Theory of Feynman-alpha technique with masking window for accelerator-driven systems

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Abstract

Recently, a modified Feynman-alpha technique for the subcritical system driven by periodically triggered neutron bursts was developed. One of the main features of this technique is utilization of a simple formula that is advantageous in evaluating the subcriticality. However, owing to the absence of the theory of this technique, this feature has not been fully investigated yet. In the present study, a theory of this technique is provided. Furthermore, the experimental conditions under which the simple formula works are discussed to apply this technique to the subcriticality monitor for the accelerator-driven system.

Keywords: Feynman-alpha technique, Subcriticality, Accelerator-driven system, Subcriticality monitor

Nomenclature

$\lambda_{\mathrm{s}} \equiv v\Sigma_{\mathrm{s}}$	=	probability that one neutron is scattered per unit time.
$\lambda_{\mathrm{c}} \equiv v\Sigma_{\mathrm{c}}$	=	probability that one neutron is captured by materials excluding neutron detector per unit time.
$\lambda_{\mathrm{f}} \equiv v\Sigma_{\mathrm{f}}$	=	probability that one neutron induces a fission reaction per unit time.

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$\lambda_{\mathrm{d}} \equiv v\Sigma_{\mathrm{d}}$	=	probability that one neutron is captured by neutron detector per unit time.
v	=	velocity of neutrons (constant value).
Σ_{s}	=	macroscopic cross section of scattering reaction.
Σ_{c}	=	macroscopic cross section of capture reaction in materials excluding neutron detector.
Σ_{f}	=	macroscopic cross section of fission reaction.
Σ_{d}	=	macroscopic cross section of capture reaction in neutron detector.
$p_{\mathrm{f}}(n,c)$	=	probability that n prompt neutrons and c delayed neutron precursors are born in one fission reaction.
$\langle \nu_{\mathrm{p}} \rangle$	=	$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} n p_{\mathrm{f}}(n,c)$, i.e., first order moment of number of prompt neutrons born in one fission reaction.
$\langle \nu_{\mathrm{d}} \rangle$	=	$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} c p_{\mathrm{f}}(n,c)$, i.e., first order moment of number of delayed neutrons born in one fission reaction.
$\langle \nu_{\mathrm{p}}(\nu_{\mathrm{p}}-1) \rangle$	=	$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} n(n-1) p_{\mathrm{f}}(n,c)$, i.e., second order factorial moment of number of prompt neutrons born in one fission reaction.
$\langle \nu_{\mathrm{p}}\nu_{\mathrm{d}} \rangle$	=	$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} nc p_{\mathrm{f}}(n,c)$, i.e., second order moment of product of numbers of prompt neutrons and delayed neutrons born in one fission reaction.
$\langle \nu_{\mathrm{d}}(\nu_{\mathrm{d}}-1) \rangle$	=	$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} c(c-1) p_{\mathrm{f}}(n,c)$, i.e., second order factorial moment of number of delayed neutrons born in one fission reaction.

α	=	$\dfrac{\beta-\rho}{\Lambda}$, i.e., neutron decay constant.
λ	=	delayed neutron time constant.
β	=	$\dfrac{\langle\nu_\mathrm{d}\rangle}{\langle\nu_\mathrm{p}\rangle+\langle\nu_\mathrm{d}\rangle}$, i.e., delayed neutron fraction.
ρ	=	$\dfrac{\lambda_\mathrm{f}\left\{\langle\nu_\mathrm{p}\rangle+\langle\nu_\mathrm{d}\rangle\right\}-(\lambda_\mathrm{c}+\lambda_\mathrm{f}+\lambda_\mathrm{d})}{\lambda_\mathrm{f}\left\{\langle\nu_\mathrm{p}\rangle+\langle\nu_\mathrm{d}\rangle\right\}}$, i.e., reactivity.
Λ	=	$\dfrac{1}{\lambda_\mathrm{f}\left\{\langle\nu_\mathrm{p}\rangle+\langle\nu_\mathrm{d}\rangle\right\}}$, i.e., neutron generation time.
$p_\mathrm{b}(q)$	=	probability that q neutrons are born in one neutron burst.
$\langle\xi\rangle$	=	$\sum_{q=0}^{\infty} q p_\mathrm{b}(q)$, i.e., first order moment of number of neutrons born in one neutron burst.
$\langle\xi(\xi-1)\rangle$	=	$\sum_{q=0}^{\infty} q(q-1)p_\mathrm{b}(q)$, i.e., second order factorial moment of number of neutrons born in one neutron burst.
τ	=	repetition period of neutron bursts.

1. Introduction

Evaluation of the subcriticality is one of the key problems to reconcile the nuclear criticality safety with the economical operation in the nuclear fuel facilities. The Feynman-alpha technique can evaluate the subcriticality through determination of the neutron decay constant, so that it has been frequently studied for this purpose (de Hoffmann, 1949; Williams, 1974). In the Feynman-alpha technique, the neutron count with respect to a certain length of counting gate, i.e., gate width, is repeatedly measured to calculate a correlation index Y that consists of the mean and variance values of thus obtained neutron count data (Feynman et al., 1956). The neutron decay constant is determined by applying a theoretical formula to the Y value as a function of the gate width, i.e., Y curve.

Investigation on the Feynman-alpha technique is still being pursued to cope with a renewed necessity, i.e., subcriticality monitoring for the accelerator-driven system (ADS). Corresponding to two kinds of operation modes of accelerators, i.e., the current mode and the pulse one, the Feynman-alpha techniques for the ADSs with respective operation modes were developed (Pázsit and Yamane, 1998, 1999; Behringer and Wydler, 1999; Muñoz-Cobo et al., 2001; Degweker, 2003; Pázsit et al., 2005; Kitamura et al., 2005, 2006; Degweker and Rana, 2007; Pázsit and Pál, 2008; Rana and Degweker, 2009). However, there is a defect in some Feynman-alpha techniques for the ADS with pulse mode developed so far; their theoretical formulae are disadvantageous in determining the neutron decay constant since they are too complicated owing to the rapid variation of neutron counting rate that is caused by the prompt decaying behaviour of neutron population.

To overcome this problem, one of the authors and his collaborators developed a modified Feynman-alpha technique for the subcritical system driven by periodically triggered neutron bursts (Misawa et al., 2014). In this technique, by introducing a masking window for omitting a period of time during which the prompt decaying behaviour of neutron population is observed, a simpler formula than the conventional ones can be applied to determine the neutron decay constant. Through an experiment performed at the Kyoto University Critical Assembly (KUCA), they further showed a possibility of suppressing the space dependency effect in evaluating the subcriticality. It is hence expected that this technique can be applied to the subcriticality monitor for the ADS with pulse mode. However, these features of this technique have not been fully investigated because the theoretical study on this technique has not been performed yet. In the present study, hence, a theory of this technique is developed by explicitly taking the periodically triggered neutron bursts and the masking window into consideration. On the basis of the theory thus developed, the experimental conditions under which the simple formula works are discussed to promote the study for developing the subcriticality monitor for the ADS.

A brief review of the modified Feynman-alpha technique is provided in the following section. The one- and two-time-point neutron detection probabilities for establishing the theoretical basis on this technique are derived in Section 3. By using the neutron detection probabilities thus derived, the theoretical formula of this technique is obtained in the same section. Finally, the conclusion is summarized in Section 5 based on the discussions given in Section 4.

2. Review of modified Feynman-alpha technique

In Fig. 1, a typical temporal response of neutron population in a subcritical system that is driven by periodic neutron bursts triggered at time $m\tau$ ($m = -\infty, \ldots, -1, 0, 1, \ldots, +\infty$) is illustrated, where τ is the repetition period of neutron bursts. In the modified Feynman-alpha technique, the masking window with a duration of w is introduced so as to omit the prompt decaying behaviour of neutron population (Misawa et al., 2014).

The modified Feynman-alpha technique utilizes the bunching procedure to efficiently obtain the Y curve (Misawa et al., 1990). As shown in Fig. 2, in the bunching procedure, successive neutron count data between w and τ with respect to a funda-

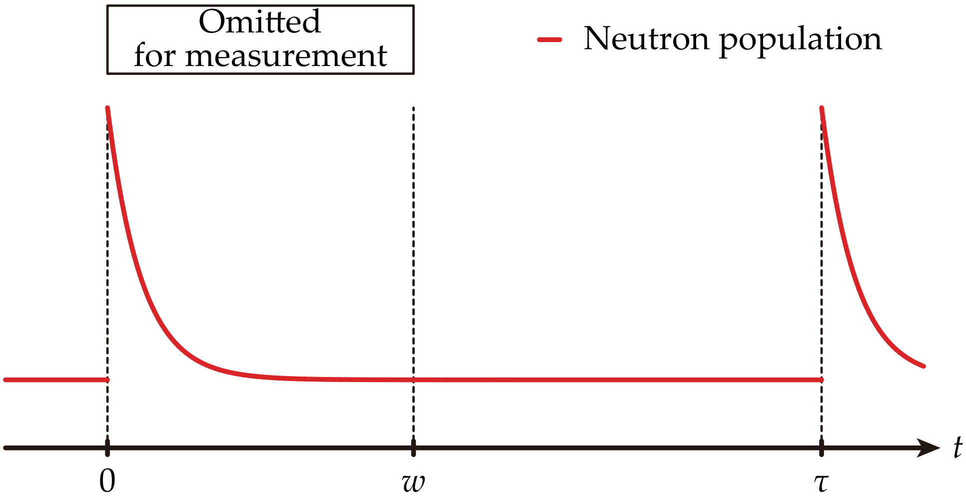

Figure 1: Illustration of masking window in modified Feynman-alpha technique.

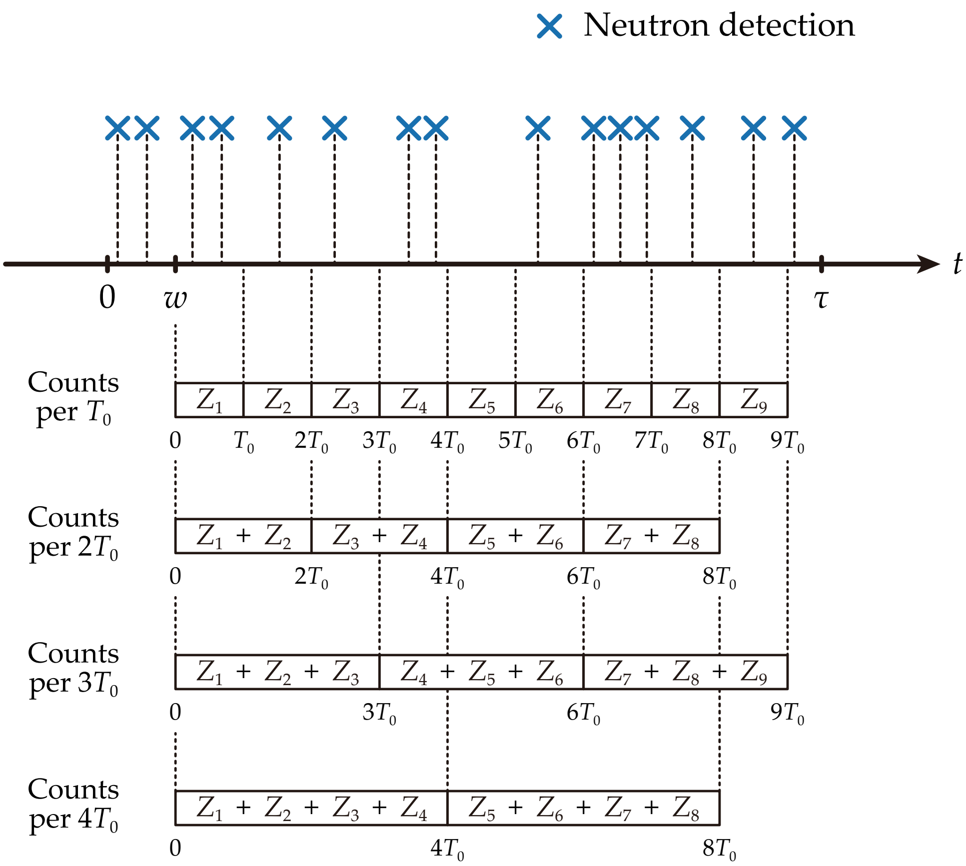

Figure 2: Data processing procedure in modified Feynman-alpha technique.

