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Numerical Predictions for Centrifuge Model Tests of a Liquefiable Sloping Ground Using a Strain Space Multiple Mechanism Model Based on the Finite Strain Theory

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ABSTRACT

This paper presents the results of numerical simulations for dynamic centrifuge model tests of a liquefiable sloping ground performed by various institutions within a framework of Class A, B, and C prediction phases of the LEAP (Liquefaction Experiments and Analysis Project). The simulations are performed by using a strain space multiple mechanism model based on the finite strain theory (including both total and updated Lagrangian formulations), in which both material and geometrical nonlinearity are considered. In the simulation, dynamic response analyses are carried out following self-weight analyses with gravity. The soil parameters of the constitutive model are determined based on the results of laboratory soil tests (e.g., cyclic triaxial tests) and some empirical formulae. The identification process of the parameters is explained in details besides the computational conditions (e.g., geometric modeling, initial and boundary conditions, numerical schemes such as time integration technique). In addition to the numerical results of the Class A prediction using a target input motion, those of the Class B and C predictions using recorded motions in the centrifuge model tests are also presented. Comparison between these predictions and measured results has revealed that the constitutive model parameters for effective stress analyses should be calibrated to well capture the shape and trend of liquefaction resistance curves, and subsequently estimate the damage of soil systems due to liquefaction with higher accuracy.

Keywords: Class A, B and C predictions, Effective stress analysis, Liquefiable sloping ground, Strain space multiple mechanism model, Finite strain theory

1. Introduction

Studies on evaluation of liquefaction-induced damage to soil-structure systems during large earthquakes have been developed through both experimental (e.g., laboratory soil test, centrifuge model test) and analytical (e.g., effective stress analysis) methods since 1970s. In particular, constitutive models of soils have been advanced by academic researchers toward the application of numerical simulation in practice since 1990s and effective stress analyses are being used increasingly in seismic design for evaluating the degree of damage to soil-structure systems due to liquefaction. The accuracy of these effective stress analyses is considered to be improving through comparison with experimental results and case histories of damage of urban infrastructures in the past large earthquakes. However, a practical process for validation of the analytical procedures including the applicability of constitutive models has not yet been established, in particular for liquefaction phenomena, as commonly recognized among geotechnical engineering community.

The necessity of validation was pointed out in VELACS project more than twenty years ago [1]. The VELACS project contributed to the development of numerical modeling on liquefiable ground, but it was revealed that there were some difficulties in obtaining reliable data for validation because the laboratory and centrifuge experimental results showed some variation among different facilities, in particular for complicated model tests.

In the same vein as the VELACS project, a new international effort called LEAP (Liquefaction Experiment and Analysis Projects) has been proposed [2-4]. The LEAP is an international research collaboration among universities (researchers) in the US, UK, Japan, China and Taiwan to evaluate the capabilities of constitutive models for liquefaction problems. One of the goals is to validate the capabilities of existing analytical procedures, including constitutive models of soils for liquefaction phenomena by using laboratory experiments and centrifuge model tests [5, 6]. As part of LEAP exercises, recently Tobita et al. [7] presented results of numerical (Class A) predictions of centrifuge model tests performed at different facilities in Japan for validation of existing effective stress analysis codes. Although valuable results were obtained, some inconsistency was recognized among the test results at different facilities, primarily because model containers (shear-beam type containers) with different size, mass, and friction characteristics were used and each centrifuge has custom earthquake simulation shaker systems. This is why validation of numerical models remains a problem yet to be solved because there were some complexities in replicating the experimental boundary condition (i.e., shear-beam type) and properly considering the effects of mass and friction of the shear-beam in simulation.

For LEAP-GWU-2015, one project within LEAP, a new validation effort with simpler boundary condition using a simpler model container has been carried out in order to circumvent the difficulties in numerical modeling associated with complex boundary conditions, and to obtain a set of reliable centrifuge test data with high quality among different centrifuge facilities, which can be used for validation of analytical procedures for liquefaction phenomena. Kutter et al. [8] presents model specifications and compare the results of the centrifuge model tests performed at Cambridge University (CU) in UK, Kyoto University (KU) in Japan, National Central University (NCU) in Taiwan, Rensselaer Polytechnic Institute (RPI) and University of California Davis (UCD) in USA, and Zhejiang University (ZU) in China.

This paper presents results of numerical simulations for the dynamic centrifuge model tests, performed at the six centrifuge facilities, within a framework of Class A, B, and C prediction (e.g., [9]) phases of the LEAP. The simulations are performed by using a strain space multiple mechanism model based on the finite strain theory (including both total and updated Lagrangian formulations) [10], in which both material and geometrical nonlinearity are considered. In this paper, the identification process of model parameters is explained in details besides the computational conditions (e.g., geometric modeling, initial and boundary conditions). In addition to the numerical results of the Class A prediction using a target input motion, those of the Class B and C predictions with recorded motions obtained in the centrifuge model tests are also presented.

