1 \smallsetminus \overline{\mu}^{-1}(1); \overline{L}|_{O_1}\right).\end{aligned} \tag{3.6}$$

The left hand side is computed as

$$\mathrm{ind}_{S^1}\left(\overline{M}_{\mu\leq n}, \overline{L}\right) = \mathrm{ind}_{S^1}\left(\mathbb{C}P^m, \mathcal{O}(1)\right) = \mathbb{C}_{(0)} \oplus m\mathbb{C}_{(1)}. \tag{3.7}$$

For $k = 1$, since $\overline{\mu}^{-1}(1) = M_n$, by Theorem 3.4 $\mathrm{ind}_{S^1}\left(O_1, O_1 \smallsetminus \overline{\mu}^{-1}(1); \overline{L}|_{O_1}\right)$ is given as

$$\begin{aligned}\mathrm{ind}_{S^1}\left(O_1, O_1 \smallsetminus \overline{\mu}^{-1}(1); \overline{L}|_{O_1}\right) &= \mathrm{ind}_{S^1}\left(O_1, O_1 \smallsetminus M_n; \overline{L}|_{O_1}\right) \\ &= \mathrm{ind}(\mathbb{C}\mathrm{P}^{m-1}; \mathcal{O}(1))\mathbb{C}_{(1)} \\ &= m\mathbb{C}_{(1)}.\end{aligned} \tag{3.8}$$

For $k = 0$, it is easy to see that $\overline{\mu}^{-1}(0) = \{[z_0 : 0 : \cdots : 0]\}$ and $(\overline{L}, \nabla^{\overline{L}})|_{[z_0:0:\cdots:0]} \cong (\mathbb{C}_{(0)}, d + \frac{1}{2}(\bar{z}dz - zd\bar{z}))$. We can take O_0 in such a way that O_0 is identified with a sufficiently small open disc $D = \{(z_1, \ldots, z_m) \in \mathbb{C}^m \colon \sum_{i=1}^m |z_i|^2 \leq \varepsilon\}$ with S^1-action $t(z_1, \ldots, z_m) = (tz_1, \ldots, tz_m)$. Then, by (3.6), (3.7), and (3.8) we obtain the following formula

$$\mathrm{ind}_{S^1}\left(D, D \smallsetminus \{0\}; D \times \mathbb{C}_0\right) = \mathbb{C}_{(0)}.$$

In the case of $m = 1$ this formula can be obtained in [1, Remark 6.10] and [5, Section 5.3.2].

Example 3.7 (Exceptional divisor). Let n and $(L, \nabla) \to (M, \omega)$ be as in Example 3.6. Then the obtained cut space $\overline{M}_{\mu\geq n}$ is the blow-up $\widetilde{\mathbb{C}}^m$ of the origin in $\mathbb{C}^m$, and M_n and L_n are the exceptional divisor $\mathbb{C}P^{m-1}$ and $\mathcal{O}(n)$, respectively. We take a sufficiently small invariant open neighborhood O of M_n. Then, by Theorem 3.4 the equivariant local index $\mathrm{ind}_{S^1}\left(O, O \smallsetminus M_n; \overline{L}|_O\right)$ is given by

$$\mathrm{ind}_{S^1}\left(O, O \smallsetminus M_n; \overline{L}|_O\right) = \mathrm{ind}\left(\mathbb{C}\mathrm{P}^{m-1}; \mathcal{O}(n)\right)\mathbb{C}_{(n)} = \binom{m-1+n}{m-1}\mathbb{C}_{(n)}.$$

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