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Long-term bridge health monitoring and performance assessment based on a Bayesian approach

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ABSTRACT: This study presents a damage detection approach for the long-term health monitoring of bridge structures. The Bayesian approach comprising both Bayesian regression and Bayesian hypothesis testing is proposed to detect the structural changes in an in-service seven-span steel plate girder bridge with Gerber system. Both temperature and vehicle weight effects are accounted in the analysis. The acceleration responses at four points of the bridge span are utilized in this investigation. The data covering three different time periods are used in the bridge health monitoring (BHM). Regression analyses showed that the autoregressive exogenous (ARX) model considering both temperature and vehicle weight effects has the best performance. The Bayesian factor is found to be a sensitive damage indicator in the BHM. The Bayesian approach can provide updated information in the real time monitoring of bridge structures. The information provided from the Bayesian approach is convenient and easy to handle compared to the traditional approaches. The applicability of this approach is also validated in a case study where artificially generated damage data is added to the observation data.

KEYWORDS: autoregressive model; Bayesian statistics; bridge health monitoring; damage detection; Kalman filter; long-term assessment; real bridge

1. INTRODUCTION

Performance assessment and inspection of civil infrastructures including bridge structures are quite important in public service for many countries. An effective maintenance strategy needs to be designed based on timely observations of the structure. Among all the existing approaches, structural health monitoring (SHM) using vibration data has been recognized as one of the promising technologies. The vibration data is utilized to estimate the modal properties of structures which will be examined in an SHM analysis (Doebling, Farrar, Prime, & Shevitz, 1996; Salawu, 1997; Ko & Ni, 2005). The fundamental concept of this technology is that modal parameters can be expressed as functions of structures' physical properties. Therefore, changes in physical properties, such as reductions in stiffness, will imply change in these modal properties.

In recent times, many techniques have been developed and devoted to identify the hidden information of structural integrity from the structural vibration data (Deraemaeker, Reynders, De Roeck, & Kullaa, 2007; Dilena & Morassi, 2011; Kim, Kawatani, & Hao, 2012; Zhang & Lam, 2015; Zhang, Beer, & Quek, 2015). From the acquired vibration data, one aims to extract damage-sensitive features and then to discriminate among features from the damaged and undamaged bridges quantitatively (Worden, Manson, & Fieller, 2000; Gul & Catbas, 2009; Döhler, Hille, Mevel, & Rücker, 2014). The former procedure can be designated as feature extraction, whereas the latter can be regarded as feature discrimination.

However, most bridge structures need to face a changing environment and several operational conditions which largely affect the structural integrity information during the long in-service period. The induced components in the signals may lurk in the measured vibration data and disguise themselves as structural responses (Sohn, Worden, & Farrar, 2003; Deraemaeker et al., 2007). Such influences could cause unreliable results in the SHM analysis. Thus, it may lead to unreliable inspection results and wrong maintenance strategies (Zhang, Kim, & Tee, 2017; Zhang, Kim, Tee & Lam, 2017).

The operational condition of bridges contains a wide range of factors including temperature, wind, traffic-induced vibrations and traffic mass effects. Focusing on short and medium span bridges, temperature and traffic influences become the dominant factors (Peeters & De Roeck, 2001; Cunha, Caetano, Magalhaes, & Moutinho, 2013). It is needed to perform accurate estimate of these environmental and operational effects for the long-term bridge health monitoring (BHM). Many former works have been developed and devoted to this challenging topic. Magalhaes et al. (2010) had applied a strategy to eliminate the environmental and operational effects in the monitoring for an arch bridge. An output-only based structural health monitoring approach for considering the changing environmental conditions is proposed by Reynders et al. (2013). Spiridonakos and Chatzi (2014) had developed a stochastic structural identification method based on vibrational and environmental data. Until recently, Spiridonakos et al. (2016) had utilized polynomial chaos expansion models for the monitoring of structures under operational variability. In general, from the recent advances in structural health monitoring, it is now widely recognized that operational conditions have to be well handled in the vibration data analysis.

This study intends to develop a long-term BHM approach which could consider the environmental effect (temperature) and operational effect (vehicle weight). The traditional approach is going to be enhanced by means of a Bayesian approach, which is capable of handling multiple environmental and operational factors. The proposed Bayesian approach consists of three steps. The first step is to identify damage indicators (DIs) from the time series model constructed for the bridge acceleration data (Nair, Kiremidjian, & Law, 2006; Kim, Isemoto, Sugiura, & Kawatani, 2013), and is described in Section 2.1. As DI is sensitive to the changes in environmental and operational conditions, the second step is to perform regression analysis of DI for removing the environmental and operational influences and is discussed in Section 2.2. This is conducted through the Bayesian regression (Kitagawa & Gersch, 1984). The final step is to perform the decision-making analysis by means of the Bayesian hypothesis testing (Kass & Raftery, 1995) which utilizes the estimated residuals of DI_{ob} in the second step. Section 2.3 describes the final step.

The paper is organized as follows. After the introduction, an overview of long-term BHM procedures will be given. The framework of conducting the long term BHM by using the vibration data is elaborated in Section 2. To improve the traditional approach, detailed information of a proposed Bayesian method will be explained in Section 3. Following that, a case study is presented in Section 4 to demonstrate the proposed procedures. Section 5 then discusses the result and compared it with the existing approaches. The last section draws the conclusion.

2. FRAMEWORK OF LONG-TERM BHM

2.1 Identification of damage-sensitive features from AR coefficients

A convenient BHM approach for detecting bridge damage is through the use of a linear time series model such as the autoregressive (AR) model for the modal parameters. However, drawbacks in modal parameter based bridge diagnosis using time series models are also obvious. The optimal time series model for vibration responses of bridge structures usually comprises higher-order terms. However, these higher order terms might not be necessary when degrees of freedom in the structure are small. The calculation based on unnecessary order terms may lead to spurious modal parameters, which results in false system frequencies and damping constants. Decisions made on these false modal parameters will be quite uncertain and might cause some unexpected consequences. A robust AR coefficient based BHM approach should be accurate enough to identify the key modal parameters.

Like most of the AR based BHM approaches, the first step is to identify the damage indicators from AR time series models constructed for the bridge acceleration responses. This paper includes only a brief description about the DI without covering detailed theories. More theoretical backgrounds of utilizing AR coefficients as a damage-sensitive feature can be found in Nair *et al*. (2006) and Kim *et al*. (2013).

Under the assumption of linear dynamic structural system, the output z at time t can be idealized using the AR model shown in the following.

$$z_t = \sum_{i=1}^{p} a_i z_{t-i} + e_t \tag{1}$$

where z_{t-i} denotes the system output at time t-i, a_i is the i-th AR coefficient, p is the optimal AR order (time lag) and e_t indicates the error at time t. Usually, the optimal AR order can be obtained by means of Akaike Information Criteria (AIC) (Akaike, 1974), which is given by:

$$\mathbf{AIC} = -2\log\left(L\right) + 2(m+1) \tag{2}$$

where L is the total likelihood of the data and m is the order of the AR model. The AIC consists of two terms; the first term is the log-likelihood function and the second term is a penalty function for the number of AR order.

The observed damage index DI_{ob} is defined by (Nair *et al*. 2006; Kim *et al*. 2013):

$$\mathbf{DI}_{ob} = \frac{|a_1|}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \tag{3}$$

where a_1, a_2 and a_3 indicate the first, second and third AR coefficients respectively.

Nair *et al*. (2006) showed that the first three AR coefficients are the significant ones among all the AR coefficients that can characterize the modal properties of a structural system. This is also agreed by Kim *et al*. (2013) who discovered that the computation of DI_{ob}, consists the first three order AR coefficients is a promising choice in BHM. In fact, depending on the kind of damage scenarios, higher order coefficients may also be required in the damage index especially for very localized damage. Moreover, if the bridge structure is very complex, the higher order mode may become dominant. This would also require higher terms in DI rather than the first three AR coefficients. This has to be realized when using the $\mathrm{DI}_{\mathrm{ob}}$.

