

## Stochastic Control of Fishery Resources Using Dynamical Models for Local Behaviour and Global Population

(局所的行動と大域的個体群の動態モデルを用いた水産資源の確率制御)

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### 1. Introduction

Ecologically-conscious fishery resources management is currently a major issue all over the world. Designing and carrying out an appropriate management plan of migratory fish species in a river, which are major inland fishery resources, requires comprehending hydrological and hydraulic characteristics of their habitats and migration processes. An example of such fish species is *Plecoglossus altivelis* (Ayu) living in Japan, which is a key aquatic species in the country from economical, ecological, and cultural viewpoints [1-3]. Huge transverse hydraulic structures installed across river cross-sections, which are often equipped with fishways to improve passage efficiency of fishes, serve as physical barriers for upstream fish migration [4]. A weir not equipped with any fishways or those with poorly designed fishways possibly fragments a river into hydraulically disconnected reaches [5]. Operating a large dam for human activities would significantly alter its downstream river environment and lead to regime shifts of hydraulic [6] and ecological conditions in rivers [7], both of which critically affect life histories of migratory fishes [8-9]. Establishment of an effective management strategy for the migratory fish species should consider complex ecological and environmental systems that affect their migration and habitation. Ecosystem dynamics is inherently subject to stochastic environmental noises resulting from complex internal and external interactions [10-11]. From this standpoint, stochastic process models can potentially serve as effective mathematical tools to analyze, predict, and control both local swimming behaviour and global population dynamics of migratory fish species.

Fish migration in water bodies have often been analyzed with individual-based models that track movements of individual fishes in water bodies [12, 13]. Most of such models have to *a priori* specify the swimming velocity as a function of hydraulic variables, which would actually be dynamically and adaptively controlled by the individual fish itself during its migration process. Yoshioka et al. [14] proposed a mathematical model of upstream fish migration of individual fishes in 1-D rivers based on a stochastic differential equation (SDE) combined with a dynamic programming principle [15]. Their mathematical model assumes that the optimal swimming velocity of individual fishes is the minimizer of a value function based on both biological and ecological considerations. Finding the optimal swimming velocity reduces to solving a Hamilton-Jacobi-Bellman equation (HJBE), which is a nonlinear and nonconservative partial differential equation (PDE) [15]. Mathematical and numerical analyses on solutions to the HJBE governing upstream fish migration from the viewpoint of viscosity solution [16] have been performed [17, 18]. The authors also analyzed stochastic population dynamics of released migratory fishes in a river system subject to predation pressure from waterfowls and fishing pressure from human using a different HJBE derived from an economic and ecological value function [19]. Both of the mathematical models, which are the migration dynamics models and the

population dynamics models, have individually been developed and analyzed; however, we consider that some links can be found between the two models, which is a main motivation of this paper.

The main objective of this paper is to present the above-mentioned migration dynamics model and population dynamics model, and to consider links between them. The rest of this paper is organized as follows. Section 2 briefly introduces a stochastic control theory to derive an HJBE from an SDE based on a dynamic programming principle. Section 3 presents the migration dynamics model and the population dynamics model discussed in this paper. Section 4 presents demonstrative computational examples of these models focusing on their applications to fishery resources management in Hii River, San-in area, Japan. Section 5 discusses links between the two models. Section 6 concludes this paper and presents future perspectives of our research.

## 2. Stochastic control theory

### 2.1 Stochastic differential equation

To formulate the upstream fish migration, all the processes and random variables are defined on the usual filtered complete probability space with a right-continuous filtration  $\mathcal{F}_t$  [15]. Consider an  $n$ -dimensional real-valued continuous stochastic process  $\mathbf{X}_t = [X_{i,t}]$  ( $1 \leq i \leq n$ ) in the bounded domain  $\Omega \in \mathbb{R}^n$  with the boundary  $\partial\Omega$  where  $t(\geq 0)$  is the time. The boundary  $\partial\Omega$  contains the absorbing boundary  $\partial\Omega_A$  and the reflecting boundary  $\partial\Omega_R$ , which do not intersect with each other. The stochastic process  $\mathbf{X}_t$  is assumed to be a controlled Markov process. It is assumed that the initial condition  $\mathbf{X}_0 = \mathbf{x} = [x_i]$  is deterministically specified at the time  $t=0$ . The governing equation of  $\mathbf{X}_t$  in the domain  $\Omega$  is the Itô's SDE