2. Brief summary of centrifuge experiments

This section briefly describes model specifications of the centrifuge experiments. The model is composed of uniform sand, with a 5 degree slope, for all six centrifuge facilities as shown in Fig. 1. For some facilities in which horizontal shaking is carried out in the plane of spinning of the centrifuge, the 5 degree slope in the shaking direction is modeled as a curved surface corresponding to the radius from the axis of rotation of the centrifuge (Fig. 1(b)). The width of the sloping ground is 20 m and the height at midpoint is 4 m in prototype scale. Figure 1 also shows the locations of accelerometers (depicted as a rectangle and triangle) and pore pressure transducers (depicted as a circle). Bold solid line symbols indicate required sensors for all centrifuge facilities, highly recommended sensors are shown in bold dashed lines, and recommended sensors are shown in non-bolded solid lines. The sloping ground is made of Ottawa F-65 sand by dry pluviation method with a target density of 1652 kg/m^3, which corresponds to a relative density of about 65%. Following the air pluviation, the ground was prepared to be fully saturated through a number of saturation techniques [8].

A series of five input motions, three of which were non-destructive and two destructive, was used for the LEAP-GWU-2015 validation experiments. All five motions were a ramped sinusoidal wave (1 Hz, 16 cycle) with a specified PGA. Figure 2 shows the first destructive motion, which is the second motion of the sequence (Motion 2) and used as an input motion for the Class A prediction as described in Section 5, with a PGA of 0.15 g. The non-destructive motions (i.e., Motions 1, 3, and 5) were intended to estimate the characteristics (e.g., stiffness) of ground after the destructive motions (i.e., Motions 2 and 4). In the following, the destructive motions are used for validation of the constitutive models of soils and analytical techniques. For further details about the centrifuge experiments, refer to [8].

3. Constitutive model of soils

In this paper, a strain space multiple mechanism model incorporating a new stress-dilatancy relationship [11], which has been extended based on the finite strain theory [10, 12], is used as an effective stress model of sands.

The original version of the strain space multiple mechanism model was proposed by Iai et al. [13] within the context of infinitesimal strain theory about twenty years ago. The model has been implemented into a finite element program, called “FLIP ROSE (Finite Element Analysis Program of LIquefaction Process/ Response Of Soil-structure

Systems during Earthquakes)”, and widely used in numerical simulation in practice for evaluating the seismic performance of soil-structure systems [14-16]. In the model, the behavior of granular materials is idealized on the basis of a multitude of virtual simple shear mechanism oriented in arbitrary directions (e.g., the virtual simple shear stress is an intermediate quantity in the upscaling process from the microscopic level to the macroscopic stress). This is why the model can take into account the evolution of induced fabric under various loading conditions, including the rotation of principal stress axis direction, the effect of which is known to play an important role in the cyclic behavior of the anisotropically consolidated sand [17, 18].

With an aim to control dilatancy in a more sophisticated manner, the strain space multiple mechanism model has been updated by introducing a new stress-dilatancy relationship [11]. In addition, the model has been recently extended within the context of the finite strain theory [10, 12] to take into account the effect of geometrical nonlinearity, and implemented in a finite strain analysis program, called “FLIP TULIP (Finite Element Analysis Program of LIquefaction Process/ Total and Updated Lagrangian Program of LIquefaction Process)”, which has been developed based on the infinitesimal strain program “FLIP ROSE”. The extended model begins to be used in numerical simulation for evaluating the seismic performance of soil-structure systems including large deformation phenomena [19, 20].

The finite strain formulation has been derived both in the reference (or undeformed) configuration corresponding to a fixed reference time (i.e., an initial time $t = 0$) and the current (or deformed) one at a subsequent time $t > 0$ [10, 12]. The Lagrangian (or material) description based on the former configuration is applied to the total Lagrangian (TL) approach, whereas the Eulerian (or spatial) description based on the latter configuration is used in the updated Lagrangian (UL) approach. The UL approach has advantages in its simplicity in formulation but has disadvantages in that numerical errors in the computed configuration at one time step will be accumulated for the following time steps. The TL approach has advantages in that the computation is always referring to the same reference configuration which is unaffected by the numerical errors but has disadvantages in its complexity in formation. Major advantages in performing both the TL and UL analyses are to confirm the reliability of the numerical results by completely different numerical scheme and formulation. The both types of formulation are available in the program "FLIP TULIP" [10, 12].

4. Detailed specification of numerical simulation (FE analysis)

4.1. Definition of Class A, B, and C predictions

In this study, Class B and C predictions are performed besides Class A prediction by using the strain space multiple mechanism model based on the finite strain theory in order to validate the applicability of the model. Preceding a detailed explanation of the analytical condition, the definition of Class A, B, and C predictions are briefly described. According to [9], the meaning of the three predictions is defined as follows:

Class A prediction

Class A prediction is based upon the planned experiment, not the actual experiment, which means Class A is a true prediction of an event made prior to the event.