mental gate width T_0 are measured to calculate the Y value with respect to T_0. The neutron count data with respect to longer gate widths $T = rT_0$ ($r = 2, 3, \ldots$) are not measured but synthesized by bunching adjacent neutron count data with respect to T_0. The Y curves as functions of w and T thus obtained are written as

$$Y(w,T) \equiv \frac{V(w,T)}{M(w,T)} - 1, \quad T = rT_0, \quad r = 1, 2, \ldots, \tag{1}$$

where $M(w,T)$ and $V(w,T)$ are the mean and variance values of neutron count data with respect to T. Denoting the xth neutron count datum with respect to T_0 by Z_x, $M(w,T)$ and $V(w,T)$ are calculated as follows:

$$M(w,T) \equiv \frac{1}{N_r} \sum_{x=1}^{N_r} \sum_{y=1}^{r} Z_{r(x-1)+y}, \tag{2}$$

$$V(w,T) \equiv \frac{1}{N_r} \sum_{x=1}^{N_r} \left\{ \sum_{y=1}^{r} Z_{r(x-1)+y} \right\}^2 - M^2(w,T), \tag{3}$$

$$N_r \equiv \text{Maximum integer that does not exceed } \frac{N}{r}, \tag{4}$$

where N is the total number of disjoint consecutive neutron count data sets with individual gate width T_0.

When the expected value of $Y(w,T)$ is written as $\langle Y(w,T) \rangle$, it is expressed as

$$\langle Y(w,T) \rangle \equiv \frac{\langle V(w,T) \rangle}{\langle M(w,T) \rangle} - 1, \tag{5}$$

where $\langle M(w,T) \rangle$ and $\langle V(w,T) \rangle$ are the expected values of $M(w,T)$ and $V(w,T)$ that are calculated as follows (Wallerbos and Hoogenboom, 1998; Kitamura et al., 2000):

$$\langle M(w,T) \rangle \equiv \left\langle \frac{1}{N_r} \sum_{x=1}^{N_r} \sum_{y=1}^{r} Z_{r(x-1)+y} \right\rangle = \frac{1}{N_r} \sum_{x=1}^{rN_r} \langle Z_x \rangle, \tag{6}$$

$$
\begin{aligned}
\langle V(w,T)\rangle &\equiv \left\langle \frac{1}{N_r}\sum_{x=1}^{N_r}\left\{\sum_{y=1}^{r} Z_{r(x-1)+y}\right\}^2 - M^2(w,T)\right\rangle \\
&= \frac{N_r-1}{N_r^2}\sum_{x=1}^{rN_r}\langle Z_x\rangle + \frac{N_r-1}{N_r^2}\sum_{x=1}^{rN_r}\langle Z_x(Z_x-1)\rangle \\
&+ \frac{2}{N_r}\sum_{x=1}^{N_r}\sum_{y=1}^{r-1}\sum_{z=1}^{r-y}\left\langle Z_{r(x-1)+y}Z_{r(x-1)+y+z}\right\rangle - \frac{2}{N_r^2}\sum_{x=1}^{rN_r-1}\sum_{y=1}^{rN_r-x}\left\langle Z_x Z_{x+y}\right\rangle .
\end{aligned} \tag{7}
$$

Hence, one understands that $\langle Y(w,T)\rangle$ can be derived by calculating the right-hand sides of Eqs. (6) and (7) that include the first-order moment $\langle Z_x\rangle$ and the second-order moments $\langle Z_x(Z_x-1)\rangle$ and $\left\langle Z_x Z_{x+y}\right\rangle$ of the neutron count data with respect to T_0. These moment quantities are calculated as follows:

$$
\langle Z_x\rangle = \int_{w+(x-1)T_0}^{w+xT_0} \mathrm{d}t_1\, P_1(t_1)\,, \tag{8}
$$

$$
\langle Z_x(Z_x-1)\rangle = 2\int_{w+(x-1)T_0}^{w+xT_0} \mathrm{d}t_1 \int_{t_1}^{w+xT_0} \mathrm{d}t_2\, P_2(t_1,t_2)\,, \tag{9}
$$

$$
\left\langle Z_x Z_{x+y}\right\rangle = \int_{w+(x-1)T_0}^{w+xT_0} \mathrm{d}t_1 \int_{w+(x+y-1)T_0}^{w+(x+y)T_0} \mathrm{d}t_2\, P_2(t_1,t_2)\,, \tag{10}
$$

where $P_1(t_1)\,\mathrm{d}t_1$ is the one-time-point neutron detection probability that one neutron is detected within an infinitesimal time interval $\mathrm{d}t_1$ around t_1, and $P_2(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2$ the two-time-point probability that a pair of neutrons are detected within respective infinitesimal time intervals $\mathrm{d}t_1$ around t_1 and $\mathrm{d}t_2$ around t_2 ($> t_1$).

In the next section, through derivation of these neutron detection probabilities for the subcritical system that is driven by periodically triggered neutron bursts, a theory of modified Feynman-alpha technique is developed.

3. Theory of modified Feynman-alpha technique

3.1. Notations

In the present study, the one- and two-time-point neutron detection probabilities $P_1(t_1)\,\mathrm{d}t_1$ and $P_2(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2$ are derived by using the de Hoffmann's formulation approach (de Hoffmann, 1949; Albrecht, 1962; Yamane and Pázsit, 1998). The theoretical framework supposed and the notations needed in deriving these probabilities are defined in this subsection.

A zero-power subcritical system that is driven by periodically triggered neutron bursts is supposed. Formulation will be thoroughly performed within the mono-energy one-point reactor model where delayed neutrons are emitted from precursors having a single time constant λ, for a simple discussion. The neutron interactions in the subcritical system (i.e., scattering, capture, fission, and detection) are dealt with the following quantities,

$$\lambda_x \equiv v\Sigma_x, \quad x = \mathrm{s, c, f, d}, \tag{11}$$

where λ_s is the probability that one neutron is scattered per unit time, λ_c the probability that one neutron is captured by materials excluding a neutron detector per unit time, λ_f the probability that one neutron induces a fission reaction per unit time, λ_d the probability that one neutron is captured by the neutron detector per unit time, Σ_x the macroscopic cross-sections for corresponding neutron interactions x $(= \mathrm{s, c, f, d})$, and v the velocity of neutrons.

The probability that n prompt neutrons and c delayed neutron precursors are born in one fission reaction is denoted by $p_\mathrm{f}(n, c)$, and is supposed to be normalized as

$$\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} p_\mathrm{f}(n, c) = 1. \tag{12}$$

Using this probability, the following moment quantities with respect to the numbers of prompt and delayed neutrons are defined as

$$\langle \nu_\mathrm{p} \rangle \equiv \sum_{n=0}^{\infty}\sum_{c=0}^{\infty} n p_\mathrm{f}(n, c), \tag{13}$$

$$\langle \nu_\mathrm{d} \rangle \equiv \sum_{n=0}^{\infty}\sum_{c=0}^{\infty} c p_\mathrm{f}(n, c), \tag{14}$$

$$\left\langle \nu_\mathrm{p}\left(\nu_\mathrm{p} - 1\right) \right\rangle \equiv \sum_{n=0}^{\infty}\sum_{c=0}^{\infty} n(n-1)\, p_\mathrm{f}(n, c), \tag{15}$$

$$\left\langle \nu_{\mathrm{p}} \nu_{\mathrm{d}} \right\rangle \equiv \sum_{n=0}^{\infty} \sum_{c=0}^{\infty} n c p_{\mathrm{f}} \left(n, c \right), \tag{16}$$

$$\left\langle \nu_{\mathrm{d}} \left(\nu_{\mathrm{d}} - 1 \right) \right\rangle \equiv \sum_{n=0}^{\infty} \sum_{c=0}^{\infty} c \left(c - 1 \right) p_{\mathrm{f}} \left(n, c \right). \tag{17}$$

The delayed neutron fraction β is naturally written as

$$\beta \equiv \frac{\left\langle \nu_{\mathrm{d}} \right\rangle}{\left\langle \nu_{\mathrm{p}} \right\rangle + \left\langle \nu_{\mathrm{d}} \right\rangle}. \tag{18}$$

Furthermore, the reactivity ρ, the neutron generation time Λ, and the neutron decay constant α are defined as follows:

$$\rho \equiv \frac{\lambda_{\mathrm{f}} \left\{ \left\langle \nu_{\mathrm{p}} \right\rangle + \left\langle \nu_{\mathrm{d}} \right\rangle \right\} - \left(\lambda_{\mathrm{c}} + \lambda_{\mathrm{f}} + \lambda_{\mathrm{d}} \right)}{\lambda_{\mathrm{f}} \left\{ \left\langle \nu_{\mathrm{p}} \right\rangle + \left\langle \nu_{\mathrm{d}} \right\rangle \right\}}, \tag{19}$$

$$\Lambda \equiv \frac{1}{\lambda_{\mathrm{f}} \left\{ \left\langle \nu_{\mathrm{p}} \right\rangle + \left\langle \nu_{\mathrm{d}} \right\rangle \right\}}, \tag{20}$$

$$\alpha \equiv \frac{\beta - \rho}{\Lambda} = \left(\lambda_{\mathrm{c}} + \lambda_{\mathrm{f}} + \lambda_{\mathrm{d}} \right) - \lambda_{\mathrm{f}} \left\langle \nu_{\mathrm{p}} \right\rangle. \tag{21}$$

As illustrated in Fig. 3, it is assumed that the neutron bursts are periodically triggered at time $m\tau$ ($m = -\infty, \ldots, -1, 0, 1, \ldots, +\infty$). All neutrons born from one neutron burst are assumed to be simultaneously injected into the subcritical system.

The probability that q neutrons are born in one neutron burst is denoted by $p_{\mathrm{b}} \left(q \right)$, and is supposed to be normalized as

$$\sum_{q=0}^{\infty} p_{\mathrm{b}} \left(q \right) = 1. \tag{22}$$

Using this probability, the following moment quantities with respect to the numbers of injected neutrons are defined as

$$\left\langle \xi \right\rangle \equiv \sum_{q=0}^{\infty} q p_{\mathrm{b}} \left(q \right), \tag{23}$$

$$\langle \xi (\xi - 1) \rangle \equiv \sum_{q=0}^{\infty} q (q-1)\, p_{\rm b} (q) \,. \tag{24}$$

3.2. One-time-point neutron detection probability

In the de Hoffmann's formulation approach, all possible event combinations that result in the neutron detection must be taken into consideration (de Hoffmann, 1949; Albrecht, 1962; Yamane and Pázsit, 1998). To complete such a task, it is helpful to draw schematic illustrations where possible event combinations are depicted. Furthermore, the population of progeny neutrons at time t originating from one ancestor neutron born at time 0 is introduced as $\mathcal{G}_{\rm p}(t)$ (see also Appendix A).