However, it is also very difficult to obtain such threshold values of DI in judging the health condition of the bridge structures. The threshold values of DI are highly depending on the structural types and damage scenarios. The values are indeed a case by case problem. It has to be determined based on more experimental study and numerical analysis.

2.2 Removing environmental and operational influences

In long-term BHM, the DI_{ob} varies quite a lot because of changes in environmental and operational conditions. In this study, influences of environmental and operational conditions on DI_{ob} are treated as exterior disturbances in the autoregressive exogenous (ARX) model. The Kalman filter is utilized to estimate the model parameters in the ARX model. A regression is then applied in combination with the Kalman filter for modeling the relationship between DI and disturbances caused by environmental and operational factors. Since the Kalman filter is adopted to provide an online updating algorithm, it is possible to utilize long-term monitoring data in a more efficient way. It is believed that the converged model parameters after updating can be considered as the optimal model parameters. A detailed procedure of this step is provided as following.

2.2.1 Kalman filter

The state space model for the Kalman filter is given by:

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_t + \mathbf{G}\mathbf{v}_t \tag{4}$$

$$\mathbf{y}_t = \mathbf{H}_t\mathbf{x}_t + \mathbf{w}_t \tag{5}$$

where $\mathbf{x}_t$ and $\mathbf{y}_t$ are the state matrix and observation at time t, $\mathbf{v}_t$ and $\mathbf{w}_t$ denote noise at time t. For a particular time series model, $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}_t$ are known. The future state can be predicted in terms of the Kalman filter as follows (Kitagawa & Gersch, 1984):

$$\mathbf{x}_{t|t-1} = \mathbf{F}\mathbf{x}_{t-1|t-1} \tag{6}$$

$$\mathbf{V}_{t|t-1} = \mathbf{F}\mathbf{V}_{t-1|t-1}\mathbf{F}^{\mathbf{T}} + \mathbf{G}\mathbf{Q}\mathbf{G}^{\mathbf{T}}, \tag{7}$$

where $\mathbf{x}_{t|t-1}$ and $\mathbf{V}_{t|t-1}$ denote the predicted mean matrix and covariance matrix at time t given the value at time t-1. $\mathbf{Q}$ stands for the covariance matrix of $\mathbf{V}_t$. The filtered state also can be estimated as follows:

$$\mathbf{x}_{t|t} = \mathbf{x}_{t|t-1} + \mathbf{K}_t\left(y_t - \mathbf{H}_t\mathbf{x}_{t|t-1}\right) \tag{8}$$

$$\mathbf{V}_{t|t} = \left(\mathbf{I} - \mathbf{K}_t\mathbf{H}_t\right)\mathbf{V}_{t|t-1}, \tag{9}$$

where $\mathbf{K}_t$ stands for the Kalman gain defined by:

$$\mathbf{K}_t = \mathbf{V}_{t|t-1}\mathbf{H}_t^{\mathbf{T}}\left(\mathbf{H}_t\mathbf{V}_{t|t-1}\mathbf{H}_t^{\mathbf{T}} + \boldsymbol{R}\right)^{-1}, \tag{10}$$

where $\boldsymbol{R}$ denotes the variance of w_t. The choosing of the matrix $\mathbf{Q}$ and $\boldsymbol{R}$ are very important in the Kalman filter. Usually, the Kalman filter is sensitive to errors in $\mathbf{Q}$ and $\boldsymbol{R}$ and its output can be unacceptable if errors are large. Fortunately, there are many recent developments in the iterative unbiased finite impulse response (FIR) filter that has the Kalman filter structure but ignores both $\mathbf{Q}$ and $\boldsymbol{R}$ (Petersen & Savkin, 1999; Gandhi & Mili, 2010; Zhao, Shmaliy, & Liu, 2016). This is utilized in the present study.

2.2.2 ARX model

The noise effect can be largely removed by using the Kalman filter, but the influences induced by environmental and operational factors still exist in the observed data. The handling of these influences is by the use of an ARX model, which can express the environmental and operational factors discretely as exterior disturbances (Bishop, 2006). This can be given by:

$$\mathbf{DI}_{ob}^{(t)} = \sum_{i=1}^{q}\alpha_i\mathbf{DI}_{ob}^{(t-i)} + \sum_{i=1}^{r}\beta_i u^{(t-i)} + w_t, \tag{11}$$

where q and r are the orders of the time series model, α_i and β_i are the associated order coefficients, $u^{(t)}$ denote the observed environmental and operational factors at time t and w_t is the white noise. In other words, by connecting to the Equations (4) and (5), the relationships among the parameters and matrixes can be given by:

$$y_t = \mathbf{DI}_{ob}^{(t)} \tag{12}$$

$$\mathbf{F}\text{=}\mathbf{I} \text{ (Identity matrix)} \tag{13}$$

$$\mathbf{G}=\mathbf{0} \text{ (Null matrix)} \tag{14}$$

$$\mathbf{x}_t=\left[\alpha_1,\ldots,\alpha_q,\beta_1,\ldots,\beta_r\right]^{\mathrm{T}} \tag{15}$$

$$\mathbf{H}_t=\left[\mathrm{DI}_{ob}^{(t-1)},\ldots,\ \mathrm{DI}_{ob}^{(t-q)},\ u^{(t-1)},\ldots,\ u^{(t-r)}\right] \tag{16}$$

$$R=1. \tag{17}$$

Moreover, the initial mean and covariance matrixes were assumed as follows,

$$\mathbf{x}_{0|0}=0 \quad \text{(Null matrix)} \tag{18}$$

$$\mathbf{V}_{0|0}=\mathbf{I} \quad \text{(Identity matrix)} \tag{19}$$

The optimal model parameters are obtained by satisfying

$$\left|\mathbf{x}_{t|t}-\mathbf{x}_{t-1|t-1}\right|\cong\mathbf{0} \tag{20}$$

2.3 Anomaly detection utilizing the Bayesian hypothesis testing

After removing all the environmental and operational effects, the final step is to identify the anomalies from the corrected data. This can be done by examining the differences between the observed DIs and the predicted DIs. The following equation can be utilized to calculate these residuals:

$$r_t=DI_{ob}(t)-DI_{pr}(t) \tag{21}$$

where, $DI_{pr}(t)=\mathbf{H}_t\mathbf{x}_{\mathrm{opt}}$. Here $\mathbf{x}_{\mathrm{opt}}$ represents the estimated ARX model parameters in Equation (11) by utilizing the former observations.

However, there still lacks an algorithm to convert this r value to a theoretical judgment. The decision making could not yet be conducted by purely the numerical values of r. In this study, a Bayesian approach is proposed to tackle this issue. This is elucidated in the following section.

3. BAYESIAN APPROACH

3.1 Bayes factor

In Bayesian statistics, it is possible to estimate the probability of a hypothesis conditionally on observed data. By linking with the damage detection analysis, if the null hypothesis (H_0) is defined as 'healthy' and the alternative hypothesis (H_1) is defined as 'damage', the Bayesian statistics could help to calculate the probability of damage that might occur in the structure. Based on the observed data **D** during monitoring, the posterior odds can be obtained by utilizing priors and marginal likelihoods as:

$$Post_{odds}=\frac{p(\mathrm{H}_1\,|\,\mathbf{D})}{p(\mathrm{H}_0\,|\,\mathbf{D})}=\frac{p(\mathbf{D}\,|\,\mathrm{H}_1)}{p(\mathbf{D}\,|\,\mathrm{H}_0)}\times\frac{p(\mathrm{H}_1)}{p(\mathrm{H}_0)} \tag{22}$$

where $p(\mathrm{H}_1)/p(\mathrm{H}_0)$ indicates the prior odds, $p(\mathbf{D}\,|\,\mathrm{H}_1)$ and $p(\mathbf{D}\,|\,\mathrm{H}_0)$ are called the marginal likelihoods.