Class B prediction

Class B prediction is performed after the experiment is completed, but without knowledge of the results and may be based upon as-built properties and measured input data. Class B is after the event, but with results unknown to the predictor.

<u>*Class C prediction*</u>

Class C prediction is carried out after the event, with results known to the predictor. The predictor may or may not iteratively adjust the model parameters to improve the quality of simulation compared to observations. Class C prediction is considered to be a comparison of the computed response with known experimental data.

4.2. Determination of model parameters

Parameters of the strain space multiple mechanism model, required for liquefaction analyses, are classified into the following three types; the first specifies volumetric mechanism, the second specifies shear mechanism, and the third controls liquefaction and dilatancy. The model parameters that specify the characteristics of liquefaction and dilatancy in the Class B and C predictions are different from those in the Class A prediction (as shown later in Tables 1 and 2). This is because the parameters in the Class B and C predictions have been adjusted according to the results of undrained cyclic triaxial tests under stress control, which were carried out after the Class A prediction. Hence, the parameters will be described later, including the determination methodology, in each section corresponding to the Class A, B, and C predictions.

The parameters for defining the characteristics of volumetric and shear deformation, which are commonly used in the Class A, B, and C predictions with only a slight modification as described below, are shown in Table 1. The mass density ρ_t is determined based on the specific gravity of sand and the void ratio, which is obtained from the target density (1652 kg/m^3) of soil in the Class A prediction. In the Class B and C predictions, the measured density at each facility [8] is used, but no significant difference in the density was recognized among different facilities, and therefore the calculated mass density is almost equal to the value shown in Table 1.

The initial (small-strain) shear modulus G_m under an arbitrary confining pressure p is evaluated by using the following equation in the program:

$$G_\mathrm{m} = G_\mathrm{ma} \left(p / p_\mathrm{a} \right)^{m_\mathrm{G}} \tag{1}$$

where p_a denotes the reference effective confining pressure, and the index m_G, which is set to be 0.5 in this study, indicates the dependency of shear modulus on confining pressure. The initial shear modulus G_ma under the confining pressure p_a is estimated using the void ratio through an empirical relation [21] as follow:

$$G_\mathrm{ma} = 7000 \frac{\left(2.17 - e\right)^2}{1+e} p_\mathrm{a}^{0.5} \tag{2}$$

As is the case with the mass density, the initial shear modulus for the Class B and C predictions is slightly different among the different facilities because the void ratio, which was determined from the measured density at each facility, is different. However, the difference in the estimated shear modulus is very small, and the shear modulus is almost identical to the calculated value, shown in Table 1, using the target density (1652 kg/m^3) of soil for the Class A prediction. In the analysis of liquefaction, Equation (1) is modified to consider the dependency on the state of liquefaction in addition to the confining pressure dependency [11].

The rest of the parameters for shear mechanism (i.e., the internal friction angle $\phi_\mathrm{f}^\mathrm{PS}$ for plane strain and the maximum damping constant h_max), shown in Table 1, are determined as follows: in order to obtain $\phi_\mathrm{f}^\mathrm{PS}$, first, the critical state frictional constant M is evaluated from a monotonic undrained triaxial compression test (Fig. 3(b)) of

Ottawa F-65 sand, of which density was set to be 1652 kg/m^3 (the target density). Then, following Appendix, the internal friction angle ϕ_f^{PS} for plane strain is given as:

$$\sin\phi_f^{PS} = \frac{1}{2}\frac{1}{\cos(\pi/6)}\mathrm{M} \tag{3}$$

The internal friction angle ϕ_f^{TR} (i.e., 35.5 degree in this case) under triaxial compression is, if necessary, calculated by [22].

$$\sin\phi_f^{TR} = \frac{3\mathrm{M}}{6+\mathrm{M}} \tag{4}$$

For the maximum damping constant h_{max}, the standard value for sands (=0.24) is used. In analogy with the shear modulus given by Equation (1), the initial bulk modulus $K_{L/U}$ under an arbitrary confining pressure p can be evaluated, by assigning the reference confining pressure p_a, as

$$K_{L/U} = K_{L/Ua}\left(p/p_a\right)^{n_K} \tag{5}$$

where $K_{L/Ua}$ is given from the shear modulus G_{ma} in Equation (2) by using the Poisson ratio of 0.33, and the index n_K, which is set to be 0.5 in this study, denotes the dependency of bulk modulus on confining pressure. In the analysis of liquefaction, the above equation is extended, by referring to the initial confining pressure p_0, as

$$K_{L/U} = r_K K_{U0}\left(p/p_0\right)^{l_K} \tag{6}$$

where r_K is a reduction factor of bulk modulus, and the power index l_K represents the confining pressure dependency of bulk modulus [11]. By changing the parameters r_K and r_{ε_d}, which controls the dilative and contractive components of dilatancy shown in Table 2, while keeping the product (i.e., $r_{\varepsilon_d} \times r_K$) constant, we can independently control volumetric characteristics due to the dissipation of E.P.W.P. following liquefaction with no change in liquefaction resistance. However, the standard value (i.e., 0.5) shown in Table 1 is used for r_K, because no laboratory test data about the volumetric characteristics (e.g., the relationship between the volumetric strain due to consolidation following liquefaction and the maximum amplitude of shear strain [23]) is obtained.