Figure 4 shows a schematic illustration for deriving the one-time-point probability $P_1(t_1)\,{\rm d}t_1$. In this figure, a possible event combination that results in the neutron detection within ${\rm d}t_1$ around t_1 is depicted; the progeny of q neutrons born in the neutron burst at $t_{\rm b}$ ($< t_1$) survive until t_1 ($< \tau$) then one of them is detected within ${\rm d}t_1$. By using the notations defined in the previous subsection and $\mathcal{G}_{\rm p}(t)$, one immediately writes the probability that a progeny neutron of the q neutrons born at $t_{\rm b}$ is detected within ${\rm d}t_1$ around t_1 as

$$\lambda_{\rm d} {\rm d}t_1 \cdot q \mathcal{G}_{\rm p} (t_1 - t_{\rm b}) \,, \tag{25}$$

where $\lambda_{\rm d}{\rm d}t_1$ is the probability that one neutron is detected within ${\rm d}t_1$, and $q\mathcal{G}_{\rm p}(t_1 - t_{\rm b})$ the population of progeny neutrons at t_1 originating from q neutrons born at $t_{\rm b}$.

Since q can take only integer values from 0 to ∞, multiplying Eq. (25) by $p_{\rm b}(q)$ and then summing over q, one obtains the probability that one neutron originating from the neutron burst at $t_{\rm b}$ is detected within ${\rm d}t_1$ around t_1, i.e., $P_1(t_{\rm b} \to t_1)\,{\rm d}t_1$, as

$$P_1 (t_{\rm b} \to t_1)\, {\rm d}t_1 \equiv \lambda_{\rm d} \sum_{q=0}^{\infty} q p_{\rm b}(q)\, \mathcal{G}_{\rm p}(t_1 - t_{\rm b})\, {\rm d}t_1 = \lambda_{\rm d} \langle \xi \rangle \mathcal{G}_{\rm p}(t_1 - t_{\rm b})\, {\rm d}t_1. \tag{26}$$

Therefore, by setting $t_{\rm b} = m\tau$ then summing up m from $-\infty$ to 0, $P_1(t_1)\,{\rm d}t_1$ can be

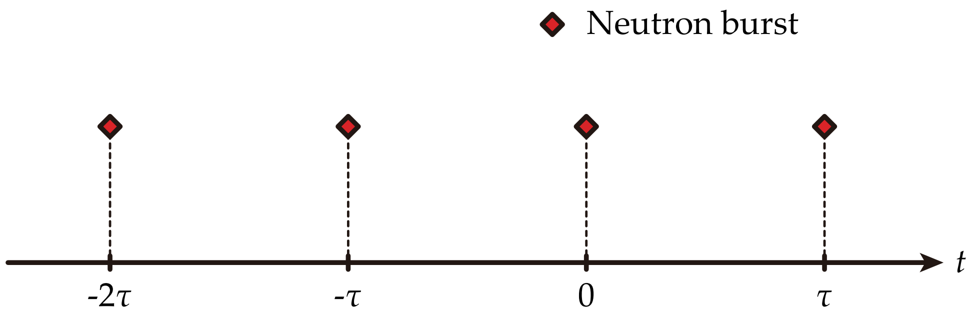

Figure 3: Timing diagram of periodically triggered neutron bursts.

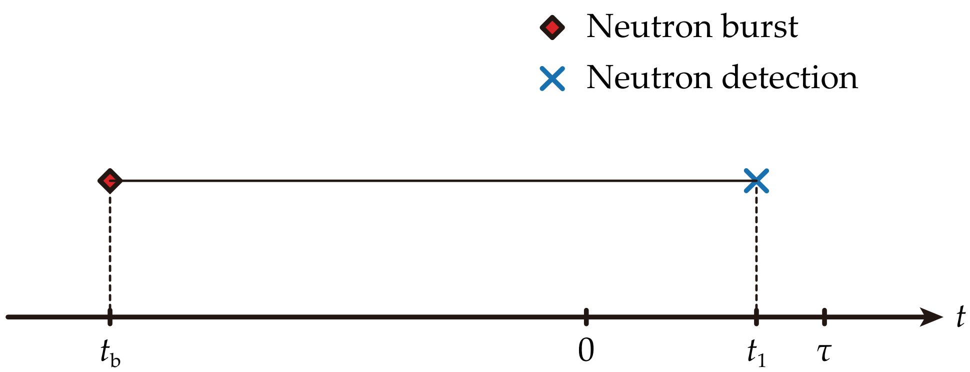

Figure 4: Neutron detection originating from neutron burst.

written as follows:

$$
\begin{aligned}
P_1(t_1)\,\mathrm{d}t_1 &= \sum_{m=-\infty}^{0} P_1(m\tau \to t_1)\,\mathrm{d}t_1 \\
&= \lambda_\mathrm{d}\langle\xi\rangle \sum_{i=\mathrm{p,d}} \Omega_{\mathrm{p},i}\left(\sum_{m=-\infty}^{0} \mathrm{e}^{\alpha_i m\tau}\right)\mathrm{e}^{-\alpha_i t_1}\mathrm{d}t_1 \\
&= \lambda_\mathrm{d}\langle\xi\rangle \sum_{i=\mathrm{p,d}} \Omega_{\mathrm{p},i}\Delta_{\alpha_i\tau}\mathrm{e}^{-\alpha_i t_1}\mathrm{d}t_1, \quad 0 \le t_1 < \tau,
\end{aligned}
\tag{27}
$$

where Δ_γ is defined as

$$
\Delta_\gamma \equiv \sum_{m=-\infty}^{0} \mathrm{e}^{m\gamma} = \frac{1}{1-\mathrm{e}^{-\gamma}}. \tag{28}
$$

3.3. *Two-time-point neutron detection probability*

3.3.1. *Components in two-time-point neutron detection probability*

de Hoffmann (1949) indicated that the two-time-point neutron detection probability $P_2(t_1, t_2)\,\mathrm{d}t_1\mathrm{d}t_2$ consists of two components, i.e.,

$$
P_2(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2 = P_2^{(\mathrm{c})}(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2 + P_2^{(\mathrm{u})}(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2, \tag{29}
$$

where the first term is the correlated one, and the second the uncorrelated one ($0 < t_1 < t_2 \le \tau$). The correlated term corresponds to cases of detecting pairs of neutrons that can be traced back to a common branching process. When a plural number of neutrons are simultaneously born from the extraneous neutron source, the correlated term is further divided into two mutually exclusive ones (Yamane and Pázsit, 1998) as

$$
P_2^{(\mathrm{c})}(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2 = P_2^{(\mathrm{c\text{-}f})}(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2 + P_2^{(\mathrm{c\text{-}b})}(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2, \tag{30}
$$

where the first and second terms are the correlated term where the nearest common branching processes are the fission reaction (see Fig. 5) and the neutron burst (see Fig. 6), respectively. On the other hand, the uncorrelated term corresponds to cases of detecting pairs of neutrons that cannot be traced back to common branching processes (see Fig. 7).

3.3.2. Correlated term due to branching by fission reaction

The probability $P_2^{(\text{c-f})}(t_1, t_2)\,\mathrm{d}t_1\mathrm{d}t_2$ is calculated with the help of the schematic illustration given in Fig. 5. Before calculating $P_2^{(\text{c-f})}(t_1, t_2)\,\mathrm{d}t_1\mathrm{d}t_2$, the population of progeny neutrons at time t originating from one ancestor delayed neutron precursor born at time 0 is introduced as $\mathcal{G}_{\mathrm{d}}(t)$ (see also Appendix B).

(a) The probability that a progeny neutron of the q neutrons born in the neutron burst at t_{b} induces a fission reaction within $\mathrm{d}t_{\mathrm{f}}$ around t_{f} ($> t_{\mathrm{b}}$):

$$\lambda_{\mathrm{f}}\mathrm{d}t_{\mathrm{f}} \cdot q\mathcal{G}_{\mathrm{p}}(t_{\mathrm{f}} - t_{\mathrm{b}}). \tag{31}$$

(b) The joint probability that a progeny neutron of one of the n prompt neutrons born at t_{f} is detected within $\mathrm{d}t_1$ around t_1 ($> t_{\mathrm{f}}$) and a progeny neutron of one of the remaining $n-1$ prompt neutrons is detected within $\mathrm{d}t_2$ around t_2:

$$\left\{\lambda_{\mathrm{d}}\mathrm{d}t_1 \cdot n\mathcal{G}_{\mathrm{p}}(t_1 - t_{\mathrm{f}})\right\} \times \left\{\lambda_{\mathrm{d}}\mathrm{d}t_2 \cdot (n-1)\,\mathcal{G}_{\mathrm{p}}(t_2 - t_{\mathrm{f}})\right\}. \tag{32}$$

(c) The joint probability that a progeny neutron of the n prompt neutrons born at t_{f} is detected within $\mathrm{d}t_1$ around t_1 and a progeny neutron of the c delayed neutron precursors born at t_{f} is detected within $\mathrm{d}t_2$ around t_2:

$$\left\{\lambda_{\mathrm{d}}\mathrm{d}t_1 \cdot n\mathcal{G}_{\mathrm{p}}(t_1 - t_{\mathrm{f}})\right\} \times \left\{\lambda_{\mathrm{d}}\mathrm{d}t_2 \cdot c\mathcal{G}_{\mathrm{d}}(t_2 - t_{\mathrm{f}})\right\}. \tag{33}$$

(d) The joint probability that a progeny neutron of the c delayed neutron precursors born at t_{f} is detected within $\mathrm{d}t_1$ around t_1 and a progeny neutron of the n prompt neutrons born at t_{f} is detected within $\mathrm{d}t_2$ around t_2:

$$\left\{\lambda_{\mathrm{d}}\mathrm{d}t_1 \cdot c\mathcal{G}_{\mathrm{d}}(t_1 - t_{\mathrm{f}})\right\} \times \left\{\lambda_{\mathrm{d}}\mathrm{d}t_2 \cdot n\mathcal{G}_{\mathrm{p}}(t_2 - t_{\mathrm{f}})\right\}. \tag{34}$$

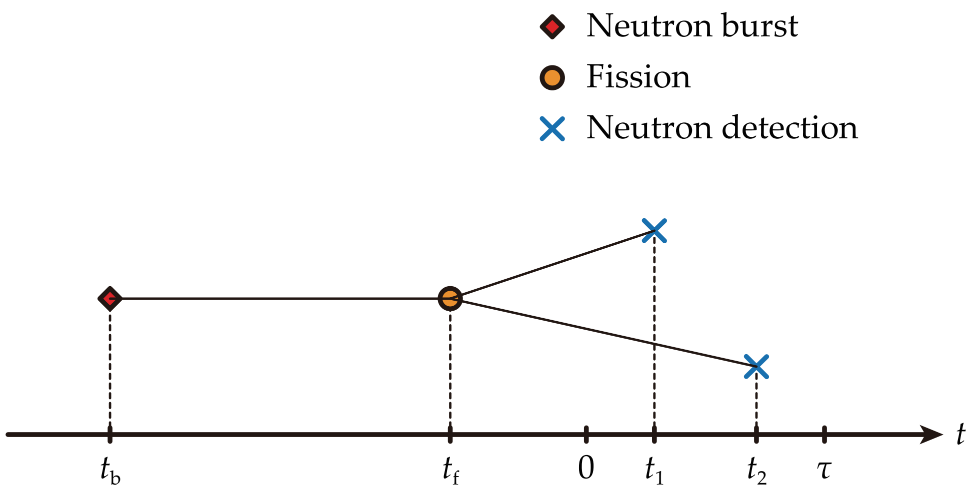

Figure 5: Neutron detection pair originating from common fission.

