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Defining Appropriate Boundary Conditions of Hydrodynamic Model from Time Series Data

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Synopsis

This study illustrates a procedure of defining precise boundary conditions for the hydrodynamic model from time series data for a particular event. As a test case, the flood event of different return periods was selected for the Brahmaputra-Jamuna River, Bangladesh. Here, the magnitude of different return period flood was chosen by best-fitted frequency distribution. This study suggests that prior to do any frequency distribution the data series should be checked for the outlier, homogeneity, randomness, dependency, and trend. The magnitude of flood obtained from the best-fitted frequency distribution should be considered as the maximum flooding level of different return period flood and river’s regular discharge hydrograph of the identical peak should be adjusted to fit the different return period floods and can be used as the upstream boundary condition of the hydrodynamic model. From the stage-discharge relationship of that particular year, the adjustment of downstream boundary condition can be estimated and implemented as the downstream boundary condition in the hydrodynamic model.

Keywords: Brahmaputra-Jamuna River, randomness, dependency, trend and frequency distribution

1. Introduction

Hydrodynamic model, particularly the morphodynamic model often needs a very precise boundary condition to reproduce the natural conditions of rivers, lake, oceans. Using directly the statistically analyzed time series data may often produce erroneous results. As an example, assessing the impacts of different return period floods are the common criteria for many engineering works. A vast range of numerical models i.e. one dimensional, two dimensional or three dimensional (Chatzirodou, Karunarathna, & Reeve, 2017; Rostand, 2007; Schuurman, Marra, & Kleinhans, 2013; Siviglia et al., 2013) models are used to assess the hydrodynamic impact of different magnitude floods. Generally, in all of the hydrodynamic model, the defining of upstream and downstream conditions are obligatory. Often the modeler needs to prepare the upstream/downstream boundary file from time series data by statistical analysis. For example, if the maximum discharge is 50000 m^3/s found from the time-series analysis of discharge and the time series analysis of water level shows the maximum water level at downstream was found 13 m using this two may be erroneous; one should use the corresponding water level for 50000 m^3/s. Nevertheless, as the river bathymetry changes every flooding season in general, hence statistically high water level may not correspond high discharge every time. Another

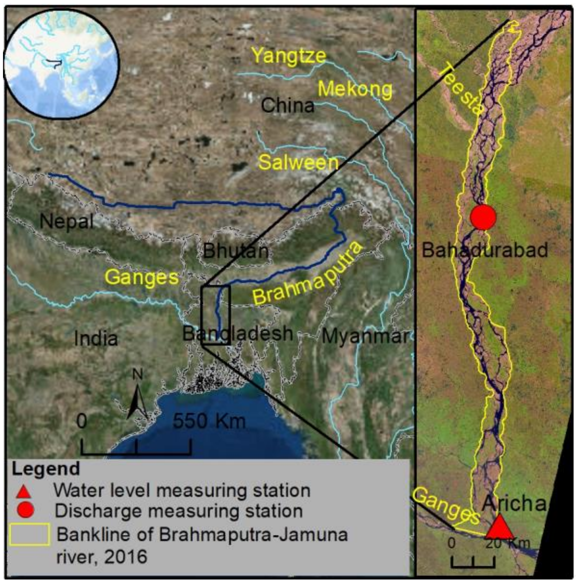

Fig. 1: Map showing the study location

circumstances may happen when one needs to assess the impacts of different return period floods, usually the statistical analysis are made calculating the magnitude in terms of discharge of different return period flood. Then at downstream boundary condition, the corresponding water level data needs to be found. The situation becomes more complicated to define downstream boundary condition when the upstream discharge hydrograph is unsteady. Hence this paper is aimed to solve this problem.

The main objective of this research is to illustrate a procedure to define the appropriate boundary conditions of hydrodynamic model for time series data for a particular event. As a test case magnitude of different floods were chosen. The specific objectives were to analysis the time series discharge and water level data of a river. Then specifying the magnitude of different return period floods- 2.33,10, 20, 50 and 100year. Next,

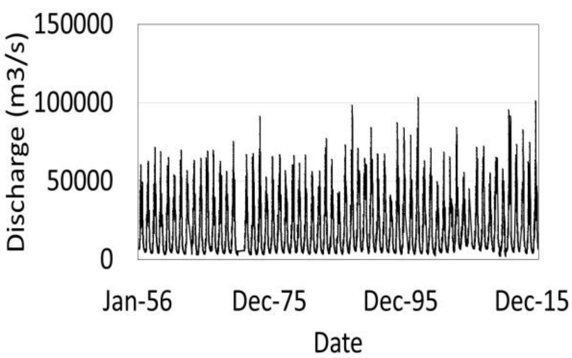

Fig. 2 Raw discharge data considered for the study

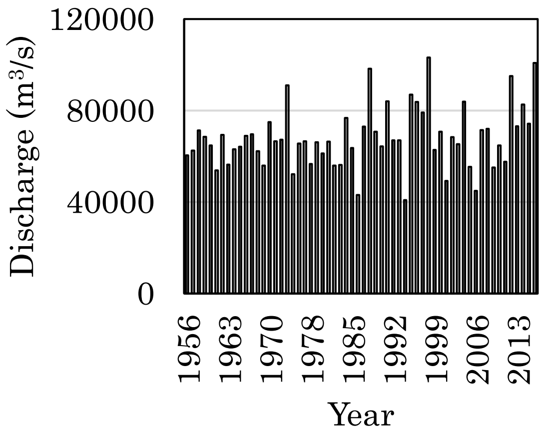

Fig. 3 Annual maximum discharge data

calculating the hydrograph for that magnitude floods. And lastly to define the downstream boundary conditions for the predefined upstream boundary condition.