In addition, the permeability of the ground, which is kept constant at 1.18×10^{-4} m/s, is determined from the results of permeability tests [8] to take into account the effect of pore water flow and migration. The bulk modulus of pore water is set to be 2.2×10^{6} kPa.

4.3. Initial and boundary conditions and input motions

The finite element (FE) analysis is carried out under a 2-dimensional plane strain condition with the same prototype dimension of the centrifuge model test, using 1701 nodes and 3200 elements including pore water elements, as shown in Fig. 4. In the simulation, 4-node quadrilateral elements are used with the selective reduced integration (SRI) technique [24]. The mesh (finite element) size must be 10-20 times smaller, as a rough guide, than the wavelength corresponding to the highest frequency of interest [25]. In this study, we use 15-20 Hz, which is relatively large compared to the natural frequency (i.e., 1 Hz) of the target motion, as the highest frequency. In order to replicate the boundary conditions of the rigid container in the model test, degrees of freedom of displacements at the base are fixed both horizontally and vertically, and only horizontal displacements are fixed at the side boundaries. Whereas the side and bottom boundaries are set to be impermeable, pore water pressure at the ground surface is specified to represent a hydrostatic condition.

Following a self-weight analysis with gravity, whose objective is to obtain the initial stress distribution before shaking, a dynamic response analysis is performed for 60 s considering pore water flow and migration. Whereas the target input motion (Motion 2) shown in Fig. 2 is given at the base of the numerical model for the Class A prediction, the recorded Motions 2 at the bottom of container during the centrifuge model tests, which are somewhat different from the target motion depending on the facilities (as described later in Fig. 16), are used as input motions for the Class C prediction. In the Class B prediction, the dynamic response under the recorded Motions 4 at each facility (shown later in Fig. 23) is simulated (with the experimental results unknown to the predictors). This follows a dynamic response analysis under the recorded Motions 2.

The numerical time integration is done by the SSpj method [26] with the standard parameters θ_1=0.6, θ_2=0.605 for the equation of motion and θ_1=0.6 for the mass balance equation of pore water flow, using a time step of 0.005 s. The both equations are written in the context of the finite strain formulation (i.e., the Lagrangian or Eulerian description) [12]. In the dynamic simulation, Rayleigh damping (α=0.0, β=0.00032), or stiffness proportional damping in this case, is used to ensure stability of the numerical solution process. The latter value is determined from the equation of $\beta = 2h_1 / \omega_1$, in which the damping ratio h_1 of the fundamental vibration mode is assumed to be 1% and the natural circular frequency of the mode is calculated as ω_1=7.55 rad/s from an eigenvalue analysis.

5. Class A prediction (for Motion 2)

5.1. Determination of liquefaction and dilatancy parameters

This section presents the numerical results of the Class A prediction for the Motion 2 shown in Fig. 2. First, details are given to describe how the model parameters (Table 2) which specify the characteristics of liquefaction and dilatancy are obtained.

In the Class A prediction, the parameters of Table 2 are determined by fitting to the results of strain-controlled cyclic undrained triaxial tests under various cyclic strain amplitudes as shown in Figs. 5 and 6 besides the results of a monotonic undrained triaxial test shown in Fig. 3. Although perfect correspondence between measured and computed results cannot be attained, particularly for the dilative component of the measured response in the monotonic test (Fig. 3), the overall tendency of the cyclic tests is reasonably simulated. In the process of parameter determination, we give priority to the degree of correspondence for the cyclic tests over that for the monotonic test, because a cyclic behavior of the sloping ground under the ramped sinusoidal wave is a final target of the prediction. The undrained shear strength q_{us} for steady state analysis is not used in this study (which corresponds to the condition that the strength is set to be a very large value), because Ottawa F-65 sand used in the centrifuge model tests is clean sand (less than 0.5% fines) with a large undrained shear strength as shown in Fig. 3.

Figure 7 presents a comparison of computed liquefaction resistance curves (under the initial confining pressure of 50 kPa), which are evaluated by using the model parameters shown in the third row of Table 2, with measured data. This data was unknown to the predictor at the stage of the Class A prediction (obtained after the Class A prediction through stress-controlled cyclic triaxial tests). This figure indicates that the computed liquefaction resistance is underestimated compared to the measured one, particularly at lower stress levels, in which a large cyclic number of loading is required for the onset of liquefaction. For instance, about 50 cyclic loads of which shear stress ratio of 0.2 are required for the occurrence of liquefaction in the laboratory test, whereas only one-tenth of the measured number can induce liquefaction under the same stress ratio in the simulation. This finding suggests that liquefaction resistance, which is generally obtained through stress-controlled cyclic shear tests, is difficult to evaluate accurately, particularly at lower stress levels, based on only strain-controlled cyclic shear tests. As described in the following section, the difference in Fig. 7 may be considered a reason to overestimate dynamic deformation in the Class A prediction compared to the results of the centrifuge model tests.