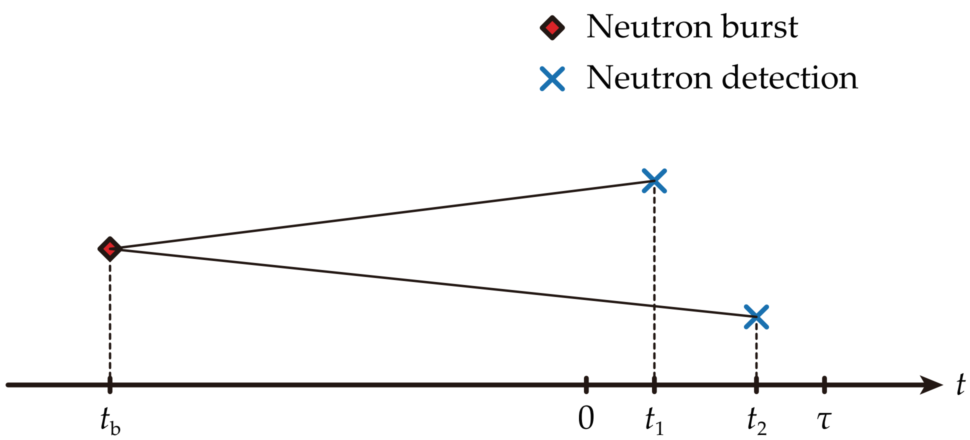

Figure 6: Neutron detection pair originating from common neutron burst.

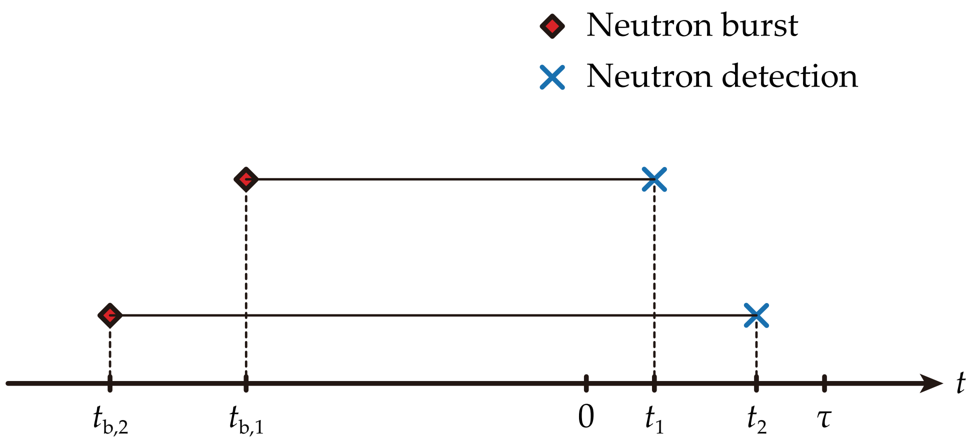

Figure 7: Neutron detection pair originating from different neutron bursts.

(e) The joint probability that a progeny neutron of one of the c delayed neutron precursors born at t_f is detected within $\mathrm{d}t_1$ around t_1 and a progeny neutron of one of the remaining $c-1$ delayed neutron precursors born at t_f is detected within $\mathrm{d}t_2$ around t_2:

$$\left\{\lambda_\mathrm{d}\mathrm{d}t_1 \cdot c\mathcal{G}_\mathrm{d}\left(t_1-t_\mathrm{f}\right)\right\} \times \left\{\lambda_\mathrm{d}\mathrm{d}t_2 \cdot \left(c-1\right)\mathcal{G}_\mathrm{d}\left(t_2-t_\mathrm{f}\right)\right\}. \tag{35}$$

Since q, n, and c can take only integer values from 0 to ∞, then one proceeds as follows: first we have added up Eqs. (32), (33), (34) and (35), and we have multiplied the result by Eq. (31). Then, the resulting equation has been multiplied by $p_\mathrm{b}\left(q\right)$ and $p_\mathrm{f}\left(n,c\right)$. Finally a summation has been performed with respect to all values of q, n, and c. This set of operations gives the probability that one fission reaction originating from the neutron burst at t_b is induced within $\mathrm{d}t_\mathrm{f}$ around t_f then a pair of neutrons originating from the fission reaction at t_f are detected within $\mathrm{d}t_1$ around t_1 and $\mathrm{d}t_2$ around t_2, i.e., $P_2^{(\text{c-f})}\left(t_\mathrm{b}\to t_\mathrm{f}\to t_1,t_2\right)\mathrm{d}t_\mathrm{f}\mathrm{d}t_1\mathrm{d}t_2$, as

$$\begin{aligned}
&P_2^{(\text{c-f})}\left(t_\mathrm{b}\to t_\mathrm{f}\to t_1,t_2\right)\mathrm{d}t_\mathrm{f}\mathrm{d}t_1\mathrm{d}t_2\\
&\quad\equiv \lambda_\mathrm{d}^2\lambda_\mathrm{f}\sum_{q=0}^{\infty} q p_\mathrm{b}\left(q\right)\mathcal{G}_\mathrm{p}\left(t_\mathrm{f}-t_\mathrm{b}\right)\\
&\quad\times\sum_{n=0}^{\infty}\sum_{c=0}^{\infty} p_\mathrm{f}\left(n,c\right)\begin{bmatrix} n\left(n-1\right)\mathcal{G}_\mathrm{p}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{p}\left(t_2-t_\mathrm{f}\right)\\ +nc\mathcal{G}_\mathrm{p}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{d}\left(t_2-t_\mathrm{f}\right)\\ +cn\mathcal{G}_\mathrm{d}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{p}\left(t_2-t_\mathrm{f}\right)\\ +c\left(c-1\right)\mathcal{G}_\mathrm{d}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{d}\left(t_2-t_\mathrm{f}\right)\end{bmatrix}\mathrm{d}t_\mathrm{f}\mathrm{d}t_1\mathrm{d}t_2\\
&\quad= \lambda_\mathrm{d}^2\lambda_\mathrm{f}\left\langle\xi\right\rangle\mathcal{G}_\mathrm{p}\left(t_\mathrm{f}-t_\mathrm{b}\right)\begin{bmatrix}\left\langle\nu_\mathrm{p}\left(\nu_\mathrm{p}-1\right)\right\rangle\mathcal{G}_\mathrm{p}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{p}\left(t_2-t_\mathrm{f}\right)\\ +\left\langle\nu_\mathrm{p}\nu_\mathrm{d}\right\rangle\begin{Bmatrix}\mathcal{G}_\mathrm{p}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{d}\left(t_2-t_\mathrm{f}\right)\\ +\mathcal{G}_\mathrm{d}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{p}\left(t_2-t_\mathrm{f}\right)\end{Bmatrix}\\ +\left\langle\nu_\mathrm{d}\left(\nu_\mathrm{d}-1\right)\right\rangle\mathcal{G}_\mathrm{d}\left(t_1-t_\mathrm{f}\right)\mathcal{G}_\mathrm{d}\left(t_2-t_\mathrm{f}\right)\end{bmatrix}\mathrm{d}t_\mathrm{f}\mathrm{d}t_1\mathrm{d}t_2.
\end{aligned}\tag{36}$$

When t_b is set to be $m\tau$ owing to the periodicity of neutron bursts, one sees from Fig. 5 that t_f can take values in the interval from $m\tau$ to t_1 and m can take only integer values from $-\infty$ to 0. Therefore, one obtains $P_2^{\text{(c-f)}}\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$ as follows:

$$P_2^{\text{(c-f)}}\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2 = \sum_{m=-\infty}^{0} \int_{m\tau}^{t_1} \mathrm{d}t_\mathrm{f}\, P_2^{\text{(c-f)}}\left(m\tau \to t_\mathrm{f} \to t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$$
$$= \lambda_\mathrm{d}^2 \lambda_\mathrm{f} \left\langle \xi \right\rangle \sum_{i=\mathrm{p,d}} \sum_{j=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \theta_{i,j,k} \left\{ \begin{array}{l} \Delta_{\alpha_i \tau} \mathrm{e}^{-\left(\alpha_i - \alpha_k\right) t_1 - \alpha_k t_2} \\ \quad - \Delta_{\left(\alpha_j + \alpha_k\right)\tau} \mathrm{e}^{-\alpha_j t_1 - \alpha_k t_2} \end{array} \right\} \mathrm{d}t_1 \mathrm{d}t_2, \tag{37}$$

where $\theta_{i,j,k}$ is defined as

$$\theta_{i,j,k} \equiv \frac{\Omega_{\mathrm{p},i}}{-\alpha_i + \alpha_j + \alpha_k} \left\{ \begin{array}{l} \left\langle \nu_\mathrm{p} \left(\nu_\mathrm{p} - 1\right) \right\rangle \Omega_{\mathrm{p},j} \Omega_{\mathrm{p},k} \\ \quad + \left\langle \nu_\mathrm{p} \nu_\mathrm{d} \right\rangle \left(\Omega_{\mathrm{p},j} \Omega_{\mathrm{d},k} + \Omega_{\mathrm{d},j} \Omega_{\mathrm{p},k}\right) \\ \quad + \left\langle \nu_\mathrm{d} \left(\nu_\mathrm{d} - 1\right) \right\rangle \Omega_{\mathrm{d},j} \Omega_{\mathrm{d},k} \end{array} \right\}. \tag{38}$$

3.3.3. Correlated term due to branching by neutron burst

The probability $P_2^{\text{(c-b)}}\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$ is calculated with the help of the schematic illustration given in Fig. 6.

The joint probability that a progeny neutron of one of the q neutrons born in the neutron burst at t_b is detected within $\mathrm{d}t_1$ around t_1 ($> t_\mathrm{b}$) and a progeny neutron of one of the remaining $q-1$ neutrons is detected within $\mathrm{d}t_2$ around t_2:

$$\left\{\lambda_\mathrm{d} \mathrm{d}t_1 \cdot q \mathcal{G}_\mathrm{p}\left(t_1 - t_\mathrm{b}\right)\right\} \times \left\{\lambda_\mathrm{d} \mathrm{d}t_2 \cdot \left(q-1\right) \mathcal{G}_\mathrm{p}\left(t_2 - t_\mathrm{b}\right)\right\}. \tag{39}$$

Since q can take only integer values from 0 to ∞, multiplying Eq. (39) by $p_\mathrm{b}\left(q\right)$ and then summing over q, one obtains the probability that a pair of neutrons originating from the neutron burst at t_b are detected within $\mathrm{d}t_1$ around t_1 and $\mathrm{d}t_2$ around t_2, i.e.,

$P_2^{\text{(c-b)}}\left(t_{\text{b}} \rightarrow t_1, t_2\right) \text{d}t_1\text{d}t_2$, as

$$
\begin{aligned}
P_2^{\text{(c-b)}}\left(t_{\text{b}} \rightarrow t_1, t_2\right) \text{d}t_1\text{d}t_2 &\equiv \lambda_{\text{d}}^2 \sum_{q=0}^{\infty} q\left(q-1\right) p_{\text{b}}\left(q\right) \mathcal{G}_{\text{p}}\left(t_1 - t_{\text{b}}\right) \mathcal{G}_{\text{p}}\left(t_2 - t_{\text{b}}\right) \text{d}t_1\text{d}t_2 \\
&= \lambda_{\text{d}}^2 \left\langle \xi\left(\xi-1\right)\right\rangle \mathcal{G}_{\text{p}}\left(t_1 - t_{\text{b}}\right) \mathcal{G}_{\text{p}}\left(t_2 - t_{\text{b}}\right) \text{d}t_1\text{d}t_2.
\end{aligned}
\tag{40}
$$

Therefore, by setting $t_{\text{b}} = m\tau$ then summing up m from $-\infty$ to 0, one calculates $P_2^{\text{(c-b)}}\left(t_1, t_2\right) \text{d}t_1\text{d}t_2$ as follows:

$$
\begin{aligned}
P_2^{\text{(c-b)}}\left(t_1, t_2\right) \text{d}t_1\text{d}t_2 &= \sum_{m=-\infty}^{0} P_2^{\text{(c-b)}}\left(m\tau \rightarrow t_1, t_2\right) \text{d}t_1\text{d}t_2 \\
&= \lambda_{\text{d}}^2 \left\langle \xi\left(\xi-1\right)\right\rangle \sum_{j=\text{p,d}} \sum_{k=\text{p,d}} \eta_{j,k} \Delta_{\left(\alpha_j+\alpha_k\right)\tau} \text{e}^{-\alpha_j t_1 - \alpha_k t_2} \text{d}t_1\text{d}t_2,
\end{aligned}
\tag{41}
$$

where

$$
\eta_{j,k} \equiv \Omega_{\text{p},j}\Omega_{\text{p},k}. \tag{42}
$$

3.3.4. Uncorrelated term

The probability $P_2^{\text{(u)}}\left(t_1, t_2\right) \text{d}t_1\text{d}t_2$ is calculated with the help of the schematic illustration given in Fig. 7.