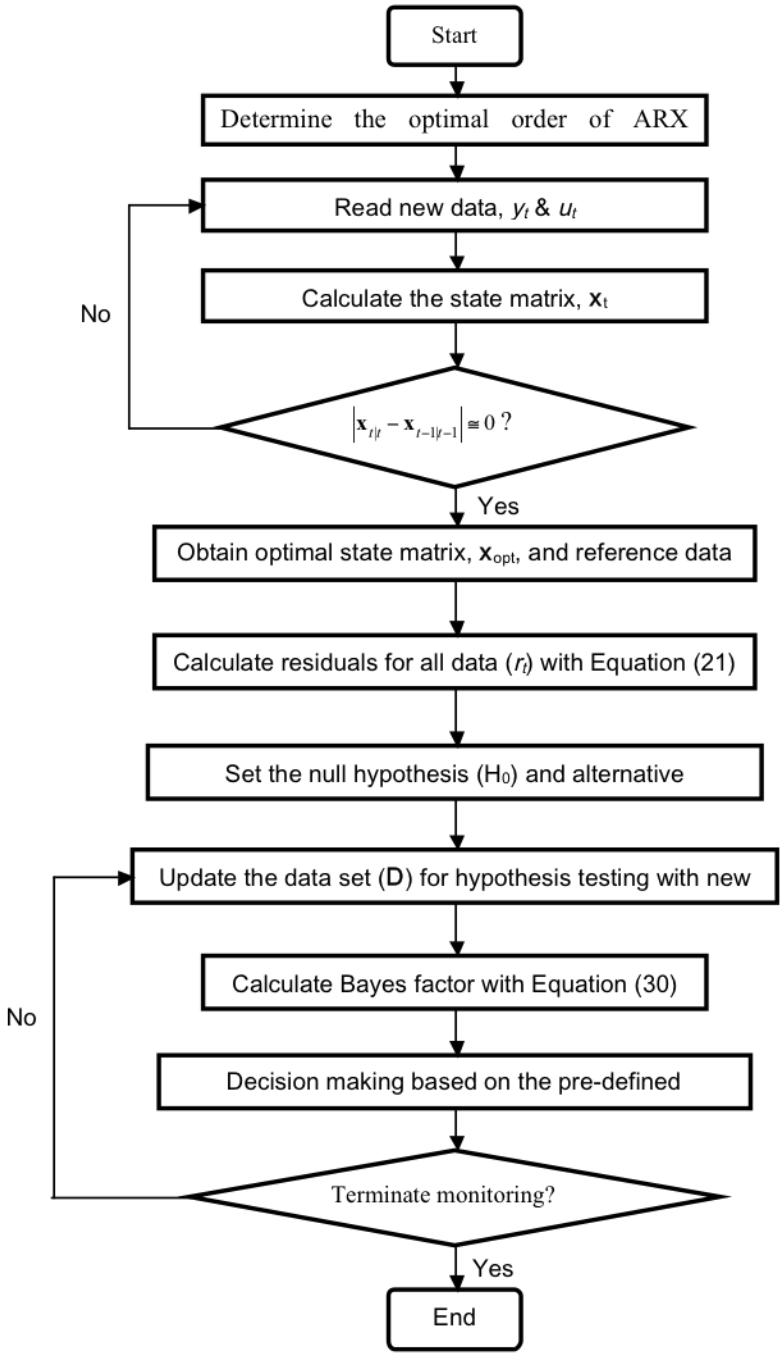

Figure 1. Flowchart of the Bayesian approach for long-term bridge health monitoring.

In this study, the ratio of the marginal likelihoods is considered as Bayes factor (*B*), which is defined as the ratio of likelihood of the two scenarios 'damage' and 'healthy' as follows (Kass & Raftery, 1995):

$$B = \frac{p(\mathbf{D}\,|\,\mathrm{H}_1)}{p(\mathbf{D}\,|\,\mathrm{H}_0)} = \frac{\int p(\mathbf{D}\,|\,\theta_1, \mathrm{H}_1)p(\theta_1\,|\,\mathrm{H}_1)\mathbf{d}\theta_1}{\int p(\mathbf{D}\,|\,\theta_0, \mathrm{H}_0)p(\theta_0\,|\,\mathrm{H}_0)\mathbf{d}\theta_0}, \tag{23}$$

where θ_0 and θ_1 are parameters under the hypotheses H_0 and H_1 .

Obviously, if the Bayes factor is greater than 1, it implies that the data favor the hypothesis H_1 and hence indicates that it is likely to be exposed to damage. If the Bayes factor is less than 1, then it is unlikely to have damage. According to the former work given by Jeffreys (1998), a Bayes factor having value from 1 to 3 means 'barely worth mentioning', 3 to 10 means 'substantial', 10 to 30 means 'strong', 30 to 100 means 'very strong', and greater than 100 means 'decisive'. In other words,

$B \leq 1$ indicates 'nothing (no damage)', $1<B\leq 3$ indicates 'very small damage', $3<B\leq 10$ indicates 'small damage', $10<B\leq 30$ indicates 'strong damage', $30<B\leq 100$ indicates 'very strong damage' and $B>100$ indicates 'decisive (totally damaged)'. In practical applications, it is necessary to provide a threshold as judging criteria for guiding the maintenance. For example, an emergency repair is needed if the Bayes factor higher than 30 is continuously observed. However, the threshold values have to be determined from more comprehensive studies. It is very difficult to consider using absolute values of a Bayes factor as judging criteria for the health monitoring. The threshold needs to be determined for different structures case by case. Even for the same structural type, the marginal value of the Bayes factor for differentiating a damage state and an intact state is different for different damaging scenarios. The estimates of these marginal values can be obtained from experimental studies or numerical analysis. For instance, a model test might be useful in determining the critical Bayes factor values.

Following the given concept, the flowchart of Bayesian approach for long-term bridge health monitoring is illustrated in Figure 1.

3.2 Updating process

Back to the BHM problem, more specifically, the data emphasized in Equation (23) is referring to the observed damage index. As introduced in Section 2, the DI_{ob} and DI_{pr} can be expressed as follows:

$$DI_{ob} = DI_{tr} + \varepsilon_{ob} \tag{24}$$

$$DI_{pr} = DI_{tr} + \varepsilon_{pr} \tag{25}$$

where DI_{tr} indicates the true DI, ε_{ob} and ε_{pr} are associated errors of DI_{ob} and DI_{pr} in this context. Therefore, the residual as given by Equation (21) can be further expressed as:

$$r = DI_{ob} - DI_{pr} = \varepsilon_{ob} - \varepsilon_{pr} = \varepsilon \tag{26}$$

where ε is believed to be normally distributed, e.g. $\varepsilon \sim N(\mu, \sigma^2)$. Therefore, the hypothesis testing can be performed based on the residuals. Following the definition given in Equation (22), the Bayesian hypothesis testing defines the null and alternative hypotheses in Equation (27) and Equation (28).

$$\mathrm{H}_0 : \mu = \mu_0 = 0 \tag{27}$$

$$\mathrm{H}_1 : \mu = \mu_1 \neq 0 \tag{28}$$

Or in other words, this hypothesis testing is testing whether the mean value of the residuals for the newly observed data equals zero. For example, the commonly utilized t-test can be employed in this hypothesis testing. The computation of t statistic is as following

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \tag{29}$$

where $\bar{x}$ is the sample mean, s is the sample standard deviation and n is the sample size. μ_0 is the specified mean value in this test and therefore equals to zero. Once the t value and degrees of freedom are determined, a p-value can be estimated by the use of Student's t-distribution. If the calculated p-value is less than the threshold chosen for statistical significance (usually the 0.05 level), then the null hypothesis is rejected in favor of the alternative hypothesis. In this way, it rejects the hypothesis that the structural health condition remains the same.