5.2. Numerical results of Class A prediction

In the Class A prediction, infinitesimal deformation analysis based on the infinitesimal strain theory is performed in addition to large deformation (finite strain) analyses, in which the effect of geometrical nonlinearity is considered by adopting both the TL and UL formulations.

Figure 8 shows the deformed configuration with contour plots of the maximum shear strain (i.e., $\gamma_{\max} = \sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \gamma_{xy}^2}$) distribution after shaking. For both the infinitesimal and finite strain analyses, a similar trend is observed in terms of the deformation mode: the upper (or left) side of the sloping ground settles down whereas the lower (or right) side upheaves due to degradation and deformation of soil under cyclic loading. In particular, almost the same response is obtained in the time history of lateral displacement at the midpoint of the ground surface (i.e., at $x=B/2$) as shown in Fig. 9(a). However, the effect of geometrical nonlinearity, which has been confirmed to become obvious when the strain goes beyond about 20% [19, 20], can be recognized in the deformed shape of the ground surface and the strain distribution. The centrifuge model tests qualitatively show the same tendencies as the finite strain analyses shown in Figs. 8(b)(c) and 9(a), but the amount of the deformation is overestimated compared to a measured range (e.g., up to 600 mm for lateral displacements at the center surface as described in the next section) among the six facilities [8], with the exception of vertical displacement at the center. This may be explained by the liquefaction resistance in the Class A prediction which is lower compared to the measured resistance as shown in Fig. 7.

Figure 9(b) presents the computed time history of horizontal response acceleration (only responses at the required sensors indicated in Fig. 1 are shown because of space limitations), which indicates no significant difference between the infinitesimal and finite strain analyses. Hence, the effect of geometrical nonlinearity is considered to be trivial in terms of acceleration. The computed time history of excess pore water pressure is shown in Fig. 9(c), in which there are small but recognizable effects of geometrical nonlinearity. The results obtained from the TL formulation are almost identical to those by the UL formulation in the figure, which indicates the formulations are numerically equivalent to each other as is the case with past studies (e.g., [19, 20]). It may be difficult to directly compare the results with measured time history at each facility because the input motion in the Class A prediction differs from the recorded motions at the bottom of container, but the measured acceleration at RPI (shown later in Fig. 18(b)) is well simulated. This is because the recorded motion at RPI is quite similar to the target motion as shown in Fig. 16. Hence, the input motion conditions of the Class A prediction may be represented by the RPI experiment. However, by comparing the response of excess pore water pressure (E.P.W.P.), the computed time history shown in Fig. 9(c) differs somewhat from the measured one (shown later in Fig. 18(c)). As mentioned previously, the discrepancy in the liquefaction resistance curves shown in Fig. 7 may be the reason.

For the Class C and B predictions in the following sections, only the numerical results obtained from the finite strain analysis based on the TL formulation are presented. This is because the finite strain analyses are superior to the infinitesimal strain analysis when a strain level becomes larger than 20% [19, 20], and the results of the Class A prediction has indicated that the TL and UL formulations are numerically equivalent to each other as described above.

6. Class C predictions (for Motion 2)

6.1. Determination of liquefaction and dilatancy parameters

In this section, the results of the Class C prediction for Motion 2 recorded at the bottom of the container at each centrifuge facility (but not the target one), are presented and compared to the measurements. First, the model parameters are calibrated, as shown in Table 2.

Stress-controlled cyclic undrained triaxial tests were performed following the Class A prediction. These test results (e.g., stress path, stress-strain relationship) were used to obtain the parameters, controlling the characteristics of liquefaction and dilatancy, including liquefaction resistance curves. The undrained shear strength q_{us} for steady state analysis was not defined as it is for the Class A prediction (because the strength is very large as shown in Fig. 10). The results obtained from the simulation for the cyclic shear tests are presented in Figs. 11-14 and compared to the measurements. Although the numerical simulations have some difficulty in perfectly replicating the measured relationships, the growth in strain amplitude in addition to the effective stress path and stress-strain relationship follow well the overall trend. Summarizing these results, the liquefaction resistance curves are obtained as shown in Fig. 15. In contrast to the result of the Class A prediction shown in Fig. 7, the measured relation between the shear stress ratio and the number of cyclic loads are well captured, including at lower stress levels (which has importance as pointed out by researchers, e.g. [27]).

The parameter r_K shown in Table 1 can be calibrated through the centrifuge test results, particularly focusing on the volumetric characteristics (i.e., settlement of the liquefied ground) due to the dissipation of E.P.W.P. following liquefaction, in the Class C prediction, even though the characteristics is not obtained from laboratory tests. However, the standard value is used as in the Class A prediction, because a variation in the parameter had no marked influence on the vertical displacement at the midpoint of the ground surface through the calibration.