The joint probability that a progeny neutron of the q_1 neutrons born in the neutron burst at $t_{\text{b},1}$ is detected within $\text{d}t_1$ around t_1 ($> t_{\text{b},1}$) and a progeny neutron of the q_2 neutrons born in the neutron burst at $t_{\text{b},2}$ is detected within $\text{d}t_2$ around t_2:

$$
\left\{\lambda_{\text{d}}\text{d}t_1 \cdot q_1 \mathcal{G}_{\text{p}}\left(t_1 - t_{\text{b},1}\right)\right\} \times \left\{\lambda_{\text{d}}\text{d}t_2 \cdot q_2 \mathcal{G}_{\text{p}}\left(t_2 - t_{\text{b},2}\right)\right\}, \quad t_{\text{b},2} \neq t_{\text{b},1}. \tag{43}
$$

We have to note here that the case $t_{\text{b},1} = t_{\text{b},2}$ must be excluded to derive the uncorrelated term because all neutrons born in one neutron burst are regarded to be correlated in the present study. Multiplying Eq. (39) by $p_{\text{b}}\left(q_1\right)$ and $p_{\text{b}}\left(q_2\right)$ and carrying out summations over q_1 and q_2 from 0 to ∞ each, one obtains the probability that a pair of neutrons originating from the different neutron bursts at $t_{\text{b},1}$ and $t_{\text{b},2}$ are detected within $\mathrm{d}t_1$ around t_1 and $\mathrm{d}t_2$ around t_2, i.e., $P_2^{(\mathrm{u})}\left(t_{\mathrm{b},1} \to t_1, t_{\mathrm{b},2} \to t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$, as

$$
\begin{aligned}
& P_2^{(\mathrm{u})}\left(t_{\mathrm{b},1} \to t_1, t_{\mathrm{b},2} \to t_2\right) \mathrm{d}t_1 \mathrm{d}t_2 \\
& \quad \equiv \lambda_{\mathrm{d}}^2 \sum_{q_1=0}^{\infty} q_1 p_{\mathrm{b}}\left(q_1\right) \sum_{q_2=0}^{\infty} q_2 p_{\mathrm{b}}\left(q_2\right) \mathcal{G}_{\mathrm{p}}\left(t_1 - t_{\mathrm{b},1}\right) \mathcal{G}_{\mathrm{p}}\left(t_2 - t_{\mathrm{b},2}\right) \mathrm{d}t_1 \mathrm{d}t_2 \\
& \quad = \lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \mathcal{G}_{\mathrm{p}}\left(t_1 - t_{\mathrm{b},1}\right) \mathcal{G}_{\mathrm{p}}\left(t_2 - t_{\mathrm{b},2}\right) \mathrm{d}t_1 \mathrm{d}t_2, \quad t_{\mathrm{b},2} \neq t_{\mathrm{b},1}.
\end{aligned} \tag{44}
$$

Therefore, by setting $t_{\mathrm{b},1} = m_1 \tau$ and $t_{\mathrm{b},2} = m_2 \tau$ then summing up m_1 and m_2 ($\neq m_1$) from $-\infty$ to 0, one obtains $P_2^{(\mathrm{u})}\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$ as follows:

$$
\begin{aligned}
P_2^{(\mathrm{u})}\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2 &= \sum_{m_1=-\infty}^{0} \sum_{\substack{m_2=-\infty \\ m_2 \neq m_1}}^{0} P_2^{(\mathrm{u})}\left(m_1 \tau \to t_1, m_2 \tau \to t_2\right) \mathrm{d}t_1 \mathrm{d}t_2 \\
&= \lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \sum_{m_1=-\infty}^{0} \left\{ \begin{array}{c} \displaystyle\sum_{m_2=-\infty}^{0} \mathcal{G}_{\mathrm{p}}\left(t_1 - m_1 \tau\right) \mathcal{G}_{\mathrm{p}}\left(t_2 - m_2 \tau\right) \\ - \mathcal{G}_{\mathrm{p}}\left(t_1 - m_1 \tau\right) \mathcal{G}_{\mathrm{p}}\left(t_2 - m_1 \tau\right) \end{array} \right\} \mathrm{d}t_1 \mathrm{d}t_2 \\
&= \lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \sum_{j=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \eta_{j,k} \left\{ \Delta_{\alpha_j \tau} \Delta_{\alpha_k \tau} - \Delta_{\left(\alpha_j + \alpha_k\right) \tau} \right\} \mathrm{e}^{-\alpha_j t_1 - \alpha_k t_2} \mathrm{d}t_1 \mathrm{d}t_2.
\end{aligned} \tag{45}
$$

3.3.5. Summary of two-time-point neutron detection probability

As mentioned before, the two-time-point neutron detection probability $P_2\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$ consists of the correlated and uncorrelated terms. Therefore, one derives $P_2\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2$ as the summation of Eqs. (37), (41), and (45), i.e.,

$$
\begin{aligned}
& P_2\left(t_1, t_2\right) \mathrm{d}t_1 \mathrm{d}t_2 \\
& \quad = \lambda_{\mathrm{d}}^2 \lambda_{\mathrm{f}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \left(\theta_{i,\mathrm{p},k} + \theta_{i,\mathrm{d},k}\right) \Delta_{\alpha_i \tau} \mathrm{e}^{-\left(\alpha_i - \alpha_k\right) t_1 - \alpha_k t_2} \mathrm{d}t_1 \mathrm{d}t_2 \\
& \quad\quad + \lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \sum_{j=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \eta_{j,k} \left\{ \chi_{j,k} \Delta_{\left(\alpha_j + \alpha_k\right) \tau} + \Delta_{\alpha_j \tau} \Delta_{\alpha_k \tau} \right\} \mathrm{e}^{-\alpha_j t_1 - \alpha_k t_2} \mathrm{d}t_1 \mathrm{d}t_2,
\end{aligned} \tag{46}
$$

where

$$
\chi_{j,k} \equiv -\frac{\lambda_{\mathrm{f}}}{\langle \xi \rangle} \frac{\theta_{\mathrm{p},j,k} + \theta_{\mathrm{d},j,k}}{\eta_{j,k}} + \frac{\langle \xi \left(\xi - 1\right) \rangle}{\langle \xi \rangle^2} - 1. \tag{47}
$$

3.4. Theoretical formula of modified Feynman-alpha technique

Using the one- and two-time-point neutron detection probabilities, i.e., Eqs. (27) and (46), one can now calculate the moment quantities given in Eqs. (8) to (10) that are needed to derive the theoretical formula of the modified Feynman-alpha technique:

$$\langle Z_x \rangle = \lambda_{\mathrm{d}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \frac{\Omega_{\mathrm{p},i} \Delta_{\alpha_i \tau}}{\alpha_i} \mathrm{e}^{-\alpha_i w} \frac{\mathrm{e}^{-\alpha_i x T_0}}{\Delta_{\alpha_i T_0} \mathrm{e}^{-\alpha_i T_0}}, \tag{48}$$

$$\begin{aligned}
\langle Z_x (Z_x - 1) \rangle = {} & 2\lambda_{\mathrm{d}}^2 \lambda_{\mathrm{f}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \frac{\left(\theta_{i,\mathrm{p},i} + \theta_{i,\mathrm{d},i}\right) \Delta_{\alpha_i \tau}}{\alpha_i^2} \mathrm{e}^{-\alpha_i w} \frac{\left(1 - \alpha_i T_0 \Delta_{\alpha_i T_0} \mathrm{e}^{-\alpha_i T_0}\right) \mathrm{e}^{-\alpha_i x T_0}}{\Delta_{\alpha_i T_0} \mathrm{e}^{-\alpha_i T_0}} \\
& + 2\lambda_{\mathrm{d}}^2 \lambda_{\mathrm{f}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \sum_{\substack{i'=\mathrm{p,d} \\ i' \neq i}} \frac{\left(\theta_{i,\mathrm{p},i'} + \theta_{i,\mathrm{d},i'}\right) \Delta_{\alpha_i \tau}}{\alpha_i \left(\alpha_{i'} - \alpha_i\right)} \mathrm{e}^{-\alpha_i w} \frac{\left(1 - \frac{\alpha_i \Delta_{\alpha_i T_0}}{\alpha_{i'} \Delta_{\alpha_{i'} T_0}}\right) \mathrm{e}^{-\alpha_i x T_0}}{\Delta_{\alpha_i T_0} \mathrm{e}^{-\alpha_i T_0}} \\
& - 2\lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \sum_{j=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \frac{\eta_{j,k} \left\{ \chi_{j,k} \Delta_{\left(\alpha_j + \alpha_k\right)\tau} + \Delta_{\alpha_j \tau} \Delta_{\alpha_k \tau} \right\}}{\alpha_j \alpha_k} \mathrm{e}^{-\left(\alpha_j + \alpha_k\right) w} \\
& \times \frac{\left\{ \frac{\alpha_k \Delta_{\alpha_k T_0}}{\left(\alpha_j + \alpha_k\right) \Delta_{\left(\alpha_j + \alpha_k\right) T_0}} - 1 \right\} \mathrm{e}^{-\left(\alpha_j + \alpha_k\right) x T_0}}{\Delta_{\alpha_k T_0} \mathrm{e}^{-\left(\alpha_j + \alpha_k\right) T_0}},
\end{aligned} \tag{49}$$

$$\begin{aligned}
\left\langle Z_x Z_{x+y} \right\rangle = {} & \lambda_{\mathrm{d}}^2 \lambda_{\mathrm{f}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \frac{\left(\theta_{i,\mathrm{p},i} + \theta_{i,\mathrm{d},i}\right) \Delta_{\alpha_i \tau}}{\alpha_i^2} \mathrm{e}^{-\alpha_i w} \frac{\alpha_i T_0 \mathrm{e}^{-\alpha_i (x+y) T_0}}{\Delta_{\alpha_i T_0} \mathrm{e}^{-\alpha_i T_0}} \\
& + \lambda_{\mathrm{d}}^2 \lambda_{\mathrm{f}} \langle \xi \rangle \sum_{i=\mathrm{p,d}} \sum_{\substack{i'=\mathrm{p,d} \\ i' \neq i}} \frac{\left(\theta_{i,\mathrm{p},i'} + \theta_{i,\mathrm{d},i'}\right) \Delta_{\alpha_i \tau}}{\alpha_i \left(\alpha_{i'} - \alpha_i\right)} \mathrm{e}^{-\alpha_i w} \frac{-\alpha_i \mathrm{e}^{-\left(\alpha_i x + \alpha_{i'} y\right) T_0}}{\alpha_{i'} \Delta_{\alpha_{i'} T_0} \Delta_{\left(\alpha_i - \alpha_{i'}\right) T_0} \mathrm{e}^{-\alpha_i T_0}} \\
& - \lambda_{\mathrm{d}}^2 \langle \xi \rangle^2 \sum_{j=\mathrm{p,d}} \sum_{k=\mathrm{p,d}} \frac{\eta_{j,k} \left\{ \chi_{j,k} \Delta_{\left(\alpha_j + \alpha_k\right)\tau} + \Delta_{\alpha_j \tau} \Delta_{\alpha_k \tau} \right\}}{\alpha_j \alpha_k} \mathrm{e}^{-\left(\alpha_j + \alpha_k\right) w} \\
& \times \frac{-\mathrm{e}^{-\left\{ \left(\alpha_j + \alpha_k\right) x + \alpha_k y \right\} T_0}}{\Delta_{\alpha_j T_0} \Delta_{\alpha_k T_0} \mathrm{e}^{-\left(\alpha_j + \alpha_k\right) T_0}}.
\end{aligned} \tag{50}$$

