RIMS Kôkyûroku Bessatsu B52

# Exponential Analysis of Differential Equations and Related Topics

edited by Yoshitsugu Take

November, 2014
Research Institute for Mathematical Sciences
Kyoto University

# RIMS Kôkyûroku Bessatsu B52

# Exponential Analysis of Differential Equations and Related Topics

October 15~18, 2013

edited by Yoshitsugu Takei

November, 2014

Research Institute for Mathematical Sciences

Kyoto University, Kyoto, Japan

The papers presented in this volume of RIMS Kôkyûroku Bessatsu are in final form and refereed. ©2014 by the Research Institute for Mathematical Sciences, Kyoto University. All rights reserved. Printed in Japan.

# PREFACE

To activate researches in the fields of microlocal analysis and asymptotic analysis, we organized the RIMS Conference

# "Exponential Analysis of Differential Equations and Related Topics"

at the Research Institute for Mathematical Sciences, Kyoto University from October 15 (Tuesday) through October 18 (Friday) in 2013. This volume of RIMS Kôkyûroku Bessatsu is a collection of research papers, announcements and survey articles contributed by invited speakers of the above conference. All papers in the volume have been reviewed and are in their final forms.

The conference was planned also to celebrate the sixtieth birthday of Professor Takashi AOKI. As is shown by the fact that many papers contained in this volume are dedicated to him, Prof. Aoki has been contributing to the developments of microlocal analysis and asymptotic analysis, and stimulating many researchers in these fields. For example, he established the so-called exponential calculus of microdifferential operators of infinite order in the field of microlocal analysis and, by using computers in an effective manner, showed the importance of the geometry of Stokes curves in the exact WKB analysis. In fact, many impressive figures of Stokes curves he wrote have brought substantial progress of exact WKB analysis. Taking this opportunity, we would like to express our deep respect and sincere gratitude to Prof. Aoki for his great efforts and achievements in these fields.

The conference was supported by RIMS. We express our heartiest thanks to the secretary staff of RIMS for their kind assistance.

Kyoto in October, 2014

Yoshitsugu TAKEI (Organizer in chief) RIMS, Kyoto University

# Program

# Exponential Analysis of Differential Equations and Related Topics

October 15 (Tue) – October 18 (Fri), 2013 Lecture Hall (Room No. 420) of RIMS, Kyoto University

# October 15, Tuesday

10:00 - 11:00	D. C. Struppa (Chapman Univ., USA) Propagation of superoscillations as solutions to the Cauchy problem for generalized Schrödinger equations
11:15 - 12:15	N. Honda (Hokkaido Univ.) and T. Kawai (RIMS, Kyoto Univ.) Sato's postulates on the S-matrix revisited through the Borel resummation of the perturbation series (a joint work with Henry P. Stapp)
14:00 - 15:00	R. Schäfke (Strasbourg, France) Loray's reduction of a vector field with nilpotent linear part and polynomial asymptotics
15:15 - 16:15	T. Koike (Kobe Univ.) On the computation of Voros coefficients via middle convolutions (joint work with K. Iwaki)
16:30 - 17:10	M. Tanda (Kinki Univ.) Parametric Stokes phenomena of the Gauss hypergeometric differential equation with a large parameter (joint work with T. Aoki)
17:20 - 18:00	T. Takahashi (Kinki Univ.) On the WKB theoretic structure of a Schrödinger operator with a Stokes curve of loop type

### October 16, Wednesday

october 10,	Viounissady
10:00 - 11:00	Y. Matsui (Kinki Univ.) Topological Radon transforms and their applications
11:15 - 12:15	A. Sebbar (Bordeaux, France) Folding paper, theta function and eta function
14:00 - 15:00	S. Kamimoto (RIMS, Kyoto Univ.), T. Kawai (RIMS, Kyoto Univ.) and T. Koike (Kobe Univ.) A happy marriage of resurgent functions and linear differential operators of infinite order in exact WKB analysis

15:15 - 16:15	A. Voros (Saclay, France) Zeta functions over zeros of zeta functions, and an exponential- asymptotic view of the Riemann Hypothesis
16:30 – 17:30	T. Aoki (Kinki Univ.) Exact WKB analysis of confluent hypergeometric equations (joint work with T. Takahashi and M. Tanda)
October 17,	Γhursday
10:00 - 11:00	Y. Ohyama (Osaka Univ.) The $q$ -Painlevé equations and Riemann-Hilbert-Birkhoff problem
11:15 – 12:15	S. Malek (Lille, France) On Gevrey asymptotics for difference-differential nonlinear PDE
14:00 - 15:00	S. Tajima (Univ. of Tsukuba) $b\text{-functions}$ and algebraic local cohomology classes attached to Lê cycles
15:15 - 16:15	S. Yamazaki (Nihon Univ.) Kernel functions and symbols of pseudodifferential operators of infinite order with an apparent parameter (joint work with T. Aoki and N. Honda)
16:30 - 17:10	K. Umeta (Hokkaido Univ.) On the sheaf of Laplace hyperfunctions in several variables (joint work with Naofumi Honda)
17:20 - 18:00	D. Tarama (Kyoto Univ.) Analytic extension of Birkhoff normal forms for Hamiltonian systems of one degree of freedom
October 18, I	Friday
10:00 - 11:00	G. Lysik (Polish Academy of Sciences, Poland) Summability of formal solutions to partial differential equations
11:15 – 12:15	M. Yoshino (Hiroshima Univ.) Connection problem for non integrable Hamiltonian system
14:00 - 15:00	A. Shudo (Tokyo Metropolitan Univ.) Toward pruning theory of the Stokes geometry for the quantum Henon map
15:15 – 16:15	Y. Umeta (Tokyo Univ. of Science) Instanton-type solutions of Painlevé hierarchies $(P_J)_m$ $(J={\rm I},{\rm II},34,{\rm IV})$
16:30 - 17:30	Y. Takei (RIMS, Kyoto Univ.) The fourth-order PI equation and coalescing phenomena of nonlinear