Moreover, the Bayes factor can be further developed to a real time updated statistic. Equation (23) is only used for computing the Bayes factor for a collected data group. It can be further integrated to a real time monitoring system. This study adopts an updating equation of B (Jiang & Mahadevan, 2008) for the BHM. The following equation is provided for B.

$$B = \frac{\int_{-\infty}^{\infty} p(\mathbf{D} \mid \mu, \mathrm{H}_1) p(\mu \mid \mathrm{H}_1) \mathrm{d}\mu}{\int_{-\infty}^{\infty} p(\mathbf{D} \mid \mu, \mathrm{H}_0) p(\mu \mid \mathrm{H}_0) \mathrm{d}\mu} = \frac{\frac{1}{\sqrt{2\pi\tau^2}} \sqrt{\frac{\pi}{A}} \exp\left(\frac{B^2}{4A} + C\right)}{\exp\left(-\frac{n\left(\bar{r}^2 + S^2\right)}{2\sigma^2}\right)} \tag{30}$$

where,

$$A = \frac{n\tau^2 + \sigma^2}{2\sigma^2\tau^2} \tag{31}$$

$$B = \frac{n\tau^2\bar{r} + \sigma^2\rho}{\sigma^2\tau^2} \tag{32}$$

$$C = \frac{-n\tau^2 S^2 - n\tau^2\bar{r}^2 - \sigma^2\rho^2}{2\sigma^2\tau^2} \tag{33}$$

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i \tag{34}$$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}\left(r_i - \bar{r}\right)^2 \tag{35}$$

where μ is assumed to follow a normal distribution with mean of ρ and standard deviation of τ (which can be determined from prior knowledge), and n denotes the number of the newly collected data and r_i represents the i-th residual. The Bayes factor is a weighted average likelihood ratio, where the weights are based on the prior distribution specified for the hypotheses (as given in Equations (27) and (28)). The likelihood ratio is evaluated at each data observation and weighted by the relative plausibility assigned to that value. Then once the weights are assigned to each ratio, the Bayes factor can be calculated by taking the average. For the ease of convenience, in this study, the weights of all the newly collected data are equally assigned. For more complicated weight assignment in Bayesian updating, this study refers to the fundamental Bayesian theory books (e.g. see Gelman et al. (1995)). Therefore, the computation of the Bayes factor can be done at any time. The reference data will provide the knowledge in the mean of residual, whereas the newly collected data will be assessed based on Equation (30). The BHM can now be continuous and the information of the Bayes factor can be given instantaneously with the observation.

In contrast to the traditional approaches, the Bayesian approach has the advantage that the detection can be updated automatically. The real time observation is highly desirable in the BHM as the damage often occurs incidentally. However, its suitability for monitoring the real bridge has not been examined. The assessment of whether the Bayesian approach can adequately describe the structural health condition has to be made and this is investigated in the following sections.

Table 1 Inspection results (2008 & 2013) of the target bridge.

	Patterns of cracks in the paint film	No. of cracks in the paint film in 2008	No. of cracks in the paint film in 2013	Cracks in the beam in 2013
Sum of all spans	Pattern 1	0	186	1
	Pattern 4	0	518	0
	Pattern 9	0	370	5
	Total	0	1074	6

Pattern 1: straight cracks, Pattern 9: curving cracks, Pattern 4: other types of cracks.
In 2008 the end beam was strengthened against fatigue damage.

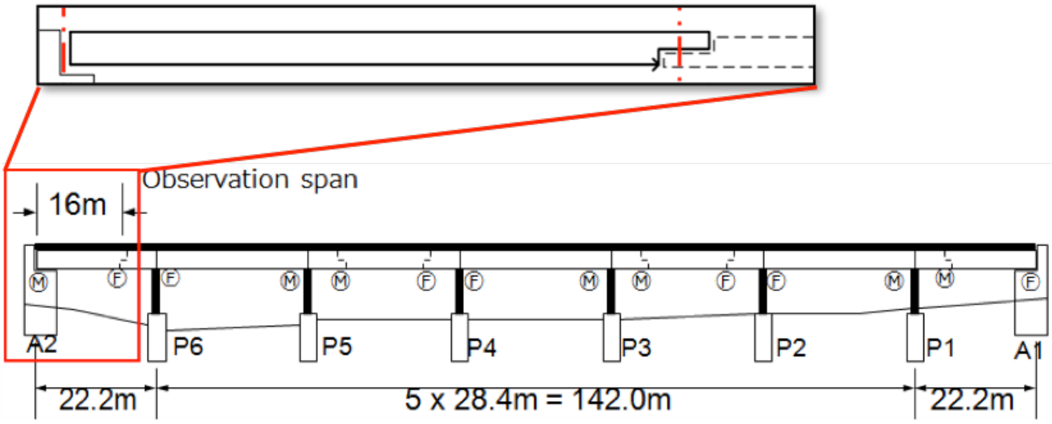

Figure 2. Observed bridge span.

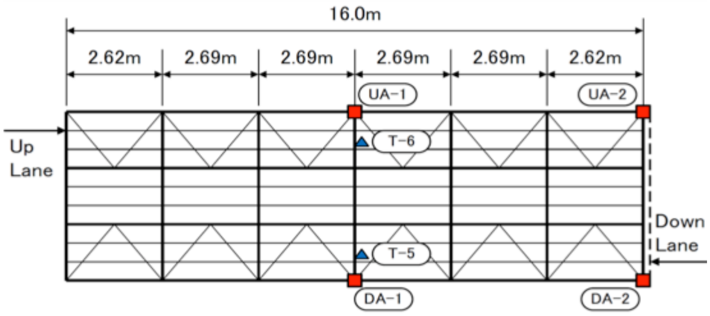

Figure 3. Sensor locations on the observation span.

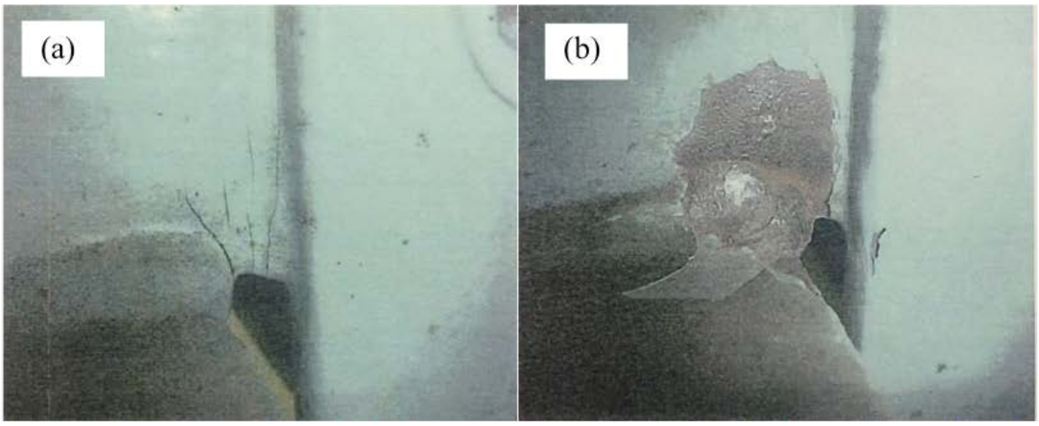

Figure 4. Photo of cracks in the paint film of the target bridge (a) pattern 3: crack length=5mm; (b) pattern 9: crack length =5mm.

