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Impact of Country-Specific Fixed Costs on the International Location of Firms, Comparative Advantage, and the Distribution of Trade Gains

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ABSTRACT

We theoretically investigate how country-specific fixed costs (CSFCs) affect the international location of firms, comparative advantage, and the distribution of trade gains, by presenting a two-country trade model of monopolistic competition with CSFCs. Key settings are that the expenditure shares of a homogeneous good and a composite differentiated good are constant, and that the only difference across countries is in terms of fixed costs (Ricardian aspect). We derive the following results. A country with smaller fixed costs (home country) has a greater-than-proportional share of the firms of differentiated goods, a comparative advantage in differentiated goods, and higher trade gains. A unilateral decrease in CSFCs of the home (foreign) country increases (reduces) these inequalities around the arbitrary trading equilibrium with incomplete specialization. When the CSFCs decrease bilaterally, the resulting impacts depend on the relative rate at which CSFCs change.

Keywords: Country-Specific Fixed Costs, International Location of Firms, Comparative Advantage, Distribution of Trade Gains.

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1 Introduction

When we consider sources of comparative advantage and the distribution of trade gains,[1] as well as the driving force of firm-level international location, we find that there is heterogeneity across countries. Trade economists have focused on three major sources: Ricardo's theory (technological differences), Heckscher–Ohlin's theory (factor proportion), and Krugman's theory (market size, factor size). However, in Ricardo's theory, it seems that the role of asymmetric fixed costs (*country-specific fixed costs*, hereafter CSFCs) has not yet been revealed; the current study endeavors to do so.

It is important to understand the impact of CSFCs on the international location of firms, comparative advantage, and the distribution of trade gains. Fixed costs play a key role in firms' decisions regarding international location. The impacts of fixed costs have been confirmed by many studies, particularly since the influential study of Melitz (2003). Notably, fixed costs differ across countries, and so they are country-specific.[2] This difference in fixed costs across countries significantly affects the international location of firms. In fact, many empirical studies have shown that CSFCs significantly impact the international location of firms (e.g., Blanes-Cristóbal et al., 2008; Gullstrand, 2011; Maurseth and Medin, 2013). In addition, fixed costs are also important policy targets.[3] In this manner, CSFCs encompass not only technology, but also government policy. However, few studies have theoretically investigated the impact of CSFCs on the international location of firms, comparative advantage, and the distribution of trade gains.

One exception is the study of Venables (1987), which presents a two-country trade model of monopolistic competition with asymmetric CSFCs across countries (Ricardian model).[4] Using such a model, Venables (1987) examines the impact of a decrease in fixed cost in one country on the international location of firms, comparative advantage, and welfare (trade gains).[5] By introducing CSFCs, Venables

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[1] For the distribution of trade gains, the conventional arguments of Mill's paradox—as shown by Chipman (1965)—and Bhagwati's immiserizing growth—as shown by Bhagwati (1958)—are particularly well known. These measure the distribution of trade gains by examining change in terms of trade in a perfectly competitive setting.

[2] For example, firms must pay fixed costs at the time of company incorporation, and for product development, divisional administration, the construction of a distribution network, and advertising in various markets. The efficiencies of these economic activities differ across countries.

[3] For example, if a government strengthens regulations concerning firm entry, then the above efficiencies may be affected.

[4] The Venables model has two sectors (perfectly and monopolistically competitive sectors) and one production factor (labor). The transport cost is positive in the monopolistically competitive sector, whereas it is 0 in a perfectly competitive sector. In the final section of Venables (1987), the trade cost variable is not transport cost but trade tax.

[5] Proposition 3 of Venables (1987) asserts that a decrease in a country's fixed costs increases both the number of firms and the welfare (trade gains) in one country while reducing them in the other country. Proposition 4 of Venables (1987), meanwhile, asserts that a decrease in a country's fixed costs increases the net exports of differentiated goods in that country.

(1987) provides an important basis for the aforementioned research line regarding Ricardo's theory.

However, the Venables model has some significant limitations. In using the Venables model, it is difficult to explicitly show whether or not a country with smaller CSFCs has a greater-than-proportional share of firms of differentiated goods, a comparative advantage in differentiated goods, and higher trade gains. Furthermore, Proposition 4 of Venables (1987) speaks to the impact of a decrease in fixed cost on net exports, but only when two countries are initially symmetric. It is unknown how this result can be extended to other equilibria. In the current study, we clarify these ambiguous points by presenting a modified Venables model.

We modify the Venables model as follows, to obtain stronger results. We assume a constant expenditure share of a homogeneous good and a composite differentiated good; Venables (1987), on the other hand, assumes a variable expenditure share. Furthermore, we assume that two countries differ only in terms of fixed costs (CSFCs), whereas Venables (1987) assumes multiple asymmetries.[6] We newly introduce the fixed cost of selling to the export market (fixed trade cost) to derive a more precise understanding of CSFCs. Here, then, CSFCs are defined by the sum of fixed costs related to selling to domestic and export markets. We assume that the home country has smaller CSFCs than does the foreign country, while the foreign country has larger CSFCs. The other aspects of our model are identical to those in the Venables model.

Under these settings, we can obtain the following results: (1) The home (foreign) country has a more (less)-than-proportional share of firms of differentiated goods (here-after, *the CSFCs effect*). (2) The home (foreign) country has a comparative advantage in differentiated goods (homogeneous goods). (3) The home (foreign) country has higher (lower) trade gains (a state that is defined as the rate of change in welfare) by opening up to trade. (4) The above statements (1), (2), and (3) are equivalent. Results (1)–(3) show that relative CSFCs determine relative firm location, new exports, and trade gains. (5) A unilateral decrease in the absolute CSFCs of a country increases the number of firms, the net exports of differentiated goods, and the trade gains in that country, while it reduces those in the other country. That is, a unilateral decrease in the CSFCs of the home (foreign) country increases (reduces) these inequalities between the two countries.

The contributions of this study to the literature are as follows. Its main contribution derives from our set of results (1)–(4). It is difficult to describe these results in terms of the Venables model, given the model's complexity. We can obtain these by using simple settings (in particular, using a constant expenditure share and focusing on CSFCs in terms of country heterogeneity). Result (5) holds true for an arbitrary trading equilibrium with incomplete specialization, whereas Venables (1987) shows a similar result for comparative advantage when two countries are

[6] The following points are asymmetric in both countries: marginal cost in the perfectly competitive sector, marginal and fixed costs in the monopolistically competitive sector, country size, and preference of households for domestic and imported brands.

initially symmetric. In this way, the model in the current study shows properties that the Venables model cannot.

The effects of CSFCs seem to correspond to the *home market effect* (hereafter HME) of Helpman and Krugman (1985, Ch. 10.4). Helpman and Krugman (1985) construct a model in which market size (population) differs across countries;[7] they also show that the larger (smaller) a country is in terms of market size, the larger (smaller) its proportional share of firms of differentiated goods will be. They refer to this result as the HME. However, the CSFCs effect and the HME differ with regards to the following two points. First, with the HME, firm sizes are identical across countries, whereas with the CSFCs effect, the firm size is smaller in the country that has smaller fixed costs. Second, governments can more easily control CSFCs than market size (population).

Results (3) and (5) above have implications regarding the distribution of trade gains, and they run counter to those found in the literature. In a model of Helpman and Krugman (1985, Ch. 10.4), trade gains are equalized across larger and smaller countries (equal distribution);[8] on the other hand, result (3) shows an unequal distribution of trade gains. More interestingly, in the conventional argument inherent in immiserizing growth with a perfectly competitive setting—as shown by Bhagwati (1958)—export-oriented technological progress reduces trade gains, in the sense that the terms of trade deteriorate; result (5), however, shows a decrease in CSFCs in a country that increases both its net exports of differentiated goods and its trade gains.

Let us look at the current study's place within the literature, and other studies that relate to it. The current study belongs to a research line that theoretically investigates the impacts of country asymmetry on a firm's international location, comparative advantage, and the distribution of trade gains.[9] As mentioned, this research line has three major approaches—namely, Ricardo's theory, Heckscher–Ohlin's theory, and Krugman's theory; these focus, respectively, on asymmetric technologies, factor proportions, and market sizes (factor volumes) across countries as determinants of comparative advantage and firm location. The current study belongs to the research line that leverages Ricardo's theory.[10]

Ricardo's theories can be decomposed into two branches that focus on asymmetric marginal and fixed costs across countries; the current study belongs to the latter branch.[11] One can also find within this branch the work of Venables (1987),

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[7] The other difference between these models is that in Helpman and Krugman (1985), the fixed export cost is 0.

[8] This property is revealed by Kikuchi (2001), who adopts the model of Helpman and Krugman (1985, Ch. 10.4).

[9] To analyze explicitly a firm's location, many studies focus on monopolistic competition as a market structure (Chamberlinian aspect).