6.2. Numerical results of Class C prediction

The input motions (Motion 2) which were recorded at the bottom of the each container are shown in Fig. 16 and compared to the target motion. The reproducibility of the achieved motions is different depending on the facilities, which indicates that the dynamic response at each facility was affected by the difference from the target motion. Thus, the Class A prediction using a target motion may not be considered as representative of the actual behavior, because the discrepancy in achieved motions cannot be ignored.

Henceforth, the results of the Class C prediction are described. First, the numerical results of the Class C prediction for ZU are shown in Fig. 17 (using the ZU input motion shown in Fig. 16). It is noted that the measured time history of lateral displacement at the midpoint of the ground surface is calculated from the acceleration time history recorded near the point (a 0.2 Hz high pass filter was applied to remove drift), and the residual lateral displacement measured at the end of the test using surface markers have been superimposed on the displacement time history. A similar technique was used to construct the measured time history of lateral displacement for UCD, KU, and CU. Whereas the midpoint moved 600 mm in the experiment, the computed lateral displacement at the point is only about 100 mm as shown in Fig. 17(a). The induced lateral displacement in the experiment is relatively large compared to the other five facilities [8], because the ground could have been looser than had been expected. The computed time histories of horizontal acceleration and E.P.W.P. show a trend similar to what was recorded at some locations (e.g., AH2 in Fig. 17(b), P4 in Fig. 17(c)) while some discrepancies are found between the measure and computed results at other locations.

Figure 18 presents the results of the Class C prediction for RPI. The measured time history of lateral displacement at the midpoint of the ground surface is obtained from the processing of the images taken by a high-speed camera with the aid of markers placed there. The observed lateral displacement at the point is well simulated (Fig. 18(a)). In contrast, the computed residual settlement is about 40 mm (i.e., 1% of the ground height at the center) smaller than that in the centrifuge test [8]. The precise cause of this discrepancy is as yet not well known, but the measuring precision remains controversial (including for other facilities) because the settlement was not measured by a displacement transducer but estimated by direct measurement of distance before and after the test. As is the case with lateral displacement, a good agreement is obtained for the horizontal acceleration and E.P.W.P. shown in Fig. 18(b) and (c), respectively. Comparison between Figs. 9(c) and 18(c) revealed that the parameter modification, based on the stress-controlled undrained cyclic tests performed after the Class A prediction, has improved the accuracy of the prediction of E.P.W.P. behavior.

The numerical results for NCU are given in Fig. 19. Figure 19(a) indicates that the computed lateral displacement at the midpoint of the ground surface is shifting towards the positive direction (i.e., the lower side) while oscillating, whereas no residual lateral displacement is induced in the experiment despite the inclination of the ground. This is because no measured displacements were available for the NCU case, and the measured time history only shows the dynamic component computed from acceleration time history. The measured horizontal acceleration and E.P.W.P. response is generally reasonably captured as shown in Fig. 19(b) and (c). However, the prediction accuracy is not as high as that in the case of RPI.

Figure 20 shows the results for UCD. Because the viscosity of pore fluid in the centrifuge experiment was set to be ten times higher than that in the other facilities, the permeability of the ground in the simulation is reduced one-tenth from the original value (section 4.2). Whereas the computed vertical displacement at the midpoint of the ground surface is underestimated compared to the observed residual settlement (about 90 mm [8]), the measured lateral displacement at the point is relatively well simulated (Fig. 20(a)). By comparing the horizontal acceleration shown in Fig. 20(b), a relatively good agreement is shown, particularly for AH1 and AH2. The amount of the computed E.P.W.P. buildup during shaking is about 60% smaller than the measurement, but the behavior in the dissipation process after shaking is reasonably simulated as shown in Fig. 20(c).

Comparison between the simulation and experiment for KU is presented in Fig. 21. The measured lateral displacement at the midpoint of the ground surface after shaking is successfully captured, but the measured vertical displacement (about 130 mm [8]**)** at this location is underestimated as shown in Fig. 21(a). The overall trend of the time history of horizontal acceleration and E.P.W.P. is generally consistent with the measurement except the amplitude of the fluctuations during shaking, which is a little overestimated due to the stronger effect of positive dilatancy compared to the experiment.

Finally, the comparison for CU is given in Fig. 22. Figure 22(a) indicates that the numerical simulation reasonably captures the lateral displacement at the midpoint of the ground surface in the experiment. However, the computed vertical displacement is evaluated about 40 mm (i.e., 1% of the ground height at the center) smaller than the measured residual settlement [8]. With regard to the horizontal acceleration shown in Fig. 22(b), the simulation provides a similar response for AH1 and AH2. For the rest, the computed response is larger than the measurements. The measured E.P.W.P. shown in Fig. 22(c) includes a significant noise, and thereby is difficult to directly compare with the numerical results, but the average trend in the experiment is generally consistent with the computed one.