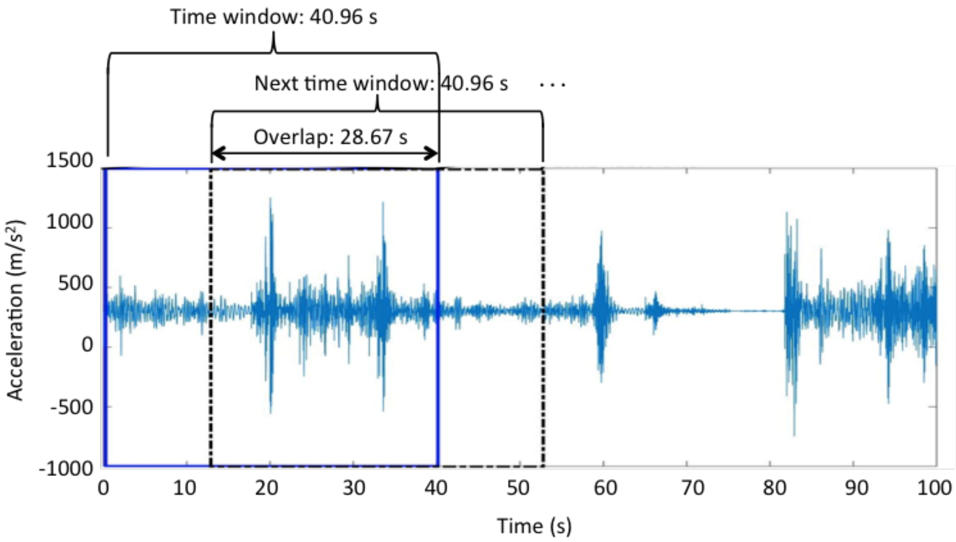

Figure 5. Plot of the adopted time window.

Table 2. Information of the training data set and test data set.

Name (n= number of data)	Monitoring period	Monitored data
Data 0 (n=1512)	5 August 2008- 17 August 2009	Accelerations, temperatures & vehicle weight
Data 1 (n=700)	6 October 2009- 29 March 2010	Accelerations & temperatures
Data 2 (n=1484)	2 July 2013- 7 July 2014	Accelerations & temperatures

Remark: Data 0 denotes the training data set; Data 1 & Data 2 indicate test data sets.

4. LONG-TERM MONITORING ON AN IN-SERVICE STEEL PLATE GIRDER BRIDGE

The dynamic data collected at a bridge is utilized in this study. In 2004 the bridge slab was reinforced by increasing thickness, and in 2008, as a preventive measure against fatigue damage, the beam end was strengthened. Furthermore, structural monitoring was thought to be necessary considering the bridge is located in a national road that is heavily occupied with traffic including around 60% of passing vehicles of heavy trucks, and thus launched for research purpose by bridge authority.

A general picture of the bridge and the observed span are shown in Figure 2. The bridge is a seven-span steel plate-girder bridge with Gerber system, which was constructed in 1960 and is still serving. The bridge is located in a national road that is heavily occupied with traffics including around 60% of passing vehicles of heavy trucks. The plan view with sensor locations on the observation span is illustrated in Figure 3. Therein, the ID number UA-1, UA-2, DA-1 and DA-2 denote the accelerometers for measuring acceleration responses of steel girders on up (UA) and down (DA) lanes. The sampling rate of these accelerometers was 200 Hz. Thermometer IDs were denoted by T-5 and T-6. Temperature was measured and recorded once every hour for the up and down lanes. A bridge weigh-in-motion (BWIM) system (Moses, 1979) is installed in the bridge to measure the vehicle weight (Heng, Oshima, & Kawano, 2011). Visual inspection is conducted every five years to the whole bridge. The latest two times were in August, 2008 and September, 2013 respectively, whose results were shown in Table 1. The photos of typical cracks detected in the inspection are illustrated in Figure 4. According to the results of the two visual inspections, no cracks in the paint film or the beam of the bridge were found in 2008 as the bridge had been strengthened against the fatigue damage in 2008. However, in 2013, 1074 and six cracks were found in the coating film and the beam of the bridge respectively. The reason that the number of cracks has increased a lot is because of the increase of transportation volume. All of these may have caused an accelerated deterioration in the bridge and thus the sign of fatigue cracks has increased quite a lot. Despite the increase in the number of cracks in the coating film, the bridge is regarded as in intact condition during the monitoring period, since no clear damage in its main structure was discovered.

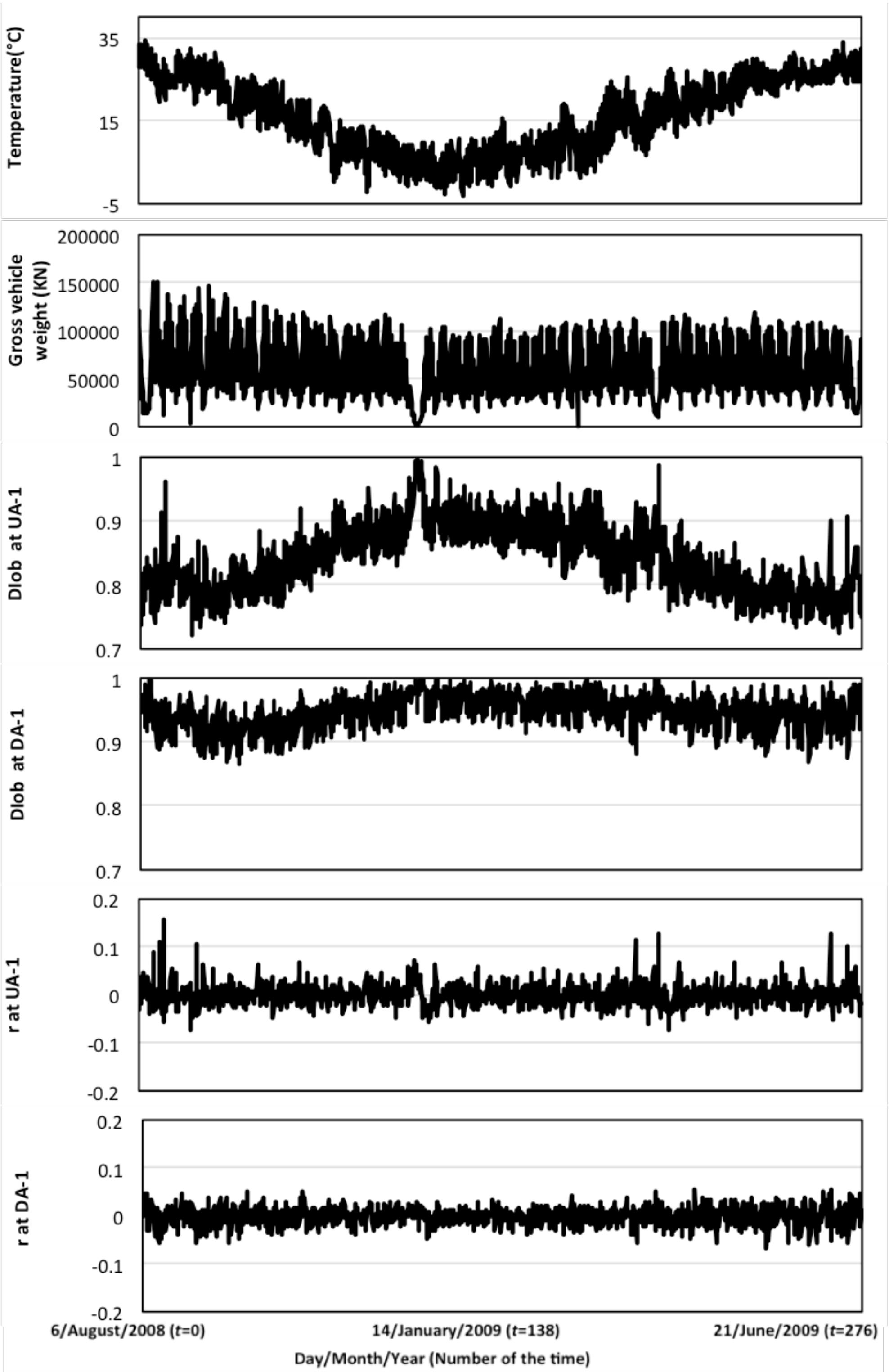

Figure 6. Temperature at T-6, gross vehicle weight estimated by the BWIM system on up lane, and examples of DI_{ob} and residuals (r) at UA-1 and DA-1.