[10] The research line of Heckscher–Ohlin's theory includes Helpman (1981), Kikuchi and Shimomura (2006), and Bernard, Redding, and Schott (2007). The research line of Krugman's theory includes Krugman (1980), Helpman and Krugman (1985, Ch. 10.4), and Medin (2003).

[11] In the former branch, one can find the studies of Okubo (2009) and Falvey, Greenaway, and Yu (2011).

Kikuchi (2004), Kikuchi, Shimomura, and Zeng (2008), and Fukushima and Kikuchi (2009); these studies consider asymmetric marginal costs across countries, in addition to CSFCs, and do not examine the impact of CSFCs on firm location, comparative advantage, or the distribution of trade gains. The current study focuses purely on CSFCs and their impact.

The remainder of this paper is organized as follows. Section 2 establishes the model and obtains the trading equilibrium. Section 3 analyzes the impacts of relative CSFCs on the international location of firms, comparative advantage, and the distribution of trade gains. Section 4 analyzes comparative statics—that is, how a decrease in absolute CSFCs affects the relationship described in Section 3. Section 5 concludes the paper. The Appendices contain the proofs of the propositions, as well as the derivations of several equations.

2 The model

2.1 Economy

There are two countries (home and foreign), two sectors (perfectly competitive sector and monopolistically competitive sector), and one factor (labor). There are L units of households in each country. Labor is a numeraire, so the wage rate is normalized to 1 in both countries. We mark the foreign country's variables with an asterisk (*).

A key feature of the current model is how it is a modification of the Venables (1987) model, in the following respects. The Venables model has a general utility function and assumes a variable expenditure share of a homogeneous good and a composite differentiated good. The Venables model also features some asymmetries across countries. In contrast, the current model assumes a constant expenditure share of the homogeneous good and the composite differentiated good, and it considers an economy in which the two countries differ only in terms of fixed costs (CSFCs). In addition, we newly introduce fixed export costs; CSFCs are then defined as the sum of fixed costs associated with selling to domestic and export markets.

2.2 Households

We describe the behavior of a household in the home country. A household supplies one unit of labor inelastically with a Cobb–Douglas utility function $U = c_0^{1-s} C^s$, where c_0 denotes the consumption of a homogeneous good, and C denotes the consumption of a composite differentiated good. Hence, the expenditure shares of the homogeneous good and the composite differentiated good are $1 - s$ and s, respectively, where $0 < s < 1$ holds true. Under zero-profit conditions, the household's income is 1, because labor income is 1. Then, $c_0 = c_0 = \ -s$ holds true. The composite differentiated good is assumed to be $C = \left[\int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega^* \in \Omega^*} c'(\omega^*)^{\frac{\sigma-1}{\sigma}} d\omega^* \right]^{\frac{\sigma}{\sigma-1}}$, where Ω and Ω^* are continuum sets and represent the mass of available differentiated goods produced by firms of the home and foreign countries, respectively; $c(\omega)$ and $c'(\omega^*)$ represent the consumption of varieties of goods of the home country ω and the foreign country ω^*, respectively; and $\sigma(>1)$ is the elasticity of substitution between any two varieties and also represents the price elasticity of demand for each variety. The price index P (dual to the aggregator C) is given by $P=\left[\int_{\omega\in\Omega}(p(\omega))^{1-\sigma}d\omega+\int_{\omega^*\in\Omega^*}\left[p_x^*(\omega^*)\right]^{1-\sigma}d\omega^*\right]^{1/(1-\sigma)}$, where p and p_x^* are the prices of varieties produced by home firms for domestic markets and by foreign firms for export markets, respectively. Firms must export $\tau(>1)$ units of good to send one unit (iceberg transport cost) to the export market. Hence, the price of varieties produced by foreign firms for export markets (home consumers) p_x^* is given by $p_x^*=\tau p^*$, where p^* is the price of varieties produced by foreign firms for the domestic market (foreign consumers).

Consumption by home-country households of domestic and foreign brands is, respectively, $c=sp^{-\sigma}P^{\sigma-1}$ and $c'=s(\tau p)^{-\sigma}P^{\sigma-1}$, where $\tau>1$ represents transport costs. Households in both countries are symmetric; hence, consumption by foreign-country households of foreign and domestic brands is, respectively, $c^*=sp^{-\sigma}P^{*\sigma-1}$ and $c^{*\prime}=s(\tau p)^{-\sigma}P^{*\sigma-1}$. In equilibrium, all firms set an identical price; hence, the price indexes in the home and foreign countries are $P^{1-\sigma}=p^{1-\sigma}\hat{M}$ and $P^{*1-\sigma}=p^{*1-\sigma}\hat{M}^*$, where $\hat{M}$ and $\hat{M}^*$ are defined, respectively, by

$$\hat{M}=M+\tau^{1-\sigma}M^*,\quad \hat{M}^*=M^*+\tau^{1-\sigma}M.$$

Here, M is the number of varieties produced in the home country, while M^* is that produced in foreign countries. That is, $\hat{M}$ is the number of varieties available in the home market (and $\hat{M}^*$, in the foreign markets) in which the number of imported brands is discounted by the transport costs. We call these the *effective numbers of varieties*.[12]

2.3 Firms

We describe firm behavior, particularly for the home country. In the perfectly competitive sector, firms can produce one unit of a homogeneous good by inputting one unit of labor. Under free entry and exit, the price of the homogeneous good, p_0, becomes 1. We focus on an economy in which the homogeneous good is produced in both countries; hence, $p_0=p_0^*=1$ is attained.

In the following paragraphs, we describe firm behavior in the monopolistically competitive sector.

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[12] We interpret $M(M^*)$ and $\hat{M}(\hat{M}^*)$ as measures of the numbers of firms. However, strictly speaking, $M(M^*)$ and $\hat{M}(\hat{M}^*)$ are masses, because we assume that Ω and Ω^* are continuum sets. However, in the real world, these are discrete sets. For analytical simplicity, we take the above interpretation.

A firm's entry process is as follows. Firms face two types of specific fixed costs—namely, fixed costs for the domestic market $f_d (> 0)$, and fixed costs for the export market $f_x (> 0)$. Firms cannot export without entering the domestic market (paying f_d), as per the definition of "exporting."

We focus on an economy in which all firms that produce a differentiated good sell in both the domestic and export markets; however, production takes place only in the domestic country.[13] Later,[14] we explicitly impose an assumption so that in trading equilibrium, no firm has an incentive to sell only to its domestic market.

We construct the firm profit as follows. Revenue from selling in the domestic market r_d is given by $r_d = py_d$, where y_d is the output for selling in the domestic market. Revenue from selling in the export market r_x is given by $r_x = p_x(y_x/\tau) = py_x$, where y_x is the output for selling in the export market. Total revenue r is given by $r = r_d + r_x$. r satisfies $r = r_d + r_x = py_d + py_x = py$, where y is the total output and is defined by $y = y_d + y_x$. Profit from selling in the domestic market π_d is given by $\pi_d = py_d - (\phi y_d + f_d)$, where $\phi (> 0)$ shows the marginal costs. Profit from selling in both the domestic and foreign markets π is given by $\pi = r - (\phi y + f)$, where f is the total fixed costs and is defined by $f = f_d + f_x$.

This profit gives the optimal pricing rule as PP: $p = \phi\sigma/(\sigma - 1)$, where $\sigma/(\sigma - 1)$ is the markup rate, and the free entry condition is FE: $p = \phi + f/y$. These conditions give the optimal output of the home country as $y = [(\sigma - 1)/\phi]f$. Similarly, the optimal pricing rule and optimal output of the foreign country are given by $p^* = \phi\sigma/(\sigma - 1)$ and $y^* = [(\sigma - 1)/\phi]f^*$, respectively. We should note that $f \neq f^*$, and thus $y \neq y^*$ holds true, whereas $p = p^*$ holds true.

2.4 Trading equilibrium

We impose market-clearing conditions for the homogeneous good, differentiated goods, and labor. These market-clearing conditions and the behavior of households and firms give rise to the trading equilibrium. The market-clearing conditions of a differentiated good produced by the home country and by foreign countries are represented by $y = Lc + \tau Lc'^*$ and $y^* = Lc^* + \tau Lc'$, respectively. For later analysis, we define the *relative CSFCs* (z) as

$$z \equiv f/f^*.$$

From $f > 0$ and $f^* > 0$, $z > 0$ holds true.

[13] We do not consider multinational enterprises that produce in both domestic and foreign countries.

[14] See Assumption 1 and Lemma 1.

