Studies on Kernel-Based System Identification

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Abstract

One of the main difficulties in constructing a model for dynamical systems is to tune the model complexity. This problem has been discussed since 1970's, and recently a new approach which is called the kernel-based system identification has been proposed. As its name suggests, the kernel-based system identification estimates the impulse response of the target system using the kernel method which has been developed in the machine learning field. Kernel-based system identification methods encode a priori knowledge on the system to positive semidefinite bivariate functions which are called kernels. A typical example of such knowledge is exponential decay of the impulse response. Many works report that the identification accuracy is significantly improved by encoding a priori knowledge.

Even though the effectiveness of kernel-based system identification methods has been reported, there are many problems to be discussed. This thesis mainly focuses on the following two problems;
1) how to encode the smoothness of the impulse response at time zero, and
2) how to design the input sequence for the kernel-based system identification methods.
Details about these problems are explained in the following.

In the former part of this thesis, we consider the estimation of continuous-time impulse responses.
Since we only consider causal systems, the impulse response is zero when the time is negative. The smoothness of the impulse response at time zero is determined by the relative degree of the system, and sometimes it is known in advance. This suggests that designing a kernel based on knowledge about the relative degree would improve the identification accuracy. This thesis proposes one simple way to encode the relative degree information to the kernel. In particular, the proposed kernel is a natural extension of the Tuned/Correlated (TC) kernel which is one of the most famous kernels in the system identification field. In more detail, we regard the TC kernel as the composition of the coordinate transformation and the spline kernel, and show that designing another coordinate transformation enables to encode the relative degree to the kernel.
In the latter part of this thesis we focus on discrete-time cases, and consider the input design problem for the kernel-based system identification. The input design plays an important role in system identification, and many methods have been proposed since 1970's. However, these methods are for classical Prediction Error Methods (PEM), and are not appropriate for kernel-based identification methods. This is because kernel methods enjoy a priori knowledge on the system. The information obtained through the

experiment depends on a priori knowledge as well known in the information theory, thus the input should also be designed based on this knowledge. There are few works on this topic, and the latter part of this thesis discusses the input design problem for the kernel-based system identification. First, the input design method based on the mutual information is proposed. The mutual information is a fundamental quantity in the information theory, and is a measure of information obtained through the experiment. It is shown that this mutual information based method is effective not only for linear systems but also Linear-Parameter-Varying (LPV) systems. Second, the goal-oriented input design method is proposed. The mutual information based method focuses on the impulse response itself, however other quantities derived from the impulse response (e.g., transient or steady response to the unit step input) are more important in some cases. In the goal-oriented method, we focus on linear functionals of the impulse response and minimize the expected errors of these values. Finally, the input design for the frequency response is proposed. The frequency response of the system is given by the Fourier transform of the impulse response and useful information for classical controller design methods. This method focuses on the gains and phases of frequency response rather than the impulse response itself. Note that the gains and phases are not linear functionals of the impulse response. This suggests that some modifications from the goal-oriented method are required. This thesis proposes these three input design methods whose purposes are all different. The effectiveness of each proposed input design method is demonstrated through numerical examples. All of them outperform the white random signal which is known to be a reasonable input for system identification.