One hence obtains $\langle M(w,T)\rangle$ and $\langle V(w,T)\rangle$ by substituting Eqs. (48) to (50) into Eqs. (6) and (7) as

$$\langle M(w,T)\rangle = \lambda_{\mathrm{d}}\langle\xi\rangle\frac{1}{N_r}\sum_{i=\mathrm{p,d}}\frac{\Omega_{\mathrm{p},i}\Delta_{\alpha_i\tau}}{\alpha_i}\mathrm{e}^{-\alpha_i w}\left(1-\mathrm{e}^{-\alpha_i N_r T}\right), \tag{51}$$

$$\begin{aligned}
\langle V(w,T)\rangle = {} & \lambda_{\mathrm{d}}\langle\xi\rangle\frac{1}{N_r}\sum_{i=\mathrm{p,d}}\frac{\Omega_{\mathrm{p},i}\Delta_{\alpha_i\tau}}{\alpha_i}\mathrm{e}^{-\alpha_i w}\left(1-\frac{1}{N_r}\right)\left(1-\mathrm{e}^{-\alpha_i N_r T}\right)\\
& + 2\lambda_{\mathrm{d}}^2\lambda_{\mathrm{f}}\langle\xi\rangle\frac{1}{N_r}\sum_{i=\mathrm{p,d}}\frac{\left(\theta_{i,\mathrm{p},i}+\theta_{i,\mathrm{d},i}\right)\Delta_{\alpha_i\tau}}{\alpha_i^2}\mathrm{e}^{-\alpha_i w}\\
& \times\left(1-\mathrm{e}^{-\alpha_i N_r T}\right)\left\{1-\frac{\alpha_i T\mathrm{e}^{-\alpha_i T}}{1-\mathrm{e}^{-\alpha_i T}}-\frac{1}{N_r}\left(1-\frac{\alpha_i N_r T\mathrm{e}^{-\alpha_i N_r T}}{1-\mathrm{e}^{-\alpha_i N_r T}}\right)\right\}\\
& + 2\lambda_{\mathrm{d}}^2\lambda_{\mathrm{f}}\langle\xi\rangle\frac{1}{N_r}\sum_{i=\mathrm{p,d}}\sum_{\substack{i'=\mathrm{p,d}\\ i'\neq i}}\frac{\left(\theta_{i,\mathrm{p},i'}+\theta_{i,\mathrm{d},i'}\right)\Delta_{\alpha_i\tau}}{\alpha_i\left(\alpha_{i'}-\alpha_i\right)}\mathrm{e}^{-\alpha_i w}\\
& \times\left(1-\mathrm{e}^{-\alpha_i N_r T}\right)\left\{1-\frac{\alpha_i}{\alpha_{i'}}\frac{1-\mathrm{e}^{-\alpha_{i'}T}}{1-\mathrm{e}^{-\alpha_i T}}-\frac{1}{N_r}\left(1-\frac{\alpha_i}{\alpha_{i'}}\frac{1-\mathrm{e}^{-\alpha_{i'}N_r T}}{1-\mathrm{e}^{-\alpha_i N_r T}}\right)\right\}\\
& - 2\lambda_{\mathrm{d}}^2\langle\xi\rangle^2\frac{1}{N_r}\sum_{j=\mathrm{p,d}}\sum_{k=\mathrm{p,d}}\frac{\eta_{j,k}\left\{\chi_{j,k}\Delta_{\left(\alpha_j+\alpha_k\right)\tau}+\Delta_{\alpha_j\tau}\Delta_{\alpha_k\tau}\right\}}{\alpha_j\alpha_k}\mathrm{e}^{-\left(\alpha_j+\alpha_k\right)w}\\
& \times\left\{1-\mathrm{e}^{-\left(\alpha_j+\alpha_k\right)N_r T}\right\}\left[\begin{aligned}&\frac{\alpha_k}{\alpha_j+\alpha_k}-\frac{1-\mathrm{e}^{-\alpha_k T}}{1-\mathrm{e}^{-\left(\alpha_j+\alpha_k\right)T}}\\ &-\frac{1}{N_r}\left\{\frac{\alpha_k}{\alpha_j+\alpha_k}-\frac{1-\mathrm{e}^{-\alpha_k N_r T}}{1-\mathrm{e}^{-\left(\alpha_j+\alpha_k\right)N_r T}}\right\}\end{aligned}\right].
\end{aligned} \tag{52}$$

The theoretical formula $\langle Y(w,T)\rangle$ can now be derived by substituting Eqs. (51) and (52) into Eq. (5). However, in the modified Feynman-alpha technique, thus obtained formula (rigorous one, hereafter) is not utilized because it is too complicated. Instead, let us introduce a wider masking window $\widetilde{w}$ that allows the following approximation,

$$\mathrm{e}^{-\alpha_{\mathrm{p}}\widetilde{w}} \simeq 0. \tag{53}$$

Equations (51) and (52) with $\widetilde{w}$, i.e., $\langle M(\widetilde{w},T)\rangle$ and $\langle V(\widetilde{w},T)\rangle$, are written as follows:

$$\langle M(\widetilde{w},T)\rangle = \lambda_{\mathrm{d}}\langle\xi\rangle\frac{1}{N_r}\frac{\Omega_{\mathrm{p,d}}\Delta_{\alpha_{\mathrm{d}}\tau}}{\alpha_{\mathrm{d}}}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}\right), \tag{54}$$

$$\begin{aligned}
\langle V(\widetilde{w},T)\rangle =& \lambda_{\mathrm{d}}\langle\xi\rangle\frac{1}{N_r}\frac{\Omega_{\mathrm{p,d}}\Delta_{\alpha_{\mathrm{d}}\tau}}{\alpha_{\mathrm{d}}}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1-\frac{1}{N_r}\right)\left(1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}\right)\\
&+2\lambda_{\mathrm{d}}^2\lambda_{\mathrm{f}}\langle\xi\rangle\frac{1}{N_r}\frac{\left(\theta_{\mathrm{d,p,d}}+\theta_{\mathrm{d,d,d}}\right)\Delta_{\alpha_{\mathrm{d}}\tau}}{\alpha_{\mathrm{d}}^2}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}\right)\\
&\times\left\{1-\frac{\alpha_{\mathrm{d}}T\mathrm{e}^{-\alpha_{\mathrm{d}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(1-\frac{\alpha_{\mathrm{d}}N_rT\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}\\
&+2\lambda_{\mathrm{d}}^2\lambda_{\mathrm{f}}\langle\xi\rangle\frac{1}{N_r}\frac{\left(\theta_{\mathrm{d,p,p}}+\theta_{\mathrm{d,d,p}}\right)\Delta_{\alpha_{\mathrm{d}}\tau}}{\alpha_{\mathrm{d}}\left(\alpha_{\mathrm{p}}-\alpha_{\mathrm{d}}\right)}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}\right)\\
&\times\left\{1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}N_rT}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}\\
&-2\lambda_{\mathrm{d}}^2\langle\xi\rangle^2\frac{1}{N_r}\frac{\eta_{\mathrm{d,d}}\left\{\chi_{\mathrm{d,d}}\Delta_{2\alpha_{\mathrm{d}}\tau}+\Delta_{\alpha_{\mathrm{d}}\tau}^2\right\}}{\alpha_{\mathrm{d}}^2}\mathrm{e}^{-2\alpha_{\mathrm{d}}\widetilde{w}}\left(1-\mathrm{e}^{-2\alpha_{\mathrm{d}}N_rT}\right)\\
&\times\left\{\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}.
\end{aligned}\tag{55}$$

One hence obtains $\langle Y(\widetilde{w},T)\rangle$ as

$$\begin{aligned}
\langle Y(\widetilde{w},T)\rangle =& \frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,d}}+\theta_{\mathrm{d,d,d}}\right)}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\\
&\times\left\{1-\frac{\alpha_{\mathrm{d}}T\mathrm{e}^{-\alpha_{\mathrm{d}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(1-\frac{\alpha_{\mathrm{d}}N_rT\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}\\
&+\frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,p}}+\theta_{\mathrm{d,d,p}}\right)}{\left(\alpha_{\mathrm{p}}-\alpha_{\mathrm{d}}\right)\Omega_{\mathrm{p,d}}}\\
&\times\left\{1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}N_rT}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}\\
&-\frac{2\lambda_{\mathrm{d}}\langle\xi\rangle\eta_{\mathrm{d,d}}\left\{\chi_{\mathrm{d,d}}\frac{\Delta_{2\alpha_{\mathrm{d}}\tau}}{\Delta_{\alpha_{\mathrm{d}}\tau}}+\Delta_{\alpha_{\mathrm{d}}\tau}\right\}}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1+\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}\right)\\
&\times\left\{\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}T}}-\frac{1}{N_r}\left(\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}N_rT}}\right)\right\}\\
&-\frac{1}{N_r}.
\end{aligned}\tag{56}$$