The definition of z, the optimal conditions of utility maximization, the optimal outputs, and the market-clearing conditions of a differentiated good give rise to (1) and (2).[15]

$$\hat{M} = \frac{sL}{\sigma f}\frac{z(1-\tau^{2(1-\sigma)})}{m_1}, \tag{1}$$

$$\hat{M}^* = \frac{sL}{\sigma f}\frac{z(1-\tau^{2(1-\sigma)})}{m_2}, \tag{2}$$

Where m_1 and m_2 are combinations of relative fixed costs and transport cost defined as

$$m_1 \equiv z - \tau^{1-\sigma}, \quad m_2 \equiv 1 - z\tau^{1-\sigma}.$$

The definitions of $\hat{M}$ $\hat{M}$ m_1 and m_2, (1), and (2) give

$$M = \frac{sL}{\sigma f}\frac{zm_3}{m_1 m_2}, \tag{3}$$

$$M^* = \frac{sL}{\sigma f}\frac{zm_4}{m_1 m_2}, \tag{4}$$

Where m_3 and m_4 are combinations of relative fixed costs and the transport cost defined as

$$m_3 \equiv (\tau^{2(1-\sigma)} + 1) - 2\tau^{1-\sigma} z, \quad m_4 \equiv (\tau^{2(1-\sigma)} + 1)z - 2\tau^{1-\sigma}.$$

We focus on an economy in which a differentiated good is produced, and the effective number of varieties is positive in both countries—that is, $M > 0$, $M^* > 0$, $\hat{M} > 0$, and $\hat{M}^* > 0$ hold true. To ensure these conditions, we need $m_1 > 0$, $m_2 > 0$, $m_3 > 0$, and $m_4 > 0$ from (1)–(4) and $\tau > 1$, where $m_3 > 0$ and $m_4 > 0$ imply that $m_1 > 0$ and $m_2 > 0$.[16]

We show the labor market-clearing conditions. We denote the labor input of firms producing a homogeneous and a differentiated good in the home (foreign) country with $l_0(l_0^*)$ and $l(l^*)$, respectively. The labor market-clearing conditions in these countries are $l_0 = L - Ml$ and $l_0^* = L - M^* l^*$, respectively. From the firm technology, $l = \phi y + f$ and $l^* = \phi y^* + f^*$ hold true. The labor market-clearing conditions and $l = \phi y + f$, $l^* = \phi y^* + f^*$, $y = [(\sigma-1)/\phi]f$, and $y^* = [(\sigma-1)/\phi]f^*$ give $l_0 = L\,[1 - (szm_3)/(m_1 m_2)]$

[15] See Appendix.A for the derivation.

[16] In fact, $m_1 > 0$ and $m_2 > 0$ are equivalent to $\tau^{1-\sigma} < z < \tau^{\sigma-1}$; then, $m_3 > 0$ and $m_4 > 0$ imply that $m_1 > 0$ and $m_2 > 0$, since $\tau^{1-\sigma} < 2\tau^{1-\sigma}/(\tau^{2(1-\sigma)} + 1)$ and $(\tau^{2(1-\sigma)} + 1)/(2\tau^{1-\sigma}) < \tau^{\sigma-1}$ hold true.

and $l_0^* = L[1-(sm_4)/(m_1m_2)]$. We focus on an economy in which the homogeneous good is produced in both countries. To ensure these conditions, we need $1 - szm_3/(m_1m_2) > 0$ and $1 - sm_4/(m_1m_2) > 0$ from the above optimal labor input of l_0 and l_0^*.[17]

We construct the export, import, and net exports of the home country in the differentiated goods market. The export value of the differentiated goods of the home country EX is defined as $EX = Mp(y - cL)$. This definition, $c = sp^{-\sigma}P^{\sigma-1}$, $P^{1-\sigma} = P^{1-\sigma}\hat{M}$, $p = \phi\sigma/(\sigma - 1)$, $y = [(\sigma - 1)/\phi]f$, (1), and (3) give $EX = [(sL\tau^{1-\sigma})/(1 - \tau^{2(1-\sigma)})](m_3/m_1)$. The import value of the differentiated goods of the home country IM is defined as $IM = M^*(\tau p^*)c'L$. Similarly, we obtain $IM = [(sL\tau^{1-\sigma})/(1 - \tau^{2(1-\sigma)})](m_4/m_2)$. The net export value of the differentiated goods of the home country NX is defined as $NX = EX - IM$. This definition, EX, and IM derive[18]

$$NX = \frac{sL\tau^{1-\sigma}(1-z^2)}{m_1m_2}. \tag{5}$$

(5) implies that the net export of differentiated goods is nonzero ($NX \neq 0$) only when $z \neq 1$ holds true.

The trade balance equilibrium can be attained from the above conditions. The net export value of the homogeneous goods of the home country, NX_0, is defined as $NX_0 = p_0(y_0 - c_0L)$. $y_0 = l_0$, $c_0 = 1 - s$, $l_0 = L[1 - (szm_3)/(m_1m_2)]$, NX of (5), and the definition of NX_0 derive the trade balance equilibrium condition $NX_0 + NX = 0$.[19]

(5) and $NX_0 + NX = 0$ imply that trade in the homogeneous good occurs only when $z \neq 1$ holds true. Hence, the trading equilibrium with incomplete specialization can be attained only when $z \neq 1$ holds true.

In the above equilibrium, we consider firms selling to both domestic and foreign markets. Do any firms sell only to the domestic market? We make the following assumption, to exclude such firms.

Assumption 1. $\left(\frac{m_1}{m_2}\right)\tau^{\sigma-1}f_x < f_d$ *and* $\left(\frac{m_2}{m_1}\right)\tau^{\sigma-1}f_x^* < f_d^*$ *hold true.*

This assumption ensures that the above equilibrium conditions construct an equilibrium—that is, in an equilibrium constructed according to the above equilibrium conditions and by Assumption 1, no firm deviates from the equilibrium.[20]

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[17] $1 - szm_3/(m_1m_2) > 0$ and $1 - sm_4/(m_1m_2) > 0$ are equivalent to $sz[(\tau^{2(1-\sigma)} + 1) - 2\tau^{1-\sigma}z] < (1 - z\tau^{1-\sigma})(z - \tau^{1-\sigma})$ and $s[(\tau^{2(1-\sigma)} + 1)z - 2\tau^{1-\sigma}] < (1 - z\tau^{1-\sigma})(z - \tau^{1-\sigma})$.

[18] Note $NX = [(sL\tau^{1-\sigma})/(1 - \tau^{2(1-\sigma)})](m_3/m_1 - m_4/m_2)$, where $(m_3/m_1 - m_4/m_2) = [(1 - \tau^{2(1-\sigma)})(1 - z^2)]/(m_1m_2)$.

[19] In detail, this is derived as follows: $NX_0 = p_0(y_0 - c_0L) = [l_0 - (1 - s)L] = sL[1 - (zm_3)/(m_1m_2)] = (sL/m_1m_2)[\tau^{1-\sigma}(z^2 - 1)] = -sL[\tau^{1-\sigma}/(1 - \tau^{2(1-\sigma)})](m_3/m_1 - m_4/m_2) = -NX$.

[20] Appendix.B provides the proof.

Lemma 1. *In the above equilibrium with* $z \neq 1$, $m_3 > 0$, $m_4 > 0$, $1 - szm_3/(m_1m_2) > 0$, *and* $1 - sm_4/(m_1m_2) > 0$, *Assumption 1 is equivalent to the condition wherein no firm has an incentive to sell only to the domestic market.*

The models of Venables (1987) and Helpman and Krugman (1985, Ch. 10.4) satisfy this assumption, because in their cases, $f_x = 0$. Hence, in their models, all firms in the monopolistically competitive sector behave in the above manner.

Lemma 1 is novel in two respects. First, Assumption 1 runs counter to an assumption of Melitz (2003)—namely, that $\tau^{\sigma-1} f_x > f_d$—to exclude a situation in which all firms sell to export markets. $\tau^{\sigma-1} f_x > f_d$ means that a combination of variable and fixed trade costs are greater than domestic fixed costs. On the other hand, Assumption 1 ensures a situation in which all firms sell to export markets. Second, Assumption 1 depends on relative CSFCs, z, through (m_1/m_2) and (m_2/m_1), while $\tau^{\sigma-1} f_x < f_d$ does not. (m_1/m_2) and (m_2/m_1) explicitly reflect asymmetry across countries, because (m_1/m_2) and (m_2/m_1) differ under $z \neq 1$, whereas $\tau^{\sigma-1} f_x/f_d$ and $\tau^{\sigma-1} f_x^*/f_d^*$ may be identical. On this point, Lemma 1 is a new result.

The presence of fixed export costs does not essentially change the trading equilibrium properties; we introduce it to understand CSFCs precisely. In doing so, we can make the comparative CSFCs statistics (Section 4) more meaningful, from an economics viewpoint.