$$d\mathbf{X}_t = \mathbf{a}(t, \mathbf{X}_t, u_t) dt + b(t, \mathbf{X}_t, u_t) d\mathbf{B}_t \quad (1)$$

where  $\mathbf{B}_t = [B_{i,t}]$  is the  $n$ -dimensional standard Brownian motion [15],  $u_t$  is the control variable that belongs to an admissible set of control  $\mathcal{U} = L^\infty([0, +\infty); U)$  with the range  $U$ ,  $\mathbf{a} = [a_i]$  is the  $n$ -dimensional drift coefficient vector,  $b = [b_{i,j}]$  is the  $n \times n$ -dimensional diffusivity matrix that is assumed to be non-negative definite. The infinitesimal generator  $A^u$  associated with the SDE(1) is expressed for generic sufficiently regular function  $\phi = \phi(s, \mathbf{x})$  as

$$A^u \phi = \sum_{i=1}^n a_i \frac{\partial \phi}{\partial x_i} + \sum_{i,j=1}^n D_{i,j} \frac{\partial^2 \phi}{\partial x_i \partial x_j} \quad (2)$$

with  $D_{i,j} = \frac{1}{2} \sum_{k=1}^n b_{i,k} b_{k,j}$ . The variable  $u_t$  is assumed to be a Markov control.

### 2.2 Value function

In the stochastic control theory, the objective function to be maximized through choosing an appropriate control variable  $u_t$  is expressed as [15]

$$v(s, \mathbf{x}, u) = \int_s^{\bar{T}} f(t, \mathbf{X}_t, u_t) dt + g(\bar{T}, \mathbf{X}_{\bar{T}}), \quad \bar{T} = \inf(T, \tau) \quad (3)$$

with the first hitting time  $\tau$  defined as

$$\tau = \inf \{t | t > s, \mathbf{X}_s = \mathbf{x}, \mathbf{X}_t \in \partial\Omega_A\} \quad (4)$$

where  $f$  and  $g$  represent the profit (or the cost) per time unit and that gained at the terminal time  $T$ , respectively. The goal of the stochastic control problem is to find an optimal control  $u = u^* \in \mathcal{U}$  that maximizes an expectation of the objective function. The maximized objective function under the expectation is referred to as the value function, which is expressed as

$$\Phi(s, \mathbf{x}) = \sup_{u \in \mathcal{U}} E[v(s, \mathbf{x}, u)] = E[v(s, \mathbf{x}, u^*)] \quad (5)$$

with

$$u^* = \arg \max_{u \in \mathcal{U}} E[v(s, \mathbf{x}, u)] \quad (6)$$

where  $E[\cdot]$  represents the expectation.

### 2.3 Hamilton-Jacobi-Bellman equation

Application of the dynamic programming principle [15] to the value function in (5) leads to the governing equation of the value function  $\Phi$ , which is the HJBE expressed as

$$\frac{\partial \Phi}{\partial s} + \sup_{u \in \mathcal{U}} \{A^u \Phi + f(s, \mathbf{x}, u)\} = \frac{\partial \Phi}{\partial s} + A^{u^*} \Phi + f(s, \mathbf{x}, u^*) = 0 \text{ in } \Omega \quad (7)$$

subject to the boundary conditions

$$\frac{\partial \Phi}{\partial \mathbf{n}}(s, \mathbf{x}) = 0 \text{ for } 0 < s < T \text{ and } \mathbf{x} \in \partial\Omega_R \quad (8)$$

and

$$\Phi(s, \mathbf{x}) = g(s, \mathbf{x}) \text{ for } 0 < s < T \text{ and } \mathbf{x} \in \partial\Omega_A, \quad (9)$$

and the terminal condition

$$\Phi(T, \mathbf{x}) = g(s, \mathbf{x}) \text{ in } \Omega \quad (10)$$

where  $\frac{\partial}{\partial \mathbf{n}} = \sum_{i=1}^n n_i D_{i,j} \frac{\partial}{\partial x_j}$  and  $\mathbf{n} = [n_i]$  is the outward normal vector on the boundary  $\partial\Omega$ . For stationary problems where the coefficients and value functions are time-independent, (7) reduces to

$$A^{u^*} \Phi + f(s, \mathbf{x}, u^*) = 0 \text{ in } \Omega \quad (11)$$

subject to the boundary conditions (8) and (9).