With a longer pulse repetition period τ than the masking window w, one can obtain a greater number of neutron count data with respect to T_0 (see also Fig. 2). Under such a condition, when one restricts the Y curve to a shorter gate width range in determining the neutron decay constant, one can introduce the following approximation (Kitamura et al., 2000),

$$N_r \simeq \frac{NT_0}{T}, \tag{57}$$

owing to a small difference between N_r and N/r. We would like to remind that N_r is the maximum integer that does not exceed N/r. Hence, one re-writes $\langle Y(\widetilde{w},T)\rangle$ as

$$\begin{aligned}
\langle Y(\widetilde{w},T)\rangle =& \frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,d}}+\theta_{\mathrm{d,d,d}}\right)}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\left(1-\frac{\alpha_{\mathrm{d}}T\mathrm{e}^{-\alpha_{\mathrm{d}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}\right)\\
&+\frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,p}}+\theta_{\mathrm{d,d,p}}\right)}{\left(\alpha_{\mathrm{p}}-\alpha_{\mathrm{d}}\right)\Omega_{\mathrm{p,d}}}\left(1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}T}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}T}}\right)\\
&-\frac{2\lambda_{\mathrm{d}}\langle\xi\rangle\eta_{\mathrm{d,d}}\left\{\chi_{\mathrm{d,d}}\frac{\Delta_{2\alpha_{\mathrm{d}}\tau}}{\Delta_{\alpha_{\mathrm{d}}\tau}}+\Delta_{\alpha_{\mathrm{d}}\tau}\right\}}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\left(1+\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}\right)\left(\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}T}}\right)\\
&-\phi(\widetilde{w})\,T,
\end{aligned} \tag{58}$$

where

$$\begin{aligned}
\phi(\widetilde{w}) \equiv& \frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,d}}+\theta_{\mathrm{d,d,d}}\right)}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\frac{1}{NT_0}\left(1-\frac{\alpha_{\mathrm{d}}NT_0\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}}\right)\\
&+\frac{2\lambda_{\mathrm{d}}\lambda_{\mathrm{f}}\left(\theta_{\mathrm{d,p,p}}+\theta_{\mathrm{d,d,p}}\right)}{\left(\alpha_{\mathrm{p}}-\alpha_{\mathrm{d}}\right)\Omega_{\mathrm{p,d}}}\frac{1}{NT_0}\left(1-\frac{\alpha_{\mathrm{d}}}{\alpha_{\mathrm{p}}}\frac{1-\mathrm{e}^{-\alpha_{\mathrm{p}}NT_0}}{1-\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}}\right)\\
&-\frac{2\lambda_{\mathrm{d}}\langle\xi\rangle\eta_{\mathrm{d,d}}\left\{\chi_{\mathrm{d,d}}\frac{\Delta_{2\alpha_{\mathrm{d}}\tau}}{\Delta_{\alpha_{\mathrm{d}}\tau}}+\Delta_{\alpha_{\mathrm{d}}\tau}\right\}}{\alpha_{\mathrm{d}}\Omega_{\mathrm{p,d}}}\mathrm{e}^{-\alpha_{\mathrm{d}}\widetilde{w}}\frac{1}{NT_0}\left(1+\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}\right)\\
&\times\left(\frac{1}{2}-\frac{1}{1+\mathrm{e}^{-\alpha_{\mathrm{d}}NT_0}}\right)\\
&+\frac{1}{NT_0}.
\end{aligned} \tag{59}$$

When one utilizes the Y curve of a shorter gate width range in determining the neutron decay constant, one can further introduce the following approximation,

$$\alpha_{\mathrm{d}} T \simeq 0. \tag{60}$$

Therefore, one can apply the following theoretical formula (simple one, hereafter):

$$\langle Y(\widetilde{w}, T)\rangle = \mathcal{Y}_0 \left(1 - \frac{1 - \mathrm{e}^{-\alpha_{\mathrm{p}} T}}{\alpha_{\mathrm{p}} T}\right) - \mathcal{Y}_1 T, \tag{61}$$

where

$$\mathcal{Y}_0 \equiv \frac{2\lambda_{\mathrm{d}} \lambda_{\mathrm{f}} \left(\theta_{\mathrm{d,p,p}} + \theta_{\mathrm{d,d,p}}\right)}{\left(\alpha_{\mathrm{p}} - \alpha_{\mathrm{d}}\right) \Omega_{\mathrm{p,d}}}, \tag{62}$$

$$\begin{aligned} \mathcal{Y}_1 \equiv \phi(\widetilde{w}) &- \frac{\lambda_{\mathrm{d}} \lambda_{\mathrm{f}} \left(\theta_{\mathrm{d,p,d}} + \theta_{\mathrm{d,d,d}}\right)}{\Omega_{\mathrm{p,d}}} \\ &- \frac{\lambda_{\mathrm{d}} \langle \xi \rangle \eta_{\mathrm{d,d}} \left\{\chi_{\mathrm{d,d}} \frac{\Delta_{2\alpha_{\mathrm{d}}\tau}}{\Delta_{\alpha_{\mathrm{d}}\tau}} + \Delta_{\alpha_{\mathrm{d}}\tau}\right\}}{2\Omega_{\mathrm{p,d}}} \mathrm{e}^{-\alpha_{\mathrm{d}} \widetilde{w}} \left(1 + \mathrm{e}^{-\alpha_{\mathrm{d}} N T_0}\right). \end{aligned} \tag{63}$$

We would like to note here that the second term $-\mathcal{Y}_1 T$ is added to the formula utilized in the subcriticality measurement experiment at the KUCA (Misawa et al., 2014) because the finite measurement time effect (Wallerbos and Hoogenboom, 1998) and the delayed neutrons were explicitly taken into consideration in the present study. However, Eq. (61) is still much simpler than those of some Feynman-alpha techniques for the ADS with pulse mode developed so far.

4. Discussions

4.1. Verification of theoretical formula by Monte Carlo calculations

In Section 3, the experimental conditions under which the simple formula Eq. (61) works were extracted; a wider masking window, a longer pulse repetition period of neutron bursts, and restricted utilization of Y curve to a shorter gate width range. Among them, the third one is not peculiar to the modified Feynman-alpha technique, rather very common in the conventional Feynman-alpha techniques. Hence, in the present section, the remaining two experimental conditions are discussed by using the rigorous formula $\langle Y(w,T)\rangle$ that is derived by substituting Eqs. (51) and (52) into Eq. (5). Prior to the discussion, verification of the rigorous formula is provided in this subsection.

To verify the rigorous formula, a series of mono-energy analogue Monte Carlo calculations, where a typical thermal subcritical system with a subcriticality of 2.9 $\%\Delta k/k$ is supposed, was carried out. The parameters for these Monte Carlo calculations are listed in Table 1. Because of the neutron life time of the order of 10^{-5} s, one understands that the system is belonging to the light water one. In these Monte Carlo calculations, the time of every neutron detection event in an imaginary neutron detector was tallied to obtain the time series data. For simplicity, the probability that n prompt neutrons and c delayed neutron precursors are born in one fission reaction, i.e., $p_{\mathrm{f}}(n,c)$, was assumed to be expressed as

$$p_{\mathrm{f}}(n,c) \equiv p_{\mathrm{f,p}}(n) \cdot p_{\mathrm{f,d}}(c), \tag{64}$$

where $p_{\mathrm{f,p}}(n)$ is the probability that n prompt neutrons are born in one fission reaction, and $p_{\mathrm{f,d}}(c)$ the probability that c delayed neutron precursors are born. In the present calculations, a binomial type distribution by Diven et al. (1956) was supposed for respective probabilities in determining the numbers of neutrons or precursors born in one fission reaction, i.e.,

$$p_{\mathrm{f,p}}(n) \equiv \frac{5!}{n!\,(5-n)!}\left[\frac{\langle \nu_{\mathrm{p}}\rangle}{5}\right]^{n}\left[1-\frac{\langle \nu_{\mathrm{p}}\rangle}{5}\right]^{5-n}, \quad n = 0,1,\ldots,5, \tag{65}$$

$$p_{\mathrm{f,d}}(c) \equiv \frac{2!}{c!\,(2-c)!}\left[\frac{\langle \nu_{\mathrm{d}}\rangle}{2}\right]^{c}\left[1-\frac{\langle \nu_{\mathrm{d}}\rangle}{2}\right]^{2-c}, \quad c = 0,1,2. \tag{66}$$

On the other hand, as seen in Table 1, the probability that q neutrons are born in one neutron burst, i.e., $p_{\mathrm{b}}(q)$, is defined on the basis of the Poisson distribution.

In the Monte Carlo calculations, 10,000 sets of the time series data where the neutron bursts with a pulse repetition period τ of 0.25 s start at time -800 s and finish at

Table 1: Parameters for Monte Carlo calculations.

Parameter:	Value
λ_{s} [s^{-1}]:	1310617.02249
λ_{c} [s^{-1}]:	33196.23153
λ_{f} [s^{-1}]:	21685.74903
λ_{d} [s^{-1}]:	334.85040
$\langle \nu_{\mathrm{p}} \rangle$ [-]:	2.4564375
$\langle \nu_{\mathrm{d}} \rangle$ [-]:	0.0185625
β [-]:	0.0075
$p_{\mathrm{f}}(n, c)$ [-]:	$p_{\mathrm{f,p}}(n) \cdot p_{\mathrm{f,d}}(c)$
$p_{\mathrm{f,p}}(n)$ [-]:	0.0340692086 ($n = 0$)
	0.1645111565 ($n = 1$)
	0.3177522660 ($n = 2$)
	0.3068682534 ($n = 3$)
	0.1481785262 ($n = 4$)
	0.0286205893 ($n = 5$)
$p_{\mathrm{f,d}}(c)$ [-]:	0.9815236416 ($c = 0$)
	0.0183902168 ($c = 1$)
	0.0000861416 ($c = 2$)
$\langle \xi \rangle$ [-]:	25000
$p_{\mathrm{s}}(q)$ [-]:	$\begin{cases} 0, & 0 \leq q < 24900, \quad 25100 \leq q \\ \dfrac{100^{(q-24900)} \mathrm{e}^{-100}}{(q-24900)!}, & 24900 \leq q < 25100 \end{cases}$
λ [s^{-1}]:	0.077
τ [s]:	0.25

time 0 s were obtained. By using the 10,000 sets of time series data between 0 s and 0.25 s, the Y curves were calculated by introducing various widths of the masking window w. Thus obtained Y curves and the corresponding rigorous formula are plotted in Fig. 8. In this figure, one confirms that the rigorous formula agree well with the Y curves by the Monte Carlo calculations. Therefore, it is considered that the rigorous formula derived in the present study was verified.

4.2. Experimental conditions for modified Feynman-alpha technique

By using the rigorous formula $\langle Y(w, T) \rangle$, various Y curves were plotted with the parameters listed in Table 1 by changing the pulse repetition period τ from $10^{+1.6}/\alpha_\mathrm{p}$ to $10^{+3.0}/\alpha_\mathrm{p}$ and the masking window width w from $10^{+0.0}/\alpha_\mathrm{p}$ to $10^{+3.0}/\alpha_\mathrm{p}$. To thus plotted Y curves ranging up to 5 ms at most, the simple formula, i.e., Eqs. (61), was applied to determine $\mathcal{Y}_0$, $\mathcal{Y}_1$, and α_p by the least square fitting technique. The deviations of the α_p values thus determined from the reference (1947.16 s^{-1}) that can be calculated by the parameters listed in Table 1 were plotted in Fig. 9.

It is seen in Fig. 9 that one cannot stably determine the α_p value within 1% error when the pulse repetition period τ is less than $10^{+2.0}/\alpha_\mathrm{p}$. At the same time, one finds that the masking window w ranging from $10^{+1.0}/\alpha_\mathrm{p}$ to $10^{+1.2}/\alpha_\mathrm{p}$ is suitable to determine the α_p value.

5. Conclusion and future work

In the present paper, a theory of the modified Feynman-alpha technique for the subcritical system driven by periodically triggered neutron bursts was developed by the de Hoffmann's formulation approach to apply this technique to the subcriticality monitor for the accelerator-driven system. Through derivation of the theoretical formula for determining the neutron decay constant α_p by this technique, the experimental conditions under which this formula works were discussed. A numerical investigation revealed that these are the pulse repetition period of neutron bursts that is longer than $10^{+2.0}/\alpha_\mathrm{p}$ and the masking window ranging from $10^{+1.0}/\alpha_\mathrm{p}$ to $10^{+1.2}/\alpha_\mathrm{p}$.