In the above analysis, unlike Venables (1987), we explicitly show the equilibrium conditions of the trading equilibrium with incomplete specialization by assuming a constant expenditure share of the homogeneous good and the composite differentiated good. In particular, $(m_1/m_2)\tau^{\sigma-1} f_x < f_d$ and $(m_2/m_1)\tau^{\sigma-1} f_x^* < f_d^*$ of Assumption 1 have key roles in ensuring the uniqueness of the trading equilibrium in which Lemma 1 holds true. In the cases of $(m_1/m_2)\tau^{\sigma-1} f_x = f_d$ and $(m_2/m_1)\tau^{\sigma-1} f_x^* = f_d^*$, to export or not is immaterial for all firms. Then, the numbers of firms to sell only to the domestic market, to the export market, and to both markets are not determined uniquely. Hence, the unique equilibrium needs Assumption 1.[21]

Proposition 1. *Under* $z \neq 1$, $m_3 > 0$, $m_4 > 0$, $1 - szm_3/(m_1m_2) > 0$, $1 - sm_4/(m_1m_2) > 0$, *and Assumption 1, we can obtain a unique trading equilibrium with incomplete specialization, in which the firms of both countries produce the homogeneous and differentiated goods and in which the volumes of interindustry trade and intraindustry trade in both goods are positive. That is,* M, M^*, l_0, *and* l_0^* *are unique and positive while* NX *and* NX^* *are unique and nonzero.*

Positive transport costs are essential to ensuring this incomplete specialization and positive intraindustry trade with asymmetric CSFCs. If the transport

[21] In the cases of $(m_1/m_2)\tau^{\sigma-1} f_x > f_d$ and $(m_2/m_1)\tau^{\sigma-1} f_x^* > f_d^*$, all firms do not export, and trade does not occur.

cost is 0, differentiated goods are not produced in either country, because all firms in the monopolistically competitive sector move to a country with smaller fixed costs.

This result runs counter to Proposition 1 of Kikuchi (2004).[22] Kikuchi (2004) assumes that the transport cost is 0 ($\tau = 1$), and then shows that incomplete specialization can be attained and intraindustry trade occurs only when fixed costs across countries are identical, if marginal costs across countries are identical. This difference between the current study and that of Kikuchi (2004) derives from the difference in how they treat transport costs.

The Venables model assumes that in the monopolistically competitive sector, a firm has a larger share of its domestic market than it does of its export market.[23] In the Venables model, this property plays a key role in ensuring the interior equilibrium and in determining the signs of comparative statistics. In comparing these models, it is important to consider whether these models share the property. Does the trading equilibrium of Proposition 1 satisfy this property? The following proposition answers "yes."[24]

Proposition 2. *Under $z \neq 1$, $1 - szm_3/(m_1m_2) > 0$, $1 - sm_4/(m_1m_2) > 0$, and Assumption 1, $m_3 > 0$ ($M > 0$) is equivalent to $r_d^* > r_x^*$ and $m_4 > 0$ ($M^* > 0$) is equivalent to $r_d > r_x$.*

Proposition 2 shows the following property. The fact that home (foreign) country firms can afford to enter markets is equivalent to the fact that a foreign (home) country firm does not penetrate the home (foreign) market better than the foreign (home) market in terms of revenue. A similar property holds true in the Venables model;[25] in this sense, these models share similar properties.

Next, we investigate trade gains. To start, we construct the indirect utility function of a household of the home country. From the utility function, $c = sp^{-\sigma}P^{\sigma-1}$, $c' = s(\tau p)^{-\sigma}P^{\sigma-1}$, $P = p\hat{M}^{1/(1-\sigma)}$, $p = \phi\sigma/(\sigma - 1)$, and $p_0 = 1$, we obtain the following indirect utility function:

$$V = (1-s)^{1-s}\left[s\frac{\sigma-1}{\sigma\phi}\hat{M}^{1/*(\sigma-1)}\right]^s.$$

[22] The model of Kikuchi (2004) is similar to the current model, in many respects; however, these models differ with regards to the following points. In the model of Kikuchi (2004), two countries differ in terms of both marginal and fixed costs (Ricardian model). The transport cost is 0. Households have a quasilinear utility function and, therefore, a variable expenditure share.

[23] This assumption is imposed in (12) of Venables (1987).

[24] The proof is given in Appendix.C.

[25] See (12) and (14) in Venables (1987).

Similarly, the indirect utility function of a household in the foreign country can be given by

$$V^* = (1-s)^{1-s}\left[s\frac{\sigma-1}{\sigma\phi}(\hat{M}^*)^{1/(\sigma-1)}\right]^s.$$

Then, we construct trade gains. Trade gains are measured in terms of a rate of welfare change engendered by opening up to trade. That is, that of the home (foreign) country is measured by V/V_A (V^*/V_A^*), where we mark subscript "A" on variables at the autarkic equilibrium.[26] The above indirect utility functions show that welfare ranking is fully characterized by a change in the effective number of varieties in the country. The effective numbers of varieties in an autarkic economy, $\hat{M}_A$ and $\hat{M}_A^*$, are equal to M_A and M_A^*, respectively. Hence, $V/V_A \alpha\ \hat{M}/M_A$ and $V^*/V_A^* \alpha\ \hat{M}^*/M_A^*$ hold true. We analyze the existence of trade gains by checking $\hat{M}/M_A$ and $\hat{M}^*/M_A^*$.

According to the above arguments, the home (foreign) country gains from opening up to trade when $\hat{M}/M_A > 1 (\hat{M}^*/M_A^* > 1)$. Then, we obtain the following proposition.[27]

Proposition 3. *Under $z \neq 1$, $m_3 > 0$, $m_4 > 0$, $1 - szm_3/(m_1m_2) > 0$, and $1 - sm_4/(m_1m_2) > 0$, the fact that no firm has an incentive to sell only to the domestic market is equivalent to the fact that each country gains from opening up to trade. That is, $(m_1/m_2)\ \tau^{\sigma-1} f_x < f_d$ and $(m_2/m_1)\tau^{\sigma-1} f_x^* < f_d^*$ are equivalent to $\hat{M}/M_A > 1$ and $\hat{M}^*/M_A^* > 1$, respectively.*

Proposition 1 of Venables (1987) also shows that trade gains occur,[28] but both the current study and that of Venables (1987) differ in the following respects. The Venables model shows trade gains by imposing $r_d > r_x$ and $r_d^* < r_x^*$, whereas the current model additionally requires Assumption 1. This difference derives from the assumption concerning the fixed export cost f_x: $f_x = 0$ holds true in the Venables model, whereas $f_x > 0$ holds true in the current model. Furthermore, by considering Lemma 1, we can know that Proposition 3 is a new result in deriving the relationship between the entry decision and trade gains.

3 Impacts of relative CSFCs

In this section, we investigate the impacts of "relative" CSFCs, z, on relative firm location, M/M^*, the net exports of differentiated goods, NX (comparative advantage), and the distribution of trade gains, $(M/M_A)/(M^*/M_A)$.

[26] This definition is identical to that of Kikuchi (2001) in adopting the model of Helpman and Krugman (1985, Ch. 10.4).
[27] The proof is given in Appendix.D.
[28] See Proposition 1 and footnote 7 in Venables (1987).

Hereafter, we focus on an economy in which the homogeneous and differentiated goods are produced in both countries, and all firms producing differentiated goods sell to both the domestic market and the export market. Hence, we hereafter impose $m_3 > 0$, $m_4 > 0$, $1 - szm_3/(m_1m_2) > 0$, $1 - sm_4/(m_1m_2) > 0$, and Assumption 1.

The following results are new and can be obtained by imposing simple model settings: the expenditure shares of the homogeneous good and the composite differentiated good are constant, and the only difference across countries is in terms of fixed costs (CSFCs).

Without loss of generality, we hereafter impose the following assumption, with the exception of Proposition 7.

Assumption 2. *The relative fixed costs are less than 1, without loss of generality:* $z < 1$.

This assumption means that the fixed costs are lower in the home country.

First, we examine the international location of firms. We can obtain the following proposition from (3) and (4).[29]

Proposition 4. *The home (foreign) country has a more (less)-than-proportional share of the firms of differentiated goods. That is,* $M/M^* > 1/z$ *holds true.*

We call this property the *CSFCs effect*. This effect seems to correspond to the HME in the model of Helpman and Krugman (1985, Ch. 10.4), in which market size is the only difference between two countries. With the HME, a country with a larger (smaller) market has a more (less)-than-proportional share of the firms of differentiated goods. Differences across countries affect a firm's international location. In this respect, the CSFCs effect and the HME are similar; however, there is a critical difference between them. Market size (population) does not affect firm size in the HME, whereas fixed costs affect firm size in the CSFCs effect; that is, firm size in the home (foreign) country is smaller (larger), as $y = [(\sigma - 1)/\phi]f$ and $y^* = [(\sigma - 1)/\phi]f^*$ reveals.