## 3. Mathematical model

### 3.1 Migration dynamics model

The SDE that governs horizontally 2-D swimming behaviour of individual fishes in a river reach is considered in this section. In the domain  $\Omega \in \mathbb{R}^2$  of shallow water flows, the 2-D stochastic process  $\mathbf{X}_t$  describes a position of an individual fish at each time  $t$ . The swimming velocity  $\mathbf{u}_t = (u_{1,t}, u_{2,t})$  of the fish is considered as a control variable. The governing equation of the process  $\mathbf{X}_t$  is given as

$$d\mathbf{X}_t = (\mathbf{V}(t, \mathbf{X}_t) - \mathbf{u}_t)dt + b(t, \mathbf{X}_t, u_t)d\mathbf{B}_t \quad (12)$$

where  $\mathbf{V}$  is the 2-D flow velocity vector and the 2-D square matrix  $b$  modulates stochastic swimming behaviour of the fish. The SDE(12) is a spatially 2-D extension of the 1-D model [17]. The positive direction of  $\mathbf{u}$  is taken opposite to that of  $\mathbf{V}$  to focus on upstream fish migration. Experimental results suggest that the diffusivity matrix  $D$  would be a decreasing function of the swimming speed  $|\mathbf{u}|$  [20]; however, here it is assumed not to depend on  $|\mathbf{u}|$  for the sake of simplicity. It is also assumed that the coefficients  $\mathbf{V}$  and  $b$  are time-independent given functions, which can be valid when considering the migration with significantly shorter timescale than hydrological changes of river flows.

The range  $U$  of the control  $\mathbf{u}$  is set as the 2-D closed ball

$$|\mathbf{u}| \leq u^{(M)}, \quad (13)$$

which is based on the trivial fact that the fish has a biologically determined maximum swimming speed  $u^{(M)}$ . Yoshioka et al. [17, 21] considered an HJBE associated with the 1-D model assuming unbounded  $u^{(M)}$  and found that to well-pose the problem requires regularization of a nonlinear term, which “artificially” truncates the swimming speed. Therefore, boundedness of  $u^{(M)}$  is a reasonable assumption from both mathematical and biological viewpoints.

The value function  $\Phi = \Phi(\mathbf{x})$ , which is assumed to be time-independent, is set as

$$\Phi(\mathbf{x}) = \sup_{\mathbf{u} \in U} E[v(\mathbf{x}, \mathbf{u})] \quad (14)$$

with the objective function

$$v(\mathbf{x}, \mathbf{u}) = -\int_0^\tau f(\mathbf{u}_t)dt + g(\mathbf{X}_\tau) \quad (15)$$

where

$$f(\mathbf{u}_t) = \frac{1}{m+1} |\mathbf{u}_t|^{m+1} \quad (16)$$

with  $m \geq 1$  represents the consumed physiological energy per time during the migration, which is assumed to be an increasing and convex function of the swimming speed following the theoretical and experimental results [22, 23], and  $g(\mathbf{X}_\tau)$  represents the profit obtained when approaching the absorbing boundary  $\partial\Omega_A$  at the first hitting time  $\tau$ . The absorbing boundary  $\partial\Omega_A$  is decomposed into the upstream boundary  $\partial\Omega_{AU}$  where the flow is inward ( $\mathbf{V} \cdot \mathbf{n} < 0$ ) and the downstream boundary  $\partial\Omega_{AD}$  where the flow is outward ( $\mathbf{V} \cdot \mathbf{n} > 0$ ). The reflecting boundary  $\partial\Omega_R$  in the present case represents the wall boundary where the flow does not penetrate ( $\mathbf{V} \cdot \mathbf{n} = 0$ ). It is also assumed that no profit is obtained at the downstream boundary  $\partial\Omega_{AD}$  ( $g = 0$ ).

Application of the dynamic programming principle to (14) leads to the HJBE

$$\sum_{i=1}^n (V_i - u_i^*) \frac{\partial \Phi}{\partial x_i} + \sum_{i,j=1}^n D_{i,j} \frac{\partial^2 \Phi}{\partial x_i \partial x_j} - \frac{1}{m+1} |u^*|^{m+1} = 0 \quad \text{in } \Omega \quad (17)$$

with the optimal swimming velocity  $\mathbf{u}^*$  as a function of the gradient of the value function  $\nabla \Phi$ :

$$\mathbf{u}^* = -\frac{\nabla\Phi}{|\nabla\Phi|} \min\left\{u^{(M)}, |\nabla\Phi|^{\frac{1}{m}}\right\} \text{ in } \Omega. \quad (18)$$

The gradient  $\nabla\Phi$  thus determines both the direction and magnitude of the optimal swimming velocity  $\mathbf{u}^*$ . Heuristically, the fish swims with the maximum swimming speed  $u^{(M)}$  where  $|\nabla\Phi|$  is larger than  $u^{(M)}$  and swims with the speed  $|\nabla\Phi|^{\frac{1}{m}}$  when  $|\nabla\Phi|$  is smaller than  $u^{(M)}$ .