To summarize the comparison of the Class C predictions with the centrifuge experiments, the measured responses (i.e., lateral displacement, horizontal acceleration, E.P.W.P.) were generally reasonably simulated. However, the simulations did not provide results that are completely consistent with the experimental ones, even though the recorded input motions were used. The measured response varied widely among the facilities, in particular for the lateral displacement, compared to the degree of variation in the input motions. This suggests that non-negligible difference may exist in the test procedure among the facilities (e.g., the setup process of the sloping ground) besides the difference in input motions. Hence, if you want to perfectly simulate the experimental results, it may be necessary to consider in more detail the variation of the test procedure in simulation.

7. Class B predictions (for Motion 4)

In the Class B prediction, the dynamic response at each facility was simulated using the recorded Motion 4 (shown in Fig. 23), with the experimental results unknown to the predictor. The Class B prediction is performed following the dynamic response analysis under the recorded Motions 2, by ignoring the non-destructive Motion 3 between the Motions 2 and 4. The same model parameters as the Class C prediction described above are used in the Class B prediction as shown in Table 2. Strictly speaking, model parameters should be modified after the liquefaction triggered by Motion 2 considering the effect of pre-shearing history (i.e., pre-liquefaction) on the dynamic properties of sands (e.g., liquefaction resistance). However, the ground condition just before the Motion 4 was not measured by using a direct method (e.g., cone penetration test under centrifugal acceleration) in the experiments, and no information about the dynamic properties is provided in order to adjust the model parameters. Hence, we use the same model parameters throughout the Class B prediction, by assuming that the relative density and the cyclic stress ratio may not change drastically through the one-time liquefaction in the case of medium dense clean sands as described in past studies [28, 29]. Because the dynamic response analysis under the Motion 2, which is carried out prior to the prediction under the Motion 4 in the Class B prediction, is assumed to be the same with the simulation in the previous section, only the results for the Motion 4 are presented below.

The numerical results of the Class B prediction for ZU are shown in Fig. 24. Comparison of the results shown in Fig. 24(a) indicates that the computed lateral displacement at the midpoint of the ground surface after shaking is about one-seventh the experimental result. This is the same tendency as in the Class C prediction, and the reason may be related to the experimental fact that the induced lateral displacement in ZU is more than twice that of other facilities [8]. This test may require an additional parameter adjustment for more accurate prediction depending on the test condition. As for the horizontal acceleration and E.P.W.P., the general trend is relatively well simulated whereas the effect of positive dilatancy is overestimated during shaking compared to the measurements.

Figure 25 presents the results of the Class B prediction for RPI, in which the target motion is well achieved at the bottom of the container shown in Fig. 23. The computed lateral displacement at the midpoint of the ground surface is generally consistent with the measured one as shown in Fig. 25(a). As in the Class C prediction, highly accurate estimates are provided for the horizontal acceleration and E.P.W.P., while a difference in the amplitude of fluctuations is recognized, as shown in Fig. 25(b) and (c).

The numerical results for NCU are given in Fig. 26. At the midpoint of the ground surface, the evaluated relative lateral displacement is a little larger than the experimental result, which shows oscillation during shaking but no residual displacement, as presented in Fig. 26(a). The cause of this discrepancy may be the same as the case of Motion 2 (Fig. 19(a)). As for the horizontal acceleration and E.P.W.P., the general trend in the experiment is relatively well simulated, in particular during the build-up process of E.P.W.P.

Comparison between the simulation and experiment for KU is presented in Fig. 27. Figure 27(a) indicates that the measured response at the midpoint of the ground surface is simulated with a high degree of accuracy both during and after shaking. As in the Class C prediction, a reasonable agreement is provided for the horizontal acceleration and E.P.W.P., although discrepancies are recognized (especially the difference in the fluctuation amplitude due to the stronger effect of positive dilatancy as shown in Fig. 27(b) and (c)).

Finally, comparison for CU is given in Fig. 28. Although the effect of positive dilatancy is overestimated, the overall tendency is captured by the simulation for the displacements and accelerations shown in Fig. 28(a) and (b), respectively. As described in the Class C prediction, the measured E.P.W.P. includes undesirable noise as shown in Fig. 28(c), and therefore is difficult to be directly compared with the simulation. However, the average trend in the experiment is generally similar to the computed results.