Proposition 4 demonstrates the magnification effect with CSFCs. For example, under $z = 1/2$, $M > 2M^*$ holds true—that is, when the fixed costs in the home country are one-half that in the foreign country, the home country attracts more than twice as many firms as the foreign country. As such, firms are not distributed in proportion to relative CSFCs—and thus, there is a magnification effect.

The mechanism behind the result is simple. We consider an imaginary autarkic economy in which firms cannot move to the other country and the fixed costs of both countries are not f_d and f_d^*, but rather f and f^*. In such an economy, the number of firms, $M_{A|f}$ and M^*_{A/f^*}, can be obtained by making τ close to infinity in (3) and (4) as $M_{A|f} = sL/\sigma f$ and $M^*_{A|f} = sL/\sigma\ f^*$, respectively; then, $M_{A|f}/M^*_{A|f^*} = 2$. Then,

[29] Appendix.E provides a proof.

opening up to trade moves firms in the foreign country to the home country—that is, $M > M_{A|f}$ and $M^* < M^*_{A/f*}$ hold true.[30] Hence, $M/M^* > 2$—which is to say, firm distribution is not proportional.

Next, we examine trade patterns. (5) shows that the net exports of differentiated goods, NX, depend on the relative CSFCs z. From this relation, we obtain the following proposition.[31]

Proposition 5. *The home country has a comparative advantage in differentiated goods, while the foreign country has a comparative advantage in homogeneous goods. That is,* $NX = NX^*_0 > 0$ *holds true.*

Proposition 5 seems to be trivial, considering CSFCs; however, it is not. This is because the fact that the home country has more firms in the monopolistically competitive sector is not directly linked to the fact that the home country has a comparative advantage (net exports) in differentiated goods. This discrepancy derives from the difference in firm size. To investigate the impact of relative CSFCs on net exports of differentiated goods, we construct a relation between net exports and total revenue in the monopolistically competitive sector. The total revenue of the monopolistically competitive sector in the home and foreign countries is Mpy and $M^*p^*y^*$, respectively. These satisfy the following equation:[32]

$$Mpy - M^*p^*y^* = 2NX. \tag{6}$$

We can explain Proposition 5 intuitively by (6). Proposition 5 and (6) show that under $z < 1$, $Mpy > M^*p^*y^*$ holds true. From $p = p^*$, we can derive the following inequality:

$$\underbrace{\left(\frac{y}{y^*}\right)}_{\text{relative firm size}} \times \underbrace{\left(\frac{M}{M^*}\right)}_{\text{relative firm location}} > 1.$$

This condition is equivalent to the condition wherein the home country has higher total revenue in differentiated goods. The relative firm size (y/y^*) is proportional to the relative fixed costs z $y = [(\sigma - 1)/\phi]f$ and $y^* = [(\sigma - 1)/\phi]f^*$: $(y/y^*) = z$. The relative firm location (M/M^*) is not proportional to the relative fixed costs z from the CSFCs effect: $(M/M^*) > 1/z$. The left-hand side of the inequality exceeds 1 because the effect of the relative firm location $(M/M^*) > 1$ dominates that of the

[30] These can be confirmed as follows. $M/M_{A|f} - 1 = (zm_3)/(m_1m_2) - 1 = (zm_3 - m_1m_2)/(m_1m_2) = \tau^{1-\sigma}(1 - z^2)/(m_1m_2) > 0$. Hence, $M > M_{A|f} > 1$ holds true $M^*/M^*_{A|f*} - 1 = (m_4)/(m_1m_2) - 1 = (m_4 - m_1m_2)/(m_1m_2) = \tau^{1-\sigma}(z^2 - 1)/(m_1m_2) < 0$. Hence, $M^*/M^*_{A|f*} < 1$ holds true.

[31] The proof is as follows. (5) shows that $NX > 0$ holds true because of the assumptions that $m_1 > 0$ and $m_2 > 0$. From $NX + NX^* = 0$, $NX^* < 0$ holds true. Q.E.D.

[32] Appendix.F provides the derivation of (6).

relative firm size $(y/y^*) < 1$. Hence, the home country has higher revenue and a comparative advantage in the monopolistically competitive sector.

Next, we examine the distribution of trade gains—that is, we compare the trade gains of the two countries. We can investigate this by checking $(\hat{M}/M_A)/(\hat{M}^*/M_A^*)$. Then, we can obtain the following proposition.[33]

Proposition 6. *The home country has higher trade gains if the relative fixed costs in autarky equals or exceeds that in trade. Equivalently,* $(\hat{M}/M_A)/(\hat{M}^*/M_A^*) > 1$ *holds true if* $z_A \geq z$ *holds true, where* z_A *is defined by* $z_A = f_d/f_d^*$

To clarify Proposition 6, we decompose trade gains in the following way, as per Kikuchi (2001). Without loss of generality, we focus on the trade gains of the home country, $\hat{M}/M_A$. From the definition of $\hat{M}$, we obtain the following equation:

$$\frac{\hat{M}}{M_A} = \underbrace{\frac{M}{M_A}}_{\text{gains from domestice brand}} + \underbrace{\frac{\tau^{1-\sigma} M^*}{M_A}}_{\text{gains from imported brand}} \tag{7}$$

$\hat{M}/M_A$ of (7) is affected through two channels. The first channel is the change rate M/M_A of (7). One advantage for households in purchasing a domestic brand is saving on the transport costs $p < p_x = \tau p$. M/M_A measures trade gains from accessing the domestic brand lacking a transport cost (gains from domestic brand).[34] Under $z_A \geq z$, $(M/M_A) > (M^*/M_A^*)$ holds true.[35] Hence, this effect is larger in the home country than in the foreign country. The second channel is the change rate $(\tau^{1-\sigma} M^*)/M_A$ of (7). In autarky, households cannot access foreign brands; however, in the trading equilibrium, they enjoy brands discounted by the transport cost. $(\tau^{1-\sigma} M^*)/M_A$ measures trade gains from accessing the import variety (gains from imported brands).[36] Under $z_A \geq z$, $(\tau^{1-\sigma} M^*/M_A) < (\tau^{1-\sigma} M/M_A^*)$ holds true,[37] and this effect is larger in the foreign country than in the home country. Proposition 6 shows that the total effect in the home country is larger than that in the foreign country. The above analysis also implies that, relatively, the foreign country gains from imports, while the home country gains from the domestic brand. In other

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[33] Appendix.G provides a proof of Proposition 6.

[34] Kikuchi (2001) calls this effect the *trade costs saving effect*.

[35] This relation can be derived as follows:

$$\frac{M/M_A}{M^*/M_A^*} - 1 = z_A \frac{m_3}{m_4} - 1 \geq z \frac{m_3}{m_4} - 1 = \frac{2\tau^{1-\sigma}(1-z^2)}{m_4} > 0.$$

[36] Kikuchi (2001) calls this effect the *variety of imports effect*.

[37] This relation can be derived as follows: $\left[(\tau^{1-\sigma} M/M_A^*)\right]/\left[(\tau^{1-\sigma} M^*/M_A)\right] = (M_A^*/M_A)(M/M^*)$ holds true. This exceeds 1 in a similar way, with gains from the domestic brand.

words, the home country gains from firm location, and the foreign country gains from trade.

Proposition 6 appears to correspond to that of HME, but it does not. Kikuchi (2001) shows that in the model of Helpman and Krugman (1985, Ch. 10.4), trade gains are equalized across countries (equal distribution), because the two effects cancel out each other; in this way, the total effect is identical in both countries. However, in the current model, this property does not hold true (which is to say, there is unequal distribution).

Propositions 4 and 5 hold true under Assumptions 1 and 2, while Proposition 6 holds true under these assumptions and $z_A \geq z$. If $z_A \geq 1$, any arbitrary z that is < 1 satisfies $z_A \geq z$. Hence, the following equivalence conditions are obtained, where we do not impose Assumption 2.[38] [39]

Proposition 7. *Under* $z_A \geq 1$ $(1 \geq z_A)$ *and* $z \neq 1$, *the following three statements are equivalent.*

(1) The home (foreign) country has a more-than-proportional share of firms of differentiated goods.
(2) The home (foreign) country has a comparative advantage in differentiated goods.
(3) The home (foreign) country has higher trade gains.

These equivalent properties are novel results.

4 Changes in absolute CSFCs

In Section 3, we examined the impact of "relative" CSFCs, z. In this section, we examine the impacts of changes to "absolute" CSFCs, f and f^*. This comparative statistic is important, because CSFCs can be partially controlled by governments; for this reason, they are important to government policy.

How do governments control CSFCs? For example, we consider a situation in which the home country government makes stronger regulations with respect to entering domestic markets, wherein all firms (home and foreign firms) entering the home country face higher fixed costs (f_d and f_x^* increase). This policy increases both f and f^*. We consider such a change "bilateral."