### 3.2 Population dynamics model

The stochastic process model for annual population dynamics of released *P. altivelis* in a river is presented following Yaegashi et al. [19]. Let  $(0, T)$  with the terminal time  $T$  be a period during which the population dynamics of released *P. altivelis* in a river is considered. The initial time  $t=0$  and the terminal time  $t=T$  are set in the spring and the autumn within a year, respectively. The total biomass of *P. altivelis* in this river at the time  $t$  is denoted as the 1-D non-negative continuous stochastic process  $X_t$ . The initial condition  $X_0$  at the time  $t=0$ , which is the release amount of the juveniles, is assumed to be deterministic. This is valid for the case where most part of the population of *P. altivelis* in the river is introduced through intensive release events. The initial population  $X_0$  is expressed as  $X_0 = NW_0$  where  $N$  is the number of individuals released at the time  $t=0$  and  $W_0$  is the average weight of the juveniles at that time. The governing Itô's SDE of the stochastic process  $X_t$  is given as

$$dX_t = (a(t, X_t) - l_t X_t) dt + b(t, X_t) dB_t \text{ with } l_t = R + k(u_t) + \chi_{\{t \geq T_c\}} c_t \quad (19)$$

and the growth-curve based Verhulst model [24, 25]

$$a(t, x) = r(1 - K^{-1}X_t)X_t \text{ and } b(t, x) = \sigma X_t \quad (20)$$

where  $B_t$  is the 1-D standard Brownian motion [15],  $R$  is the natural mortality rate,  $k(u_t)$  is the predation pressure from a waterfowl as a function of the effort to reduce the pressure  $u = u_t$ ,  $c$  is the fishing pressure from human,  $T_c < T$  is the opening time of harvesting *P. altivelis*, and  $\chi_S$  is the characteristic function for generic set  $S$ . The capacity  $K$ , the intrinsic growth rate  $r$ , and the noise intensity  $\sigma$  are positive model parameters. The noise intensity  $\sigma$  implicitly models influences of the natural and artificial environmental changes in the river to the population dynamics as a multiplicative noise [26]. Assuming that the population dynamics is limited by the released amount rather than by the environmental capacity, the parameter  $K$  is expressed as [19]

$$K = mX_0 = mNW_0 \quad (21)$$

with a positive constant  $m$ . This assumption can be valid for a river where the population of released *P. altivelis* dominates the natural counterparts and they do not saturate in the habitat.

The variables  $u$  and  $c$  are taken as the controls variables in the model. The admissible set of the controls  $u$  and  $c$  are  $\mathcal{U} = L^\infty([0, +\infty); U)$  and  $\mathcal{C} = L^\infty([0, +\infty); C)$ , respectively. The range  $U$  and  $C$  of the controls  $u$  and  $c$  are specified as

$$U = \{u | 0 \leq u \leq 1\} \text{ and } C = \{c | 0 \leq c \leq c_M\}, \quad (22)$$

respectively where  $c_M$  is the maximum fishing pressure. The objective function to be maximized

$v = v(s, x, u, c)$  is assumed to represent the profit of the local fishery cooperatives. The objective function is proposed as

$$v = \int_s^T \left\{ -f(u_t) + \chi_{\{t \geq T_i\}} \theta c_t X_t \right\} dt - \eta X_0 \quad (23)$$

where  $f(\geq 0)$  with  $f(0) = 0$  is an increasing function,  $\theta$  is the price of matured *P. altivelis* per weight and  $\eta$  is the cost of larval *P. altivelis* per weight, both of which are assumed to be constant. The first and second integrands of Eq.(23) are the cost of operating countermeasures to exterminate the predator and the total benefit of harvesting *P. altivelis*, respectively. The term  $-\eta X_0$  represents the cost to buy larval *P. altivelis* to be released at the time  $t = 0$ . The two functions  $k(u)$  and  $f(u)$  on the control variable  $u$  are simply specified as

$$k(u) = k_0(1 - \alpha u) \text{ and } f(u) = \omega u, \quad (24)$$

respectively where  $k_0$  is the predation pressure from the waterfowl without any countermeasures,  $\alpha$  with  $0 < \alpha \leq 1$  modulates efficiency to decrease the predation pressure with the control  $u$ , and  $\omega$  modulates the cost to decrease the predation pressure.