8. Conclusions

This paper presented the numerical results for dynamic centrifuge model tests of a liquefiable sloping ground performed by a number of institutions within a framework of LEAP (Liquefaction Experiments and Analysis Project). The simulations were performed by using a strain space multiple mechanism model based on the finite strain theory, including both the TL and UL formulations, besides the infinitesimal strain analysis. In addition to a Class A prediction (a true blind prediction of an event) with a target input motion, Class C prediction (a prediction after an event, with the results known to the predictors) was carried out using the recorded input motions in the centrifuge test. Besides, the results of Class B prediction (a prediction after an event, with the experimental results unknown to the predictors), which was performed with different recorded motions following the Class C prediction, were also presented. Primary conclusions of this study are summarized as follows:

(1) Although the measured deformation was qualitatively simulated by the finite strain analyses in the Class A prediction, the magnitudes of the lateral movement were overestimated. This is not attributed to the difference between the target and recorded input motions, but to the fact that the model parameters, which was determined through the results of strain-controlled cyclic shear tests, didn’t capture the liquefaction resistance characteristics obtained from stress-controlled cyclic shear tests after the prediction.

(2) In the Class C prediction, the parameter adjustment by using the liquefaction resistance curves improved the quality of simulation, with no further iterative adjustment based on the centrifuge experimental results (e.g., displacement, excess pore water pressure), compared to the Class A prediction. This suggests that the constitutive model parameters for liquefaction analysis should be determined and calibrated so as to well capture the general shape of liquefaction resistance curves,

including at low stress levels, in order to precisely estimate the damage of soil-structure systems due to liquefaction.

(3) In the Class A through C predictions, a comparison between the finite and infinitesimal strain analyses has demonstrated that the effect of geometrical nonlinearity is of importance in precisely evaluating the dynamic behavior of liquefiable sloping ground, particularly in a large deformation regime (only the comparison for the Class A prediction is shown in this paper). Besides, it has been shown that the TL formulation is not only theoretically but also numerically equivalent to the UL formulation in the finite strain analyses.

(4) Numerical simulation has the capability to predict the general trend of dynamic behavior in shaking table model tests (e.g., centrifuge) by using input model parameters, determined through laboratory experiments, and the observed waveform at the shaking table as an input motion, but may have difficulty providing completely consistent results with the model tests. This is because the measured response is more or less affected by a series of test procedures (e.g., soil deposition method) in addition to the difference in input motions. However, the variation among different facilities is difficult to quantify in advance of simulation although the effect cannot be ignored. Thus, how to reduce or estimate the variability is a critical issue that remains to be resolved.

Appendix

The critical states in triaxial compression tests are given by [22]

$$q = \mathrm{M}p \tag{A1}$$

where

$$q = \sigma'_1 - \sigma'_3 \tag{A2}$$

$$p = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} \tag{A3}$$

$$\mathrm{M} = \frac{6\sin\phi_{\mathrm{f}}^{\mathrm{TR}}}{3 - \sin\phi_{\mathrm{f}}^{\mathrm{TR}}} \tag{A4}$$

According to [30], the difference between the maximum and minimum principal stresses are given by using the Lode angle θ as follows:

$$\frac{\sigma'_1 - \sigma'_3}{2} = \sqrt{J_2}\cos\theta \tag{A5}$$

where

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \tag{A6}$$

$$s_{ij} = \sigma'_{ij} - \frac{1}{3}\delta_{ij}\sigma'_{kk} \tag{A7}$$

Substitution of the condition of triaxial compression (i.e., $\theta = \pi / 6$) into Equation (A5) yields the stress difference under triaxial compression as follows:

$$\frac{\sigma'_1 - \sigma'_3}{2} = \sqrt{J_2}\cos\frac{\pi}{6} \tag{A8}$$

From Equations (A1) and (A8), the critical states under general stress states are given as

$$\sqrt{J_2}\cos\frac{\pi}{6} = \frac{1}{2}\mathrm{M}p \tag{A9}$$

In the case of two-dimensional (2D) problems, the Mohr-Coulomb failure criterion can be defined as

$$\tau = p^{\mathrm{2D}}\sin\phi_{\mathrm{f}}^{\mathrm{2D}} \tag{A10}$$

where

$$\tau = \frac{\sigma'_1 - \sigma'_3}{2} \tag{A11}$$

$$p^{\mathrm{2D}} = \frac{\sigma'_1 + \sigma'_3}{2} \tag{A12}$$

Using Equation (A5), the failure criterion is rewritten in terms of the stress invariant as

$$\sqrt{J_2}\cos\theta = p^{\mathrm{2D}}\sin\phi_{\mathrm{f}}^{\mathrm{2D}} \tag{A13}$$

By assuming that two-dimensional analyses under plane strain (PS) condition are equivalent to the condition of pure shear (i.e., $\theta = 0$ and $\sigma'_2 = \left(\sigma'_1 + \sigma'_3\right)/2$), Equation (A13) can be written as

$$\sqrt{J_2} = p \sin \phi_{\mathrm{f}}^{\mathrm{PS}} \tag{A14}$$

Comparison of Equations (A9) and (A14) yields the relation between the critical state frictional constant M and the internal friction angle $\phi_{\mathrm{f}}^{\mathrm{PS}}$ for plane strain as follows:

$$\sin \phi_{\mathrm{f}}^{\mathrm{PS}} = \frac{1}{2} \frac{1}{\cos\left(\pi / 6\right)} \mathrm{M} \tag{A15}$$

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