On the other hand, we consider a situation in which the home country government targets only foreign firms, by creating stronger regulations regarding entry to the domestic market; then, only f_x^* increases. This policy increases only the f^*. We consider such a change "unilateral."

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[38] In Proposition 7, we impose $z \neq 1$ to obtain a unique trading equilibrium with incomplete specialization.

[39] From the proof of Proposition 6 (Appendix.G), under $z > 1$ and $z \geq z_A$, the foreign country has higher trade gains. Hence, under $1 \geq z_A$, the three statements in Proposition 7 are equivalent.

In this manner, a government's policy target does not frequently relate to CSFCs for the domestic country, and is not trivial when there is a fixed export cost. Then, it is worth analyzing both unilateral and bilateral changes in CSFCs.

There is a crucial difference between unilateral and bilateral changes in CSFCs. A unilateral change in CSFCs certainly changes the relative fixed costs; however, this property does not hold true for a bilateral change in CSFCs when f and f^* change at the same rate. Hence, we analyze these changes separately. Let us analyze unilateral change first.

Unilateral change in CSFCs

We analyze an impact of unilateral change in absolute CSFCs, as per Venables (1987). From (1)–(5), we obtain the following proposition.[40]

Proposition 8. *A decrease in absolute CSFCs has the following impacts around the arbitrary trading equilibrium with incomplete specialization:*

(1) A decrease in the absolute CSFCs of a country increases the number of firms, while reducing the same in the other country. That is, $dM/df < 0$, $dM/df^* > 0$, $dM^*/df > 0$, *and* $dM^*/df^* < 0$ *hold true.*
(2) A decrease in the absolute CSFCs of a country increases the net exports of differentiated goods, while reducing the same in the other country. That is, $dNX/df < 0$, $dNX/df^* > 0$, $dNX^*/df > 0$, *and* $dNX^*/df^* < 0$ *hold true.*
(3) A decrease in the absolute CSFCs of a country increases the welfare, while reducing the same in the other country. Equivalently, $d\hat{M}/df < 0$, $d\hat{M}/df^* > 0$, $d\hat{M}^*/df > 0$, *and* $d\hat{M}^*/df^* < 0$ *hold true.*

We can interpret (3) of Proposition 8 as follows. We assume that after opening up to trade, the CSFCs change. That is, M_A and M_A^* are fixed. Then, (3) implies a change in trade gains. Hence, this proposition means that a unilateral decrease in the absolute CSFCs of the home (foreign) country increases (reduces) inequalities in terms of the number of firms, net exports, and trade gains between the two countries.

Results similar to (1) and (3) of Proposition 8 are seen with Proposition 3 of Venables (1987). Figure 2 of Venables (1987) shows the impact on the number of firms; this explains the impact on welfare.

Result (2) of Proposition 8 is similar to Proposition 4 of Venables (1987), but these results differ with regards to the equilibrium scope. Result (2) of Proposition 8 holds true around the "arbitrary" trading equilibrium with incomplete specialization, while Proposition 4 of Venables (1987) shows a similar result for comparative advantage when the two countries are initially symmetric. On this point, (2) of Proposition 8 makes a contribution.

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[40] Appendix.H provides a proof.

The combination of the results of (2) and (3) shows that a decrease in the CSFCs of a country increases both the net exports of the differentiated goods and the trade gains in the country. This finding runs counter to the conventional argument inherent in Bhagwati's (1958) immiserizing growth with a perfectly competitive setting; export-oriented technological progress reduces trade gains, in the sense that the terms of trade deteriorate.

We can interpret these results more easily by decomposing dM, dM^*, $d\hat{M}$, and $d\hat{M}^*$ as follows: $dM = (\partial M/\partial f)df + (\partial M/\partial z)dz$, $dM^* = (\partial M^*/\partial f^*)df^* + (\partial M^*/\partial z)dz$, $d\hat{M} = (\partial \hat{M}/\partial f)df + (\partial \hat{M}/\partial z)\,dz$, and $d\hat{M}^* = (\partial \hat{M}^*/\partial f^*)df^* + (\partial \hat{M}^*/\partial z)dz$.[41] We call the first term of the right-hand side of these equations the "absolute effect"; the second term of those is the "relative effect." On the other hand, *NX* depends only on the relative CSFCs, z; dNX has only a relative effect.

We can interpret these decompositions as follows. The absolute effect reflects an effect that is independent of the international firm's movement (imaginary autarkic), while the relative effect reflects an effect that is dependent on that.[42] The absolute effect has a mechanism similar to that of an autarkic economy. The relative effect has a mechanism similar to those of Propositions 4–6.

The total effect can be explained as follows. A decrease in the CSFCs of the home (foreign) country has positive impacts on the number of firms and the welfare of the home (foreign) country in both absolute and relative terms, while it has negative impacts on those of the foreign (home) country. A decrease in the CSFCs of the home (foreign) country has similar impacts on the net exports of differentiated goods in relative terms, but it does not affect it in absolute terms. Hence, signs regarding the total effect are determined unambiguously.

Bilateral change in CSFCs

In the presence of fixed export costs, a government's policy targets often comprise not only the domestic market of the home firms, but also the export market of the foreign firms (both f_d and f_x^*). Then, the government can cause bilateral change in the CSFCs. We analyze the impact of this bilateral change.

When the absolute CSFCs of both countries change bilaterally, the relative CSFCs behave as

$$dz = z\left(\frac{df}{f} - \frac{df^*}{f^*}\right). \tag{8}$$

In the case of unilateral change, $dz/z = df/f$ or $dz/z = -df^*/f^*$ always holds true; however, in the case of bilateral change, it does not generally hold true. In

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[41] These decompositions can be obtained from (1), (3), $M^* = \left[(sL)/(\sigma f^*)\right]\left[m_4/(m_1 m_2)\right]$ (which is derived from (4)), and $\hat{M}^* = \left[(sL)/(\sigma f^*)\right]\left[(1-\tau^{2(1-\sigma)}/m_2)\right]$ (which is derived from (2)).

[42] Note that $\lim_{\tau\to\infty} M = \lim_{\tau\to\infty} \hat{M} = (sL/\sigma f) = M_{A|f}$ and $\lim_{\tau\to\infty} M^* = \lim_{\tau\to\infty} \hat{M}^* = (sL/\sigma f^*) = M^*_{A|f^*}$.

particular, when $df/f = df^*/f^*$ holds true, $dz/z = 0$ holds true, and relative CSFCs are not important. This property contrasts with that in Section 3.

Hence, we obtain the following proposition.[43]

Proposition 9. *When a decreasing rate of change in the home (foreign) CSFCs is larger than that seen in the foreign (home) country, the following properties hold true:*

(1) The net exports of the home (foreign) country increase (decrease), and
(2) The number of home (foreign) firms and the amount of home (foreign) welfare both increase, while such changes in the foreign (home) country are ambiguous. Equivalently, $df/f < (>) df^*/f < (>) 0$ implies that $dM(dM^*) > 0$ and $d\hat{M}(d\hat{M}^*) > 0$, although the signs of $dM^*(dM)$ and $d\hat{M}^*(d\hat{M})$ are ambiguous.

The directions of change in the net exports are clear, because these depend only on the relative effect. However, changes in the number of firms and in the amount of welfare are not clear, because these depend on both absolute and relative effects.

This proposition shows that when CSFCs change bilaterally, we should check the magnitude relation between the rate of change in CSFCs in the home and foreign countries. In particular, when both rates of change in CSFCs are the same, changes in the number of firms and in the amount of welfare are clear. Otherwise, changes in those of one country will be ambiguous while those of the other country will be clear. This point has not received attention, despite it being important to empirical analysis.

5 Conclusion

This paper presents a simple model in which two countries differ only in terms of fixed costs; its objective is to investigate the impact of country-specific fixed costs (CSFCs) on the international location of firms, comparative advantage, and the distribution of trade gains (Ricardian theory with asymmetric fixed costs across countries).

The core result is the CSFCs effect (i.e., a country with lower fixed costs will have more firms than a country with higher fixed costs). Comparative advantage and the distribution of trade gains can be determined through the CSFCs effect (i.e., a country with lower (higher) fixed costs is a net exporter (importer) of differentiated goods and has higher (lower) trade gains). In these results, relative CSFCs determine relative firm location, net exports, and the distribution of trade gains.

These results are novel; it is difficult to express them by way of the Venables (1987) model, given its complexity. The main contribution of this study is with regards to this point. Two types of assumption play key roles in deriving the

[43] Appendix.H shows a proof of Proposition 9.

aforementioned results. First, we assume that the expenditure shares of the homogeneous good and a composite differentiated good are held constant. Second, we assume that the only difference across countries is fixed costs.