The value function is presented as

$$\Phi(s, x) = \sup_{u \in \mathcal{U}, c \in \mathcal{C}} E[v(s, x, u, c)] \quad (25)$$

whose governing HJBE is

$$\frac{\partial \Phi}{\partial s} + \sup_{u \in \mathcal{U}, c \in \mathcal{C}} \left\{ A^{u,c} \Phi - f(u) + \chi_{\{s \geq T_i\}} \theta c x \right\} = \frac{\partial \Phi}{\partial s} + A^{u^*,c^*} \Phi - f(u^*) + \chi_{\{s \geq T_i\}} \theta c^* x = 0 \text{ in } \Omega \quad (26)$$

where the optimal controls  $u^*$  and  $c^*$  via the value function  $\Phi$  as

$$u^* = 1 (k_0 \alpha \gamma - \omega \geq 0), u^* = 0 \text{ (Otherwise)} \text{ and } c^* = c_M (\theta x - \gamma \geq 0), c^* = 0 \text{ (Otherwise)}, \quad (27)$$

respectively where  $\gamma = x \frac{\partial \Phi}{\partial x}$ . Both of the optimal controls are thus the bang-bang type. The domain of the population of *P. altivelis*  $x$  is set as  $\Omega = (0, L)$  with  $L (> 0)$  determined later. Substituting  $u = u^*$  and  $c = c^*$  into (26) fully specifies the HJBE in the domain  $\Omega$ . The terminal condition for the HJBE is set as  $\Phi_{s=T} = 0$  and the boundary conditions as  $V_{x=0} = 0$  and  $\left. \frac{\partial \Phi}{\partial x} \right|_{x=L} = 0$ . Solving the HJBE yields the optimal controls  $u^*$  and  $c^*$  over  $(0, T) \times (0, L)$ .

## 4. Applications

### 4.1 Study area

Demonstrative application examples of the migration dynamics model and population dynamics model focusing on *P. altivelis* in Hii River, San-in area, Japan are presented. Population of the fish in the river is thought to have been considerably decreasing due to degradation of river environment by dam and weir constructions and excessive predation pressure from waterfowls: *Phalacrocorax carbo* (Great Cormorant) [27, 28] (Photo. 1). The total length of the mainstream and the catchment area of Hii River are 153 (km) and 2,070 (km<sup>2</sup>), respectively [29]. Hydrological characteristics of Hii River is



**Photo 1:** *P. altivelis* (left panel) and *P. carbo* (right panel).  
(The photo of *P. carbo* is taken from <http://www.yunphoto.net>).

described and analyzed in detail in Sato et al. [30, 31], and not explained in this paper. Two downstream brackish lakes named Lake Shinji and Lake Nakaumi from upstream are connected to the river. Being different from the cases of the rivers that directly flow into the sea, hatched larvae of *P. altivelis* are thought to descend to Lake Shinji; however, it is not well-known where they survive during winter season.

#### 4.2 Migration dynamics model

The HJBE for the migration dynamics model is applied to numerical computation of spatial distribution of the optimal swimming velocity of individual *P. altivelis* in the downstream area of Yoshii Weir installed at a midstream reach of Hii River, which is a weir to control channel erosion with the width of 80 (m) and the height of 3 (m) (**Photo 2**). A field survey carried out by the authors during June, 2015 found that the mainstream of Hii River has a number of weirs and Yoshii Weir is relatively larger one among them. **Photo 3** shows fishways currently that are installed to Yoshii Weir. The previous Yoshii fishway consisted solely of the pool-type fishway. Downstream pools of this fishway suffered from severe depositions of fine soil particles. This might be due to its geometry of shaping flow recirculation that traps the particles. Such depositions were considered to be a cause of degrading fluid transport capacity of the fishway, which might have further lead to degradation of its attraction ability and passage efficiency for migratory fish species. The downstream part of the pool-type fishway was renovated in 2013 in order to reduce the sediment deposition. Around the same time, the nature-like fishway was installed at the same side of the weir so that migratory fishes can more efficiently find the way to passage the weir. Currently, Hii River Fisheries Cooperatives, who manage fishery resources in middle and upstream reaches of Hii River, are concerned with attraction ability and passage efficiency of the renovated Yoshi Weir. Assessing attraction ability and passage efficiency of the weir is therefore a highly important issue.