turning points

# LIST OF PARTICIPANTS

Kana Ando Chiba University
Takashi Aoki Kinki University

Yasuo Chiba Tokyo University of Technology

Setsuro FUJIIÉ Ritsumeikan University
Keiko FUJITA Toyama University
Yuichi IKE University of Tokyo
Ryuichi ISHIMURA Chiba University

Kohei IWAKI RIMS, Kyoto University Shingo KAMIMOTO RIMS, Kyoto University Akira KANEKO Ochanomizu University

Kazuo Kaneko Seki Kowa Institute of Mathematics

Kiyoomi Kataoka University of Tokyo

Takahiro KAWAI RIMS, Kyoto University

Tatsuya Koike Kobe University

Masahiro Kubo Nagoya Institute of Technology

Naoto Kumanogo Kogakuin University

Grzegorz Łysik Polish Academy of Sciences

Stephane Malek Université de Lille 1
Fumitsuna Maruyama Keio University
Yutaka Matsui Kinki University

Shinya MORITOH Nara Women's University
Takahiro MOTEKI RIMS, Kyoto University

Yasuyuki NAGATOMO Meiji University
Hajime NAGOYA Rikkyo University
Tomoki NAKANISHI Nagoya University
Yasuo Ohno Kinki University
Yousuke Ohyama Osaka University
Yasunori Okada Chiba University

Shinji Sasaki RIMS, Kyoto University

Reinhard Schäfke IRMA, Université de Strasbourg

Ahmed Sebbar Université de Bordeaux I

Akira Shirai Sugiyama Jogakuen University
Akira Shudo Tokyo Metropolitan University

Daniele C. STRUPPA Chapman University
Hidetoshi Tahara Sophia University

Shinichi Tajima University of Tsukuba

Toshinori Takahashi Kinki University

Yoshitsugu Takei RIMS, Kyoto University

Mika TANDA Kinki University
Daisuke TARAMA Kyoto University

Kouichi Toda Toyama Prefectural University

Motoo UCHIDA Osaka University
Koichi UCHIYAMA Sophia University
Kohei UMETA Hokkaido University

Yoko Umeta Tokyo University of Science

André Voros CEA Saclay

Seiichiro WAKABAYASHI University of Tsukuba
Takuya WATANABE Ritsumeikan University

Hideshi Yamane Kwansei Gakuin University

Susumu Yamazaki Nihon University

Hiroshi Yamazawa Shibaura Institute of Technology

Kunio Yoshino Tokyo City University

Masafumi Yoshino Hiroshima University

# Contents

	pa	age
Prei	FACE	i
Pro	GRAM	ii
List	OF PARTICIPANTS	iv
Art	ICLES	
1.	Domenico Napoletani and Daniele C. Struppa, A phase integral approach to irrationality	1
2.	Naofumi Honda, Takahiro Kawai and Henry P. Stapp, On the geometric aspect of Sato's postulates on the S-matrix	11
3.	Kohei IWAKI and Tatsuya KOIKE, On the computation of Voros coefficients via middle convolutions	55
4.	Toshinori Takahashi, On the WKB theoretic structure of a Schrödinger operator with a Stokes curve of loop type	71
5.	Yutaka Matsui, An inversion formula of Schapira type for topological Radon transforms of definable functions	85
6.	Ahmed Sebbar, Paperfolding and modular functions	97
7.	Shingo Kamimoto, Takahiro Kawai and Tatsuya Koike, Resurgent functions and linear differential operators of infinite order — Their happy marriage in exact WKB analysis	.27
8.	André Voros, Zeta functions over zeros of Zeta functions and an exponential-asymptotic view of the Riemann Hypothesis	47
9.	Takashi Aoki, Toshinori Takahashi and Mika Tanda, Exact WKB analysis of confluent hypergeometric differential equations with a large parameter	.65
10.	Shinichi Tajima, On $b$ -functions and algebraic local cohomology classes attached to hypersurfaces with line singularities	75
11.	Takashi Aoki, Naofumi Honda and Susumu Yamazaki, Kernel functions and symbols of pseudodifferential operators of infinite order with an apparent parameter	.93

12.	Naofumi Honda and Kohei Umeta, On the sheaf of Laplace hyperfunctions in several variables	213
13.	Daisuke Tarama and Jean-Pierre Françoise, Analytic extension of Birkhoff normal forms for Hamiltonian systems of one degree of freedom — Simple pendulum and free rigid body dynamics —	219
14.	Grzegorz Łysik, Summability of formal solutions to partial differential equations	237
15.	Masafumi Yoshino, Semi-formal solution and monodromy of some confluent hypergeometric equations	255
16.	Akira Shudo, The Stokes geometry of the quantum Hénon map in the horseshoe regime	263
17.	Yoko UMETA, Multiple-scale analysis for some class of systems of non-linear differential equations	283
18.	Yoshitsugu Takei, On the fourth order PI equation and coalescing phenomena of nonlinear turning points	301