The impact of changes to absolute CSFCs is given as follows: a unilateral decrease in absolute CSFCs in the home (foreign) country increases (reduces) inequalities between the two countries in terms of the number of firms, net exports, and trade gains. This property runs counter to the conventional argument inherent in Bhagwati's (1958) immiserizing growth.

In the presence of fixed export costs, the government can create bilateral changes in CSFCs. This comparative statistic implies that when CSFCs change bilaterally, we should check the magnitude relation between the rates of change in the home and foreign countries. This point is important to empirical analysis.

This paper presents a Ricardian model that purely emphasizes CSFCs and derives various basic results. Hence, this study can be benchmarked to investigate the impact of CSFCs on international firm location, comparative advantage, and the distribution of trade gains.

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Appendices

Appendix.A: Derivation of (1) and (2)

The market-clearing condition $y = Lc + \tau Lc'^{*}$ can be rewritten as follows, $y = [(\sigma - 1)/\phi] f$, $c = sp^{-\sigma} P^{\sigma-1}$, $c^{*\prime} = s(\tau p)^{-\sigma} P^{*\sigma-1}$, $P^{1-\sigma} = P^{1-\sigma}\hat{M}$, $P^{*1-\sigma} = P^{*1-\sigma}\hat{M}^{*}$, $p = p^{*} = \phi\sigma/(\sigma - 1)$, and the definitions of $\hat{M}$ and $\hat{M}^{*}$:

$$\frac{\sigma f}{sL} = \frac{\hat{M}^{*} + \tau^{1-\sigma}\hat{M}}{\hat{M}\hat{M}^{*}}. \tag{A.1}$$

Similarly, the market-clearing condition $y^{*} = Lc^{*} + \tau Lc'$ can be rewritten as follows:

$$\frac{\sigma f^{*}}{sL} = \frac{\tau^{1-\sigma}\hat{M}^{*} + \hat{M}}{\hat{M}\hat{M}^{*}}. \tag{A.2}$$

In dividing (A.1) by (A.2), we obtain

$$z = \frac{\hat{M}^{*} + \tau^{1-\sigma}\hat{M}}{\tau^{1-\sigma}\hat{M}^{*} + \hat{M}}. \tag{A.3}$$

We solve (A.3) for $\hat{M}/\hat{M}^{*}$, and then obtain

$$\frac{\hat{M}}{\hat{M}^{*}} = \frac{m_2}{m_1}. \tag{A.4}$$

We define $\hat{M}_W$ as $\hat{M}_W \equiv \hat{M} + \hat{M}^*$. The definition of $\hat{M}_W$, (A.4), and $m_1 + m_2 = (1-\tau^{1-\sigma})(1+z)$ give

$$\hat{M} = \frac{m_2}{(1-\tau^{1-\sigma})(1+z)}\hat{M}_w, \tag{A.5}$$

$$\hat{M}^* = \frac{m_1}{(1-\tau^{1-\sigma})(1+z)}\hat{M}_w, \tag{A.6}$$

By substituting $\hat{M}$ of (A.5) and $\hat{M}^*$ (A.6) for those of (A.1), we obtain

$$\hat{M}_w = \frac{sL}{\sigma\ f}\frac{z(z+1)(1+\tau^{1-\sigma})(1-\tau^{1-\sigma})^2}{m_1 m_2}. \tag{A.7}$$

(A.5), (A.6), and (A.7) give (1) and (2). Q.E.D.

Appendix.B: Proof of Lemma 1

In a monopolistically competitive sector, the following final good market-clearing conditions are satisfied: $y_d = cL$, $y_x/\tau = c'^*L$, $y_d^* = c^*L$, and $y_x^*/\tau = c'L$.

We prove that $\tau^{\sigma-1} f_x(m_1/m_2) < f_d$ is equivalent to the fact that no firm producing in the home country has an incentive to sell only to the domestic market. That is, we prove that $(m_1/m_2)\tau^{\sigma-1} f_x < f_d$ is equivalent to $\pi_d < 0$, where π_d is defined as $\pi_d = py_d - (\phi y_d + f_d)$. From $y_d = cL$, $y_x/\tau = c'^*L$, $c = sp^{-\sigma}P^{\sigma-1}$, $c^{*\prime} = s(\tau\ p)^{-\sigma}P^{*\sigma-1}$, $P^{1-\sigma} = p^{1-\sigma}\hat{M}$, $P^{*1-\sigma} = p^{*1-\sigma}\hat{M}^*$, $p = p^* = \phi\sigma/(\sigma-1)$, (1), and (2), we obtain

$$\begin{aligned}\frac{y_x}{y_d} &= \frac{\tau\, s(\tau\ p^*)^{-\sigma} P^{*\sigma-1}}{sp^{-\sigma}P^{\sigma-1}} \\ &= \tau^{1-\sigma}\frac{\hat{M}}{\hat{M}^*} \\ &= \tau^{1-\sigma}\frac{m_2}{m_1}.\end{aligned} \tag{B.1}$$

(B.1) $y = [(\sigma-1)/\phi]f$ derive

$$\frac{\phi}{\sigma-1}y_d = \frac{f_d + f_x}{1+\tau^{1-\sigma}\dfrac{m_2}{m_1}}. \tag{B.2}$$

(B.2) and $\pi_d = py_d - (\phi y_d + f_d)$ give

$$\begin{aligned}\pi_d &= \phi\left(\frac{1}{\sigma-1}\right)y_d - f_d \\ &= \frac{f_d + f_x}{1+\tau^{1-\sigma}\dfrac{m_2}{m_1}} - f_d \\ &= \frac{f_x - \tau^{1-\sigma}\dfrac{m_2}{m_1}f_d}{1+\tau^{1-\sigma}\dfrac{m_2}{m_1}}.\end{aligned} \tag{B.3}$$

(B.3) shows that $\pi_d < 0$ is equivalent to $(m_1/m_2)\tau^{\sigma-1}f_x < f_d$.

In the same manner, we can prove that $\pi_d^* < 0$ is equivalent to $(m_2/m_1)\tau^{\sigma-1}f_x^* < f_d^*$. Hence $\tau^{\sigma-1}f_x^*(m_2/m_1) < f_d^*$ is equivalent to the fact that no firm producing in a foreign country has an incentive to sell only to the domestic market. Q.E.D.

Appendix.C: Proof of Proposition 2

We prove that when s, L, σ, f, z, m_1, and m_2 are positive, $M > 0$ is equivalent to $r_d^* > r_x^*$ as follows:

$$\begin{aligned} r_d^* > r_x^* &\leftrightarrow 1 > \frac{y_x^*}{y_d^*} \\ &\leftrightarrow 1 > \tau^{1-\sigma}\frac{m_1}{m_2} && \text{by } y_x^*/y_d^* = \tau^{1-\sigma}(m_1/m_2) \\ &\leftrightarrow \tau^{2(1-\sigma)} + 1 > 2\tau^{1-\sigma}z && \text{by Definition of } m_1 \text{ and } m_2 \\ &\leftrightarrow m_3 > 0 && \text{by Definition of } m_3 \\ &\leftrightarrow M > 0. \end{aligned}$$

In the same manner, we can prove that $M^* > 0$ is equivalent to $r_d > r_x$ when s, L, σ, f, z, m_1, and m_2 are positive. Q.E.D.

Appendix.D: Proof of Proposition 3

From (1), (2), $M_A = (sL)/(\sigma f_d)$, $M_A^* = (sL)/(\sigma f_d^*)$, we can obtain the following equations:

$$\frac{\hat{M}}{M_A} = \frac{f_d}{f}\frac{z(1-\tau^{2(1-\sigma)})}{m_1}, \tag{D.1}$$

$$\frac{\hat{M}^*}{M_A^*} = \frac{f_d^*}{f^*}\frac{z(1-\tau^{2(1-\sigma)})}{m_1}, \tag{D.2}$$

We prove that $\hat{M}/M_A > 1$ is equivalent to $(m_1/m_2)\tau^{\sigma-1} f_x < f_d$. From (D.1), we obtain the following equations:

$$\begin{aligned}\frac{\hat{M}}{M_A} - 1 &= \frac{f_d z(1-\tau^{2(1-\sigma)}) - fm_1}{fm_1} \\ &= \frac{f_d \tau^{1-\sigma} m_2 - f_x m_1}{fm_1}.\end{aligned} \tag{D.3}$$

(D.3) shows that $\hat{M}/M_A > 1$ is equivalent to $(m_1/m_2)\tau^{\sigma-1} f_x < f_d$.