To demonstrate the proposed approach, the data has to incorporate the environmental and operation conditions. The record of temperature and vehicle weight measured from 5th August 2008 to 17th August 2009 is extracted as the training data for establishing the ARX model. Another two data sets are extracted and considered as test data for the damage detection. The information of these training data set and test data sets are provided in Table 2, where Data 0 denotes the training data set, Data 1 and Data 2 denote the test data sets. The influences of those environmental and operational factors on regression are investigated by comparing the following three cases: consider both temperature and vehicle weight effects, consider only temperature effect, and consider only vehicle weight effect.

Table 3. Different regression models.

Name	Regression model	Considered factors	Utilized data
ARX-T	ARX	Temperature	Acceleration & temperature
AR	AR	Not considered	Acceleration
ARX-TW	ARX	Temperature & vehicle weight	Acceleration, temperature & vehicle weight
ARX-W	ARX	Vehicle weight	Acceleration & vehicle weight

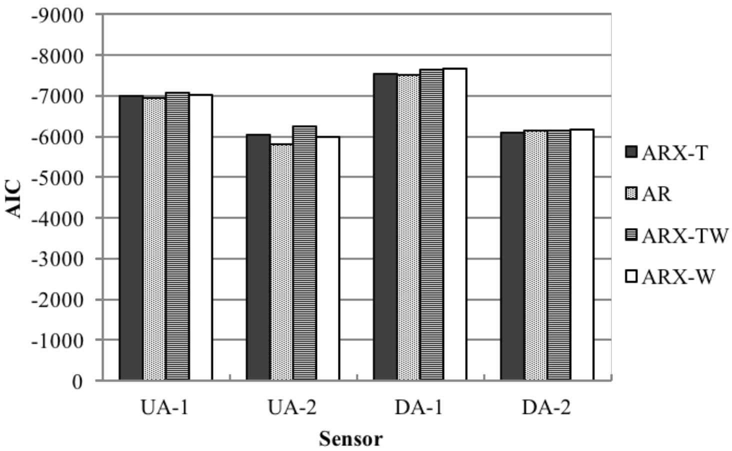

Figure 7. AIC values of all candidate ARX models at the sensors of UA-1, UA-2, DA-1 and DA-2 in Data 0.

5. APPLICATION AND DISCUSSION

5.1 Regression analysis to consider environmental and operational changes

The DI identified in this study is based on the acceleration data through a moving time window, as shown in Figure 5. A time length of 40.96s is adopted for the moving window. The window size is judged from a preliminary study that considers different lengths of moving windows such as 20.48s, 40.96s and 81.92s. For the overlap window, 70% overlap of the time length of 40.96 s (28.67s, in other words 12.29s is the interval of two adjacent time windows) was adopted in this study. The choosing of window size is quite important in this step. On one hand, a smaller one may neglect important information in the time series data, on the other hand, a larger one would lead to unnecessary computations. The chosen one is based on a comparison between a larger and a smaller window size. The selected window size would lead to a stationary value of the corrected DI which would suffer less influence from the window size effect.

This study focuses on changes in lower modes which are not easily affected by noise than higher modes and are relatively easy to identify. This generally implies the lower order AR coefficients are useful terms to the structural health conditions. Thus, the DI is computed based on Equation (3) by using only the first three AR coefficients. Examples of DI_{ob}, residuals (r) of Data 0, temperature and gross vehicle weight are plotted in Figure 6. The Akaike information criterion (AIC) is a useful index for assessing the performance of a regression model. A smaller AIC value implies a better model. The AIC results of all the models considering different environmental and operational inputs, as shown in Table 3, are computed and compared in Figure 7. One should note that the DI discussed in this study (Equation (11)) is not exactly the same as defined in Equation (3). This study first estimates the DI from the AR coefficients identified from the time history of the moving window as shown in Equation (3), and then the ARX model for the time histories of the collected DIs (see Equation (11)) is examined because the DIs in the long-term monitoring are fluctuated by environmental and operation influences. Therefore, the ARX model is trying to remove the noises contained in AR.

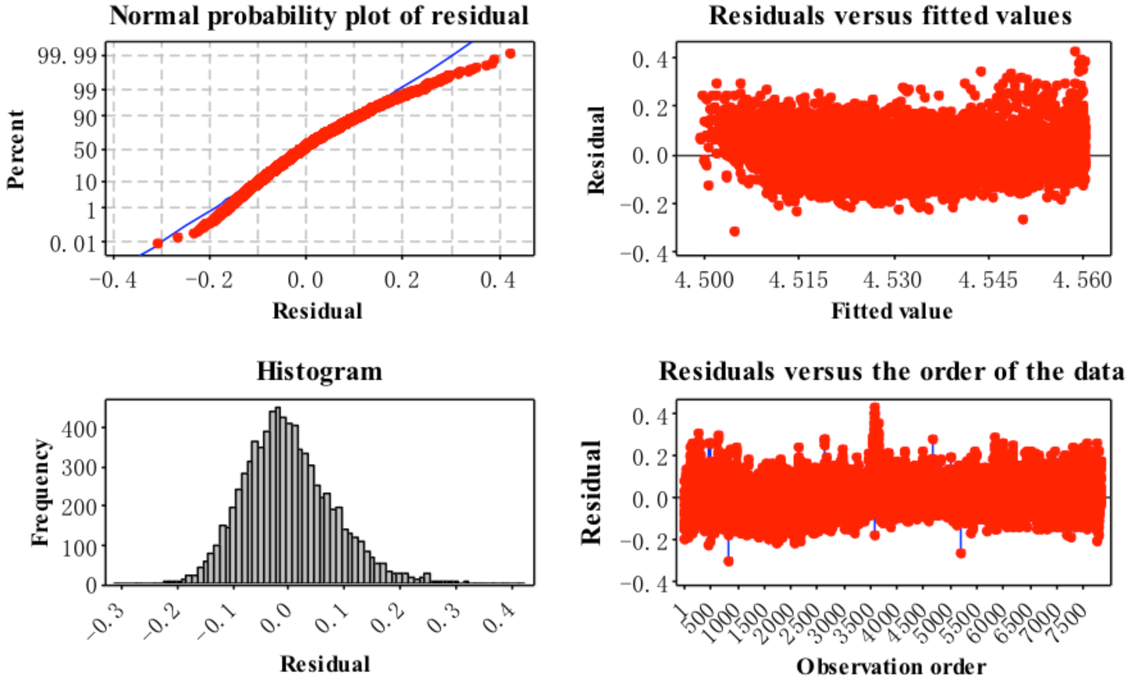

Figure 8. Residual plot of ARX-T model.

The results show that the ARX model considering both temperature and vehicle weight effects (ARX-TW) performs the best among all the candidate models. Meanwhile, it is noticed the ARX model considering only the temperature effect (ARX-T) or vehicle weight effect (ARX-W) will not improve the regression model too much. The construction of ARX model needs to consider both effects. However, since data of vehicle weight are only available during the first year of monitoring, ARX-T is used in the following analysis. The validity of the adopted ARX model can be checked from the residual plot as shown in Figure 8. The well-fitting in the normal probability plot proves the feasibility of the adopted model.