A 2-D time-independent shallow water flow field in the domain is numerically computed with the 2-D shallow water equations using the verified finite element/volume scheme [32]. The computational domain was extracted from Google Earth (Google Inc., Mountain View, Calif.) has been discretized into 57,443 triangular elements and 113,609 nodes using the free software named VORO (available at <http://www.yss-aya.com/voro.html>). The HJBE is numerically solved with a 2-D counterpart of the Petrov-Galerkin finite element scheme [33] that has already been verified with simpler HJBEs. The

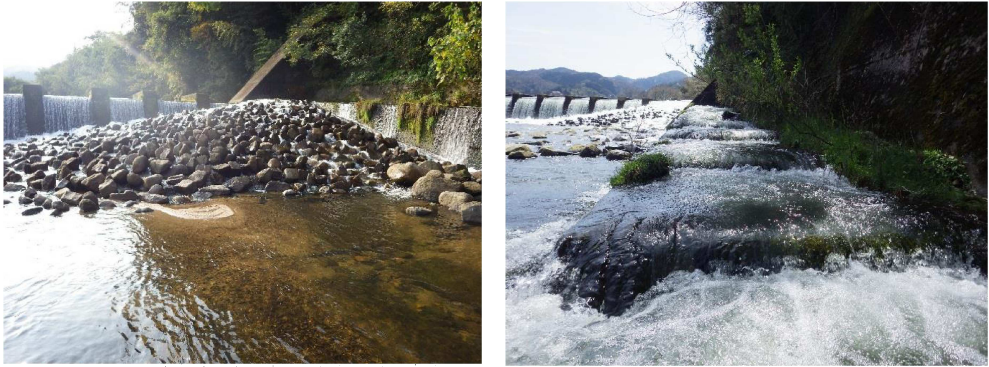
maximum swimming speed  $u^{(M)}$  is set as 2 (m/s), which is approximately same with the burst swimming speed observed at laboratorial experimental results [34]. The parameter  $m$  for the cost function is set to be 3 considering the identification result based on laboratorial experiments [17]. The downstream-end of the domain is considered as the downstream boundary  $\partial\Omega_{AD}$ . The entrance of the pool-type fishway and the top of the nature-like fishway where the individual fishes can passage are considered as the upstream boundary  $\partial\Omega_{AU}$ . The remaining parts of the boundary  $\partial\Omega$  is considered as the reflecting boundary  $\partial\Omega_R$ . The value function  $\Phi$  is specified as  $\Phi=0$  on  $\partial\Omega_{AD}$  and  $\Phi=P_0$  ( $=const > 0$ ) on  $\partial\Omega_{AU}$  where  $P_0$  is the ecological profit to be gained on  $\partial\Omega_{AU}$ . The parameter  $P_0$  is set as  $P_0 = 20,000$  ( $m^4/s^3$ ). Preliminary computational investigations implied that the optimal swimming velocity  $\mathbf{u}^*$  does not significantly depend on the  $P_0 > 10,000$  ( $m^4/s^3$ ); namely,  $\mathbf{u}^*$  is almost independent of  $P_0$  when it is sufficiently large. This tendency of  $\mathbf{u}^*$  is consistent with the theoretical analysis results for the 1-D model [18]. A steady state numerical solution is computed with a Picard iteration method for solving nonlinear equations.

Figures 1(a) through 1(d) show the computational results with the migration dynamics model. Figures 1(a) and 1(b) present the computed flow field based on the shallow water equations, which qualitatively agree with field observation results. Figure 1(c) shows the computed value function whose maximum value is normalized to 1 and the optimal swimming velocity vectors. Figure 1(d) shows the expectation of the first hitting time  $\tau$  of the individual fishes from each point in the domain  $\Omega$  to either of the two open boundaries (the entrance of the pool-type fishway and the top of the nature-like fishway). Figure 1(d) indicates that the computational results show that the first-hitting time has a sharp interior layer along the downstream of Yoshii Weir. As indicated in Figure 1(c), this is considered to be due to that individual fishes are attracted to the currents from Yoshi Weir serving as a reflecting boundary. The computational results imply that the current from the pool-type fishway is too weak to attract the migrating fishes. The individual fishes at downstream of the layer may be able to find the entrance of the pool-type fishway with non-zero, but very small probability. In addition, the flow speed in the nature-like fishway is too high for fishes to ascend: exceeding almost 3 (m/s), which is larger than  $u^{(M)} = 2$  (m/s). The presented computational results suggest a possibility that the current fishways are not effectively working as passages for *P. altivelis*.

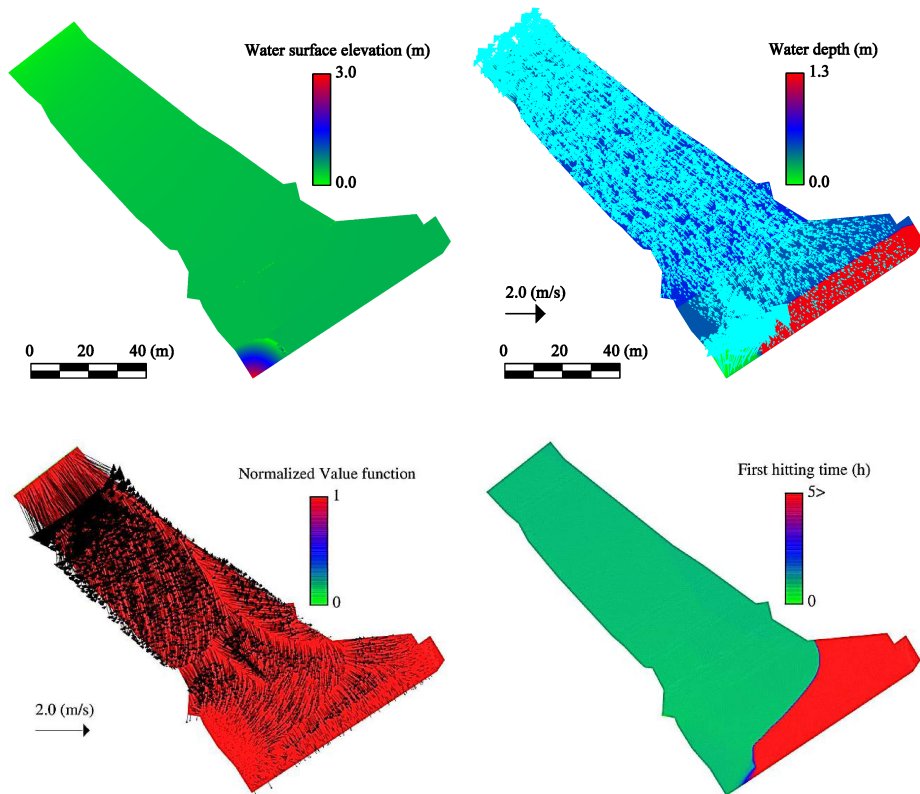


Photo 2: Plane view of Yoshii Weir and associated fishways.





**Photo 3:** The nature-like fishway (left panel) and the pool-type fishway (right panel). Both of the photographs are taken from downstream.



**Figure 1:** Computed water depth (upper left panel), water surface elevation and horizontal flow velocity vectors (upper right panel), value function and the optimal swimming velocity vectors (lower left panel), and first hitting time to the fishways (lower right panel).

### 4.3 Population dynamics model

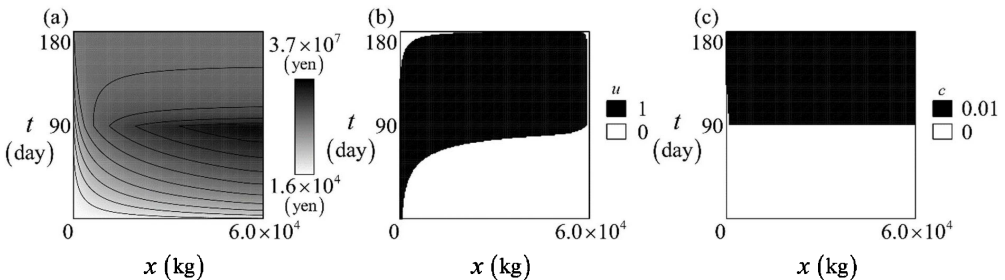
The population dynamics model is applied to numerical computation of optimal strategy for harvesting and extermination of *P. carbo* in order to sustain released *P. altivelis* population in Hii River. **Table 1** summarizes the parameter values determined based on information from officers in Hii River Fishery Cooperatives. On the other parameters, the natural mortality rate  $R$  is estimated as  $4.6 \times 10^{-3}$  (1/day) [35]. The value of  $\alpha$  is estimated as 0.50 [19]. The maximum fishing pressure  $c_M$  is set as 0.01 (1/day) following Yaegashi et al. [19] and it has preliminarily been checked that this parameter does not significantly affect quality of the value function and the controls than the other parameters. Assuming that the SDE (19) with  $k = c_M = 0$  admits a non-trivial and non-singular probability density for large  $T$  [36], the range of the noise intensity  $\sigma$  is set as  $0.04 \leq \sigma \leq 0.32$  (1/day<sup>1/2</sup>). The value of the parameter  $k_0$  is estimated from an exact solution to a deterministic counterpart of the SDE(19) with  $c_M = 0$  and given  $X_0$  for each computation [19]. Based on the weights of matured and released *P. altivelis* [37, 38] the parameters  $r$ ,  $\sigma$ , and  $X_0$  are set as  $6.8 \times 10^{-2}$  (1/day), 0.18 (1/day<sup>1/2</sup>), and 2,250 (kg), respectively. The domain of the population of *P. altivelis*  $x$  is set as  $\Omega = (0, 6.0 \times 10^4)$  (kg) and is uniformly discretized into a mesh with 300 elements and 301 nodes.

The time increment for temporal integration is 0.01 (day). The HJBE(26) is numerically solved with the verified Petrov-Galerkin finite element scheme [33]. Increasing computational resolution in space and time does not significantly affect the computational results presented below.

The optimal management strategy is numerically computed for a set of parameters as a demonstrative example. **Figures 2(a)** through **2(c)** plot the computed value function  $\Phi$  and the optimal controls  $u^*$  and  $c^*$ , respectively. **Figure 2(a)** shows that the value function  $\Phi$  does not have spurious oscillations, indicating that the numerical scheme can reasonably handle the HJBE. **Figure 2(b)** shows that the optimal strategy for the extermination of *P. carbo* is to intensively reduce the predation pressure mainly after the opening time  $T_c$  except the terminal time  $T$ , and before the opening time  $T_c$  for the case of small biomass of *P. altivelis*. **Figure 2(c)** indicates that the optimal harvesting strategy is to harvest *P. altivelis* after the opening time  $T_c$  except for small population  $x$ .

**Table 1:** Specified values of the model parameters in the population dynamics model.

$T$ (day)	$T_c$ (day)	$m$ (-)	$W_0$ (kg)	$\eta$ (yen/kg)	$\theta$ (yen/kg)
180	90	5.4	$9.4 \times 10^{-3}$	$4.4 \times 10^3$	$4.0 \times 10^3$



**Figure 2:** Computational results of (a) value function  $\Phi$ , (b) optimal effort to reduce the predation pressure  $u^*$ , and (c) optimal fishing pressure  $c^*$ .

### **5. Links between the two models**

The migration dynamics model and the population dynamics model deal with the dynamics of fishery resources in rivers based on different spatial and temporal scales. The migration dynamics model tracks local swimming behaviour of migratory individual fishes in river flows, while the population dynamics model assesses temporal evolution of the population of the fishes in rivers as habitats. There should be some links between the two models since they describe the same dynamics from different viewpoints. The authors consider that it is possible to see the population dynamics model as a coarse-scaled migration dynamics model. Spatially-distributed statistics, such as ascending probability and first hitting time of individual fishes for physical barriers, computed with the migration dynamics model would be used for identifying the model parameters  $r$ ,  $K$ , and  $\sigma$  in the population dynamics model. In addition, the above-mentioned two statistics are directly related with migration rate of the fishes between habitats partially fragmented by physical barriers. Considering hydraulic regimes at and around the physical barriers can support reasonably evaluating their attraction ability and passage efficiency. A multi-habitat counterpart of the population dynamics model, as presented in the literature [39], can then be developed with such local and crucial ecological information.

### **6. Conclusions**

A stochastic control theory was applied to dynamic mathematical modelling of local migration and global population dynamics of migratory fishes in river environment. The migration dynamics model assumed an energy minimization principle to determine the optimal swimming velocity of individual fishes in shallow water flow fields. The population dynamics model considers temporal biomass changes of released fishery resources in river environment subject to harvest by human and predation from waterfowls. Linkages between the two models were discussed focusing on physical barriers installed across river cross-sections.

Future researches will address mathematical and numerical analyses of the models, their further practical applications, and their improvements for more realistic analysis of fishery resources dynamics. For example, objective functions, coefficients, and parameters involved in each mathematical model should more close to realistic based on available field survey data and testimonies from field workers. So far, Yoshioka et al. [40] addressed identification of more biologically reasonable cost function for the migration dynamics model, which has been applied to a number of migratory fish species.

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