We prove that $\hat{M}^*/M_A^* > 1$ is equivalent to $(m_2/m_1)\tau^{\sigma-1} f_x^* < f_d^*$. From (D.2), we obtain the following equations:

$$\begin{aligned}\frac{\hat{M}^*}{M_A^*} - 1 &= \frac{f_d^*(1-\tau^{2(1-\sigma)}) - f^* m_2}{f^* m_2} \\ &= \frac{f_d^* \tau^{1-\sigma} m_1 - f_x^* m_2}{f^* m_2}.\end{aligned} \tag{D.4}$$

(D.4) implies that $\hat{M}^*/M_A^* > 1$ is equivalent to $(m_2/m_1)\tau^{\sigma-1} f_x^* < f_d^*$. Q.E.D.

Appendix.E: Proof of Proposition 4

By dividing M of (3) by M^* of (4), we can obtain $M/M^* = m_3/m_4$. This equation and the definitions of m_3 and m_4 give $(M/M^*) - (1/z) = [2\tau^{1-\sigma}(1-z^2)]/(zm_4)$. Hence, under $z < 1$, $M/M^* > (1/z)$. Q.E.D.

Appendix.F: Derivation of (6)

$p = \phi\sigma/(\sigma-1)$, $y = [(\sigma-1)/\phi]f$, and (3) give

$$Mpy = sL\frac{zm_3}{m_1 m_2}. \tag{F.1}$$

$p^* = \phi\sigma/(\sigma-1)$, $y^* = [(\sigma-1)/\phi]f^*$ and (4) give

$$M^* p^* y^* = sL\frac{m_4}{m_1 m_2}. \tag{F.2}$$

(F.1) and (F.2) derive

$$Mpy - M^* p^* y^* = \frac{sL}{m_1 m_2} 2\tau^{1-\sigma}(1-z^2). \tag{F.3}$$

(F.3) and (5) imply (6). Q.E.D.

Appendix.G: Proof of Proposition 6

(D.1), (D.2), and the assumption of $z_A \geq z$ derive the following relations:

$$\begin{aligned}\frac{\hat{M}/M_A}{\hat{M}/M_A^*} - 1 &= z_A \frac{m_2}{m_1} - 1 \\ &\geq z\frac{m_2}{m_1} - 1 \qquad\qquad \text{by } z_A \geq z \\ &= \frac{\tau^{1-\sigma}(1-z^2)}{m_1}.\end{aligned}$$

Hence, under $z < 1$, $(\hat{M}/M_A)/(\hat{M}^*/M_A^*) > 1$. Q.E.D.

Appendix.H: Proofs of Propositions 8 and 9

$dNX/dz < 0$ **and** $dNX^*/dz > 0$
(6) can be rewritten as $NX = (1/2)(Mpy - M^*p^*y^*)$. The total differential of this is given by

$$dNX = \frac{1}{2}\left[\frac{d(Mpy)}{dz} - \frac{d(M^*p^*y^*)}{dz}\right]dz. \tag{H.1}$$

By differentiating (F.1) by z, we can obtain

$$\frac{d(Mpy)}{dz} = sL\frac{d}{dz}\left(\frac{zm_3}{m_1m_2}\right). \tag{H.2}$$

$\frac{d}{dz}\left(\frac{zm_3}{m_1m_2}\right)$ can be rewritten as follows.

$$\begin{aligned}\frac{d}{dz}\left(\frac{zm_3}{m_1m_2}\right) &= \frac{(zm_3)'m_1m_2 - (zm_3)(m_1m_2)'}{(m_1m_2)^2} \\ &= \frac{m_3(m_1m_2 - zm_3)}{(m_1m_2)^2} - \frac{2\tau^{1-\sigma}zm_1m_2}{(m_1m_2)^2} \\ &= -\frac{m_3\tau^{1-\sigma}(1-z^2)}{(m_1m_2)^2} - \frac{2\tau^{1-\sigma}zm_1m_2}{(m_1m_2)^2} < 0.\end{aligned} \tag{H.3}$$

From (H.2) and (H.3), $d(Mpy)/dz < 0$ holds true.

By differentiating (F.2) by z, we can obtain

$$\frac{d(M^*p^*y^*)}{dz} = sL\frac{d}{dz}\left(\frac{m_4}{m_1m_2}\right). \tag{H.4}$$

$\frac{d}{dz}\left(\frac{m_4}{m_1 m_2}\right)$ can be rewritten as follows.

$$\begin{aligned}\frac{d}{dz}\left(\frac{m_4}{m_1 m_2}\right) &= \frac{(m_4)' m_1 m_2 - m_4 (m_1 m_2)'}{(m_1 m_2)^2} \\ &= \frac{(\tau^{2(1-\sigma)}+1) m_1 m_2 - m_4 m_3}{(m_1 m_2)^2} \\ &= \frac{(\tau^{2(1-\sigma)}+1)\tau^{1-\sigma}(1+z^2) - 4\tau^{2(1-\sigma)} z}{(m_1 m_2)^2} \\ &= \frac{\tau^{1-\sigma}(m_1^2 + m_2^2)}{(m_1 m_2)^2} > 0.\end{aligned} \tag{H.5}$$

From (H.4) and (H.5), $d(M^* p^* y^*)/dz > 0$ holds true.

(H.1), $d(Mpy)/dz < 0$, and $d(M^* p^* y^*)/dz > 0$ imply $dNX/dz < 0$. $dNX/dz > 0$ and $NX + NX^* = 0$ imply $dNX^*/dz > 0$. Q.E.D.

dM* and *dM*

From (3), the total differential $dM = (\partial M/\partial f)df + (\partial M/\partial z)dz$ can be rewritten as follows:

$$dM = \left[\left(\frac{zm_3}{m_1 m_2}\right)\frac{d}{df}\left(\frac{sL}{\sigma f}\right)\right]df + \left[\left(\frac{sL}{\sigma f}\right)\frac{d}{dz}\left(\frac{zm_3}{m_1 m_2}\right)\right]dz, \tag{H.6}$$

where $(d/dz)[(zm_3)/(m_1 m_2)] < 0$ is shown in (H.3).

We can rewrite M^* of (4) as follows: $M^* = [(sL)/(\sigma f^*)]\ [m_4/(m_1 m_2)]$. From this, the total differential $dM^* = (\partial M^*/\partial f^*)df^* + (\partial M^*/\partial z)dz$ can be rewritten as follows:

$$dM^* = \left[\left(\frac{m_4}{m_1 m_2}\right)\frac{d}{df^*}\left(\frac{sL}{\sigma f^*}\right)\right]df^* + \left[\left(\frac{sL}{\sigma f^*}\right)\frac{d}{dz}\left(\frac{m_4}{m_1 m_2}\right)\right]dz, \tag{H.7}$$

where $(d/dz)[(m_4)/(m_1 m_2)] > 0$ is shown in (H.5).

$d\hat{M}$ and $d\hat{M}^*$

From (1), the total differential $d\hat{M} = (\partial \hat{M}/\partial f)df + (\partial \hat{M}/\partial z)dz$ can be rewritten as follows:

$$d\hat{M} = \left[\left(\frac{z(1-\tau^{2(1-\sigma)})}{m_1}\right)\frac{d}{df}\left(\frac{sL}{\sigma f}\right)\right]df + \left[\left(\frac{sL}{\sigma f}\right)\frac{d}{dz}\left(\frac{z(1-\tau^{2(1-\sigma)})}{m_1}\right)\right]dz, \tag{H.8}$$

where $(d/dz)\left[z(1-\tau^{2(1-\sigma)})/m_1\right] = -\left[\tau^{1-\sigma}(1-\tau^{2(1-\sigma)})\right]/m_1^2 < 0$ holds true.

We can rewrite $\hat{M}^*$ of (2) as follows: $\hat{M}^* = [(sL)/(\sigma f^*)][(1-\tau^{2(1-\sigma)})/m_2]$. From this, the total differential $d\hat{M}^* = (\partial M^*/\partial f^*)df^* + (\partial \hat{M}^*/\partial z)dz$ can be rewritten as follows:

$$d\hat{M}^* = \left[\left(\frac{1-\tau^{2(1-\sigma)}}{m_2}\right)\frac{d}{df^*}\left(\frac{sL}{\sigma f^*}\right)\right]df^* + \left[\left(\frac{sL}{\sigma f^*}\right)\frac{d}{dz}\left(\frac{1-\tau^{2(1-\sigma)}}{m_2}\right)\right]dz, \qquad \text{(H.9)}$$

where $(d/dz)\,[(1-\tau^{2(1-\sigma)})/m_2] = \tau^{1-\sigma}(1-\tau^{2(1-\sigma)})/m_2^2 > 0$ holds true.

Proof of Proposition 8

When the absolute CSFCs of the home or foreign country change unilaterally, the relative CSFCs behave, respectively, as $dz = z(df/f)$ or $dz = -z(df^*/f^*)$. These equations and (H.1)–(H.9) imply Proposition 8. Q.E.D.

Proof of Proposition 9

(8) and (H.1)–(H.9) imply Proposition 9. Q.E.D.