After the ARX model is established, the Kalman filter is applied to the time series data. The quality of the applied Kalman filter can be seen from the plot comparing the original observation and the fitted observation as shown in Figure 9. The well-fitting in the figure proves the validity of the Kalman filter.

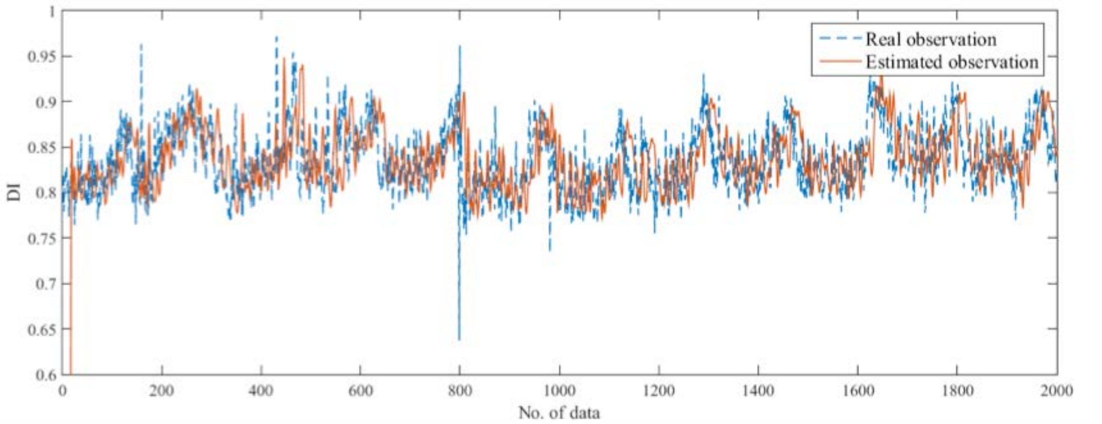

Figure 9. Comparison between the real observation and the estimated observation from Kalman filter.

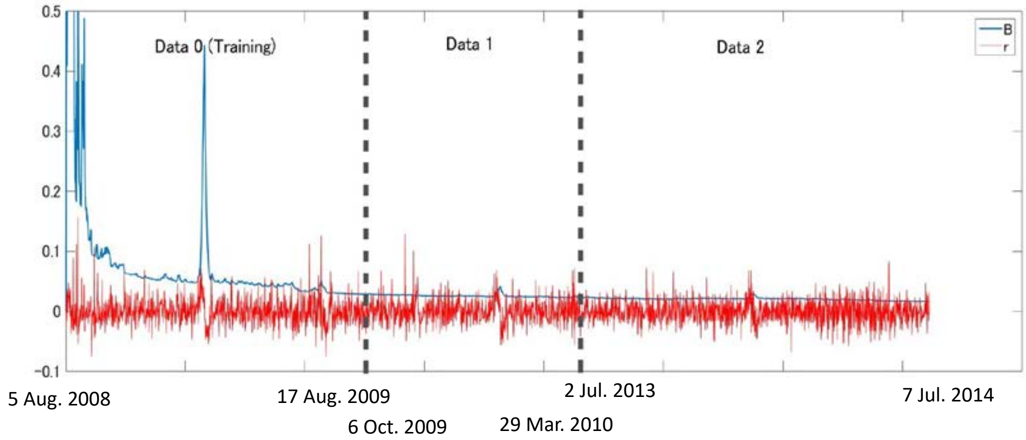

Figure 10. Bayes factor (B) and residual (*r*) of DA1 in time order.

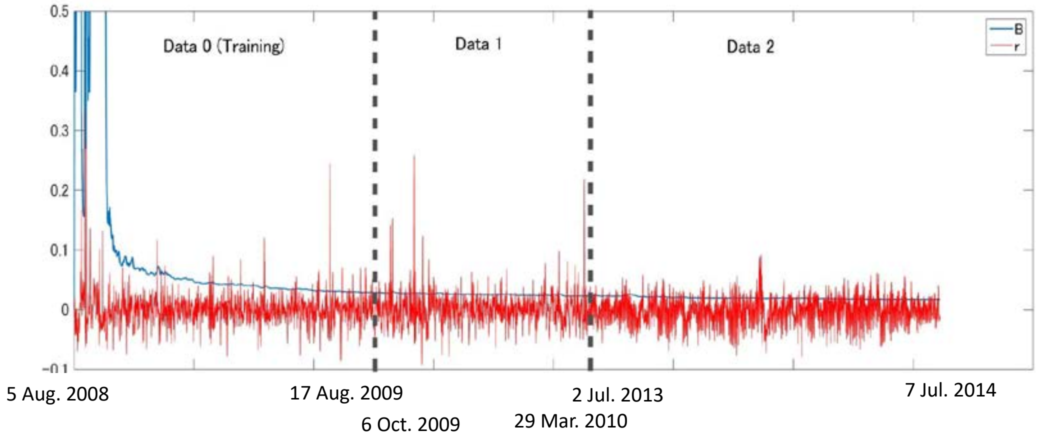

Figure 11. Bayes factor (B) and residual (*r*) of DA2 in time order.

5.2 Bayes factor for damage detection

5.2.1 With healthy data

Since the optimal regression model is determined, the next step is to detect the anomalies from the observations. The residuals calculated with monitored data together with the Bayes factors focusing on them at all four sensors are plotted in Figures 10-13. It can be seen the values of Bayes factors along all the periods are very low, less than 0.5 for the test data sets. It indicates little possibility of damage occurred in the structure. An interesting observation is that small peaks of the Bayes factor were observed at Data 1 and Data 2 of DA1 (see Figure 10) and UA1 (see Figure 12) sensors. This is mainly because of the drastically decreased heavy traffics during new-year holidays, i.e. beginning of January of 2010 and 2014. However, after new-year holidays, those changes disappeared. It implies a sudden increase of Bayes factor, which will not last too long, is only resulted from the holiday event. But if the value keeps very long, it may indicate possibility of damage in the bridge. However, those small peaks of the Bayes factors were not clear for the DA 2 and UA 2 sensors. It is explainable that the sensors at the hinge (DA 2 and UA 2) are dominated by higher frequency characteristics and might be less affected by gross traffic weight than those deployed at the span center (DA1 and UA 1). One should note the calculated Bayes factor values are very unstable at the beginning of the training period. It comes out several peak values in the Figures 10-13. The reason is that the baseline is yet not fully mature at this stage. Therefore, the training is still in its infancy. That is also why the value of Bayes factor is very sensitive to the traffic changes during new-year holiday. In that sense, the peak values during this period do not have too much meaning. The peak values in the training stage are not a reflection of the structural health condition changes. It does not imply an alarm in the bridge structural system.

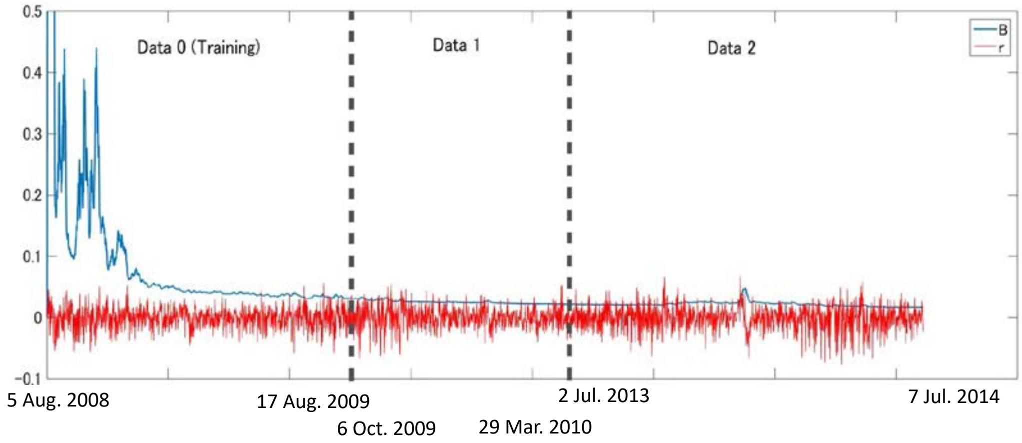

Figure 12. Bayes factor (B) and residual (*r*) of UA1 in time order.

