Regularity of solutions to the stationary transport equation with the incoming boundary data

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Summary

In this thesis, we discuss regularity of solutions to a boundary value problem of the Stationary Transport Equation (STE), which is a mathematical model for describing the stationary state of density of particles with absorption and scattering in a media, and also discuss an inverse problem to determine an unknown coefficient from discontinuous boundary measurements.

The Radiative Transport Equation (RTE) is known to be an equation which describes propagation of particles, such as neutrons and photons, with absorption and scattering in a media. In this thesis, we regard RTE as a governing equation of the Diffused Optical Tomography (DOT), which is an medical imaging technique in next generations using near infrared light, and in particular we focus on regularity of solutions to STE, which is the stationary case of RTE, in order to solve an inverse problem to determine an unknown coefficient with boundary measurements. We propose a mathematical way of solving this inverse problem by observing jumps of boundary measurements and using the X-ray transform. In order to apply our analysis to DOT, we should consider the case where the coefficients are bounded and piecewise continuous, which corresponds to discontinuity of biological tissues, and where the incoming boundary data is also bounded.

In this thesis, we consider a boundary value problem of the stationary transport equation in a bounded convex domain with the incoming boundary condition, and we obtain two results on regularity of its solutions. The first result is to describe discontinuity of the solution which arises from discontinuous points of the incoming boundary condition, and we show the exponential decay of a jump of the solution on a discontinuous point. The decay gives an idea for solving an inverse problem to determine an unknown coefficient from boundary measurements of the solution to the boundary value problem. The second one is to give a precise estimate of solutions to STE in Sobolev spaces in order to justify its numerical analysis with the discontinuous Galerkin method. We consider a boundary value problem in a bounded convex domain of two dimensions, and we obtain a $W^{1,p}$ estimate for the solution with $1 \leq p < p_m$ under suitable assumptions. Here the constant p_m depends only on the shape of the domain, and $p_m > 2$ if the domain is strictly convex. This result states that we give a sufficient condition for the solution to STE to belong to the Sobolev space H^1, which is required in previous numerical analysis of the discontinuous Galerkin method.