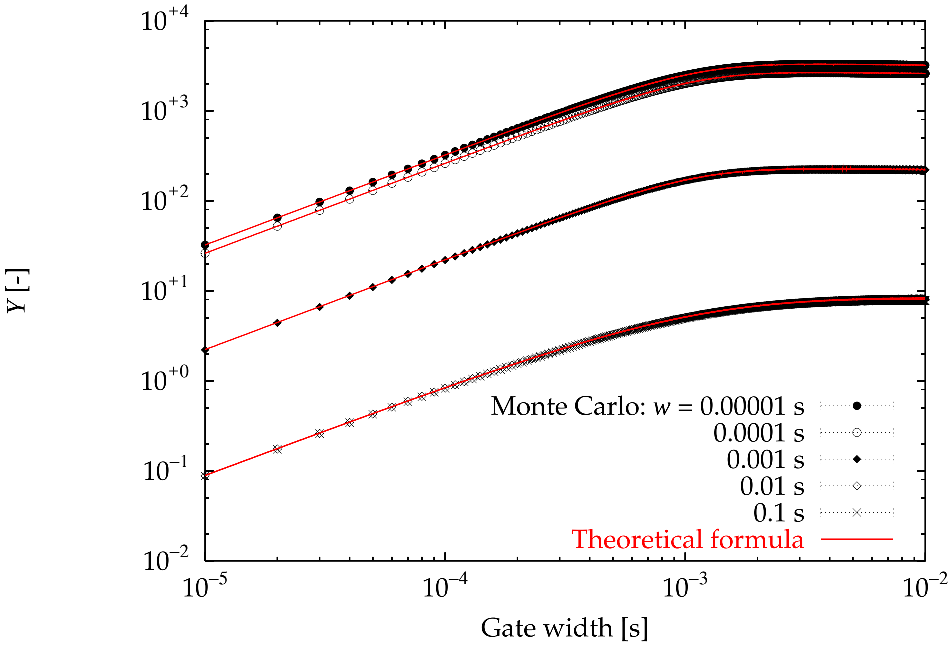

Figure 8: $\langle Y(w, T) \rangle$ curves by Monte Carlo calculations and theoretical formula.

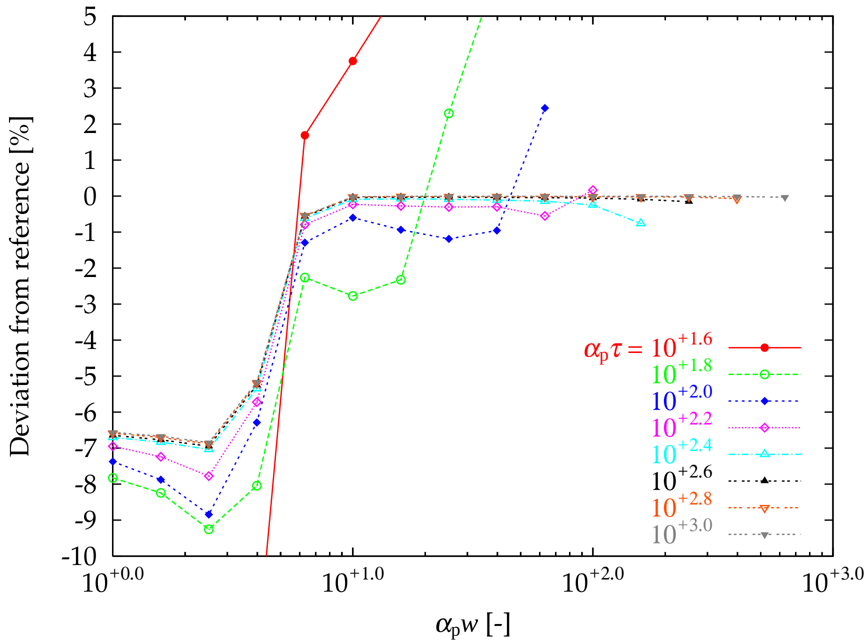

Figure 9: Deviation of α_{p} determined by modified Feynman-alpha technique from reference.

As mentioned before, the other feature of this technique is a possibility of suppressing the space dependency effect in evaluating the subcriticality. In a forthcoming paper, hence, discussions on this feature will be reported.

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Appendix A. Population of progeny neutrons originating from one ancestor neutron

In the de Hoffmann's formulation approach, the population of progeny neutrons at time t originating from one ancestor neutron born at time 0, i.e., $\mathcal{G}_{\mathrm{p}}(t)$, plays a very essential role, so that it is derived here.

The reactor kinetic equations with one group of delayed neutron precursors are written as follows:

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = -\alpha N(t) + \lambda C(t), & \text{(A.1)} \\ \dfrac{\mathrm{d}C(t)}{\mathrm{d}t} = -\lambda C(t) + \dfrac{\beta}{\Lambda} N(t), & \text{(A.2)} \end{cases}$$

where $N(t)$ and $C(t)$ are the populations of neutrons and delayed neutron precursors in the system of interest, respectively. Replacing $N(t)$ with $\mathcal{G}_{\mathrm{p}}(t)$ and $C(t)$ with $\mathcal{H}_{\mathrm{p}}(t)$ then adding the Dirac's delta $\delta(t)$ to the right-hand side of Eq. (A.1), one obtains

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{G}_{\mathrm{p}}(t)}{\mathrm{d}t} = -\alpha \mathcal{G}_{\mathrm{p}}(t) + \lambda \mathcal{H}_{\mathrm{p}}(t) + \delta(t), & \text{(A.3)} \\ \dfrac{\mathrm{d}\mathcal{H}_{\mathrm{p}}(t)}{\mathrm{d}t} = -\lambda \mathcal{H}_{\mathrm{p}}(t) + \dfrac{\beta}{\Lambda} \mathcal{G}_{\mathrm{p}}(t), & \text{(A.4)} \end{cases}$$

where $\mathcal{H}_{\mathrm{p}}(t)$ is the population of progeny delayed neutron precursors at t originating from one ancestor neutron born at 0. Since the initial conditions of $\mathcal{G}_{\mathrm{p}}(t)$ and $\mathcal{H}_{\mathrm{p}}(t)$ are read as

$$\begin{cases} \mathcal{G}_{\mathrm{p}}(0) = 0, & \text{(A.5)} \\ \mathcal{H}_{\mathrm{p}}(0) = 0, & \text{(A.6)} \end{cases}$$

their Laplace transforms, i.e., $\widehat{\mathcal{G}}_{\mathrm{p}}(s)$ and $\widehat{\mathcal{H}}_{\mathrm{p}}(s)$, are calculated as

$$\begin{cases} \widehat{\mathcal{G}}_{\mathrm{p}}(s) = \dfrac{s + \lambda}{s^2 + (\alpha + \lambda) s - \frac{\lambda \rho}{\Lambda}}, & \text{(A.7)} \\ \widehat{\mathcal{H}}_{\mathrm{p}}(s) = \dfrac{\frac{\beta}{\Lambda}}{s^2 + (\alpha + \lambda) s - \frac{\lambda \rho}{\Lambda}}. & \text{(A.8)} \end{cases}$$

By introducing

$$\alpha_{\mathrm{p}} \equiv \frac{(\alpha+\lambda)\,\Lambda + \sqrt{(\alpha+\lambda)^2\,\Lambda^2 + 4\lambda\Lambda\rho}}{2\Lambda}, \tag{A.9}$$

$$\alpha_{\mathrm{d}} \equiv \frac{(\alpha+\lambda)\,\Lambda - \sqrt{(\alpha+\lambda)^2\,\Lambda^2 + 4\lambda\Lambda\rho}}{2\Lambda}, \tag{A.10}$$

$$\Omega_{\mathrm{p},i} \equiv \frac{\alpha_i - \lambda}{\alpha_i - \alpha_{i'}}, \quad i = \mathrm{p}, \mathrm{d}, \quad i' \neq i, \tag{A.11}$$

one re-writes Eq. (A.7) as

$$\widehat{\mathcal{G}}_{\mathrm{p}}(s) = \sum_{i=\mathrm{p,d}} \frac{\Omega_{\mathrm{p},i}}{s + \alpha_i}. \tag{A.12}$$

Therefore, $\mathcal{G}_{\mathrm{p}}(t)$ is derived as follows:

$$\mathcal{G}_{\mathrm{p}}(t) = \sum_{i=\mathrm{p,d}} \Omega_{\mathrm{p},i} \mathrm{e}^{-\alpha_i t}. \tag{A.13}$$

Appendix B. Population of progeny neutrons originating from one ancestor delayed neutron precursor

Here, derivation of the population of progeny neutrons at time t originating from one ancestor delayed neutron precursor born at time 0, i.e., $\mathcal{G}_{\mathrm{d}}(t)$, is provided.

By replacing $N(t)$ with $\mathcal{G}_{\mathrm{d}}(t)$ and $C(t)$ with $\mathcal{H}_{\mathrm{d}}(t)$ then adding the Dirac's delta $\delta(t)$ to the right-hand side of Eq. (A.2), one obtains

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{G}_{\mathrm{d}}(t)}{\mathrm{d}t} = -\alpha\mathcal{G}_{\mathrm{d}}(t) + \lambda\mathcal{H}_{\mathrm{d}}(t), & \text{(B.1)} \\ \dfrac{\mathrm{d}\mathcal{H}_{\mathrm{d}}(t)}{\mathrm{d}t} = -\lambda\mathcal{H}_{\mathrm{d}}(t) + \dfrac{\beta}{\Lambda}\mathcal{G}_{\mathrm{d}}(t) + \delta(t), & \text{(B.2)} \end{cases}$$

where $\mathcal{H}_{\mathrm{d}}(t)$ is the population of progeny delayed neutron precursors at t originating from one ancestor delayed neutron precursors born at 0. Since the initial conditions of $\mathcal{G}_{\mathrm{d}}(t)$ and $\mathcal{H}_{\mathrm{d}}(t)$ are read as

$$\begin{cases} \mathcal{G}_{\mathrm{d}}(0) = 0, & \text{(B.3)} \\ \mathcal{H}_{\mathrm{d}}(0) = 0, & \text{(B.4)} \end{cases}$$

their Laplace transforms, i.e., $\widehat{\mathcal{G}}_{\mathrm{d}}(s)$ and $\widehat{\mathcal{H}}_{\mathrm{d}}(s)$, are calculated as

$$\begin{cases} \widehat{\mathcal{G}}_{\mathrm{d}}(s) = \dfrac{\lambda}{s^2 + (\alpha + \lambda)\, s - \frac{\lambda\rho}{\Lambda}}, & \text{(B.5)} \\ \widehat{\mathcal{H}}_{\mathrm{d}}(s) = \dfrac{s + \alpha}{s^2 + (\alpha + \lambda)\, s - \frac{\lambda\rho}{\Lambda}}. & \text{(B.6)} \end{cases}$$

By introducing

$$\Omega_{\mathrm{d},i} \equiv -\frac{\lambda}{\alpha_i - \alpha_{i'}}, \quad i = \mathrm{p}, \mathrm{d}, \quad i' \neq i, \tag{B.7}$$

one re-writes Eq. (B.5) as

$$\widehat{\mathcal{G}}_{\mathrm{d}}(s) = \sum_{i=\mathrm{p,d}} \frac{\Omega_{\mathrm{d},i}}{s + \alpha_i}. \tag{B.8}$$

Therefore, $\mathcal{G}_{\mathrm{d}}(t)$ is derived as follows:

$$\mathcal{G}_{\mathrm{d}}(t) = \sum_{i=\mathrm{p,d}} \Omega_{\mathrm{d},i} \mathrm{e}^{-\alpha_i t}. \tag{B.9}$$