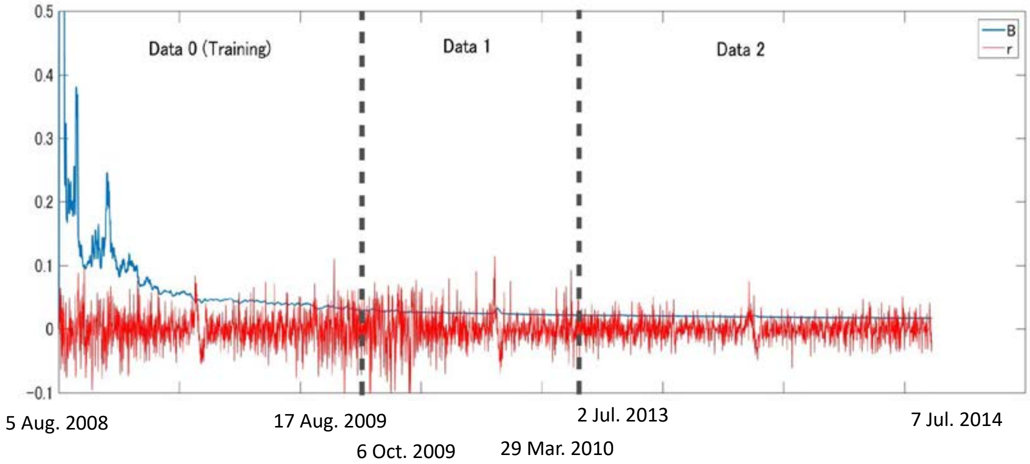

Figure 13. Bayes factor (B) and residual (*r*) of UA2 in time order.

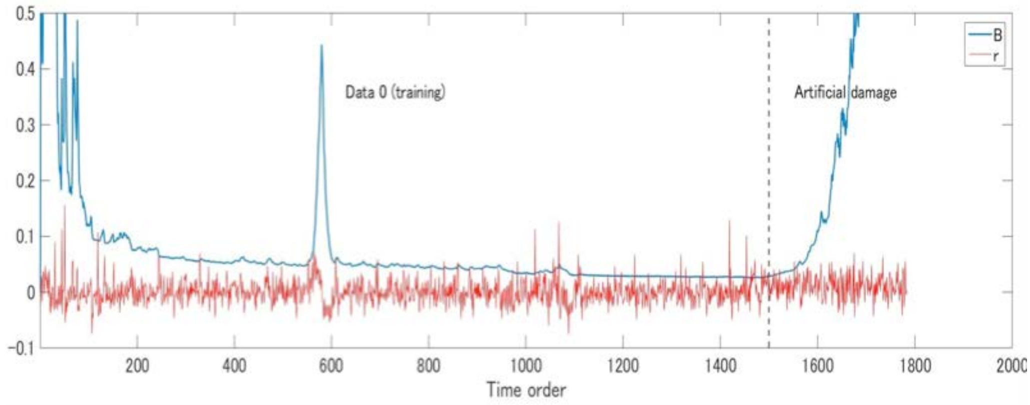

Figure 14. Bayes factor (B) and residual (r) of DA1 in time order (artificial damage included).

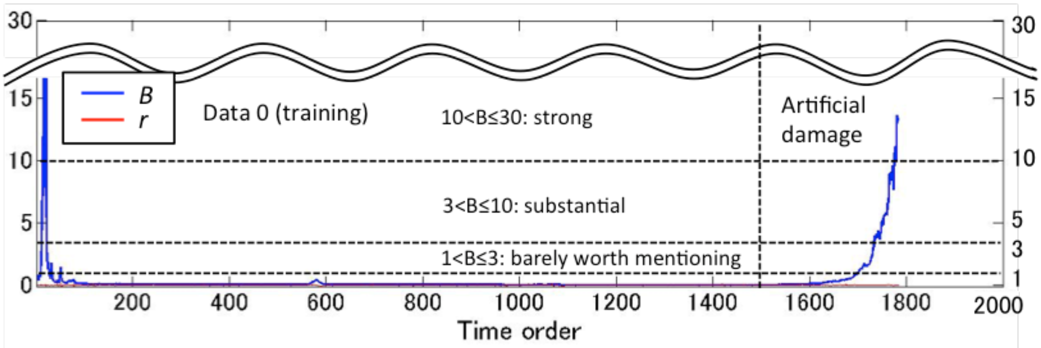

Figure 15. Bayes factor (B) and residual (r) of DA1 in a full-range vertical scale with evidence against H_0 (artificial damage included).

5.2.2 With artificially made damaged data

In the final part of the study, a short investigation is made to an extension to consider an extra artificially made damage data. 300 random residuals were artificially generated by means of the Monte Carlo simulation and treated as the damage data. The generated data were assumed to follow a normal distribution with mean of 0.01 (not 0 deliberately) and standard deviation of 0.02. They were then added to the observation data that can be seen in Figure 14 showing the result of DA1 as an example. These data were treated as continued data after the observation period. Note the artificially made damage data in this case does not exactly have a natural meaning. The assumed distribution of the damage data is obviously made very arbitrarily. It is not corresponding to the real cracks happened to the bridge. We also believe using the real damage data could be more persuasive. However, as here we aim to know how sensitive the Bayes factor is to the extraordinary observations, the natural meaning of the artificial data is not very critical. The more important thing is to know how Bayes factor respond to the anomalies. In that sense, we made the artificial data in quite a simplified way.

The Bayes factor values are estimated and also plotted in Figure 14. It shows that the Bayes factor substantially increased in the region of damaged data, which implies possibility of the Bayes factor for damage detection. The redrawn plot of the Figure 14 in a full-range vertical scale is shown in Figure 15 in which the Bayes factor increased to 13.8 meaning evidence against the H_0 is "strong" (Jeffreys, 1998). Obviously, the applicability of the proposed Bayesian approach is indirectly validated. It can be seen such concept is very useful in the structural damage identification. The value change can be easily observed from Bayes factor and the process of updating is useful to track changes in the Bayes factor.

6. CONCLUSIONS

In this study, a Bayesian approach is proposed to detect the structural damage from long-term monitoring data taken from an in-service plate-girder bridge with Gerber system. The Bayesian approach consists of three steps: Step 1 is to calculate the damage index from coefficients of AR model utilizing bridge accelerations; Step 2 is to establish ARX models for damage index with consideration of environmental and operational changes by means of the Kalman filter; Step 3 is to identify the anomalies from the corrected damage index values based on the residuals utilizing the Bayesian hypothesis testing. The concept is applied to the data covering three different time periods. Several autoregressive exogenous models are considered to model both temperature and vehicle weight effects.

Observations through this study could be summarized as follows.

(1) The adoption of ARX model to consider environmental and operational changes led to higher accuracy compared the linear regression without consideration of the effects.
(2) Bayes factors of the observed bridge along all the periods were very low, less than 0.5, which indicates little possibility of damage occurred in the structure.
(3) The value change can be easily observed from Bayes factor and the process of updating is useful to track changes in the Bayes factor.
(4) The Bayes factor is a useful and sensitive indicator for the BHM, and the updating process can be quite useful in the real-time structural monitoring.
(5) The identification of structural damage can be replaced by the identification of the changes in the Bayes factor, and the detection process can be largely simplified by the Bayesian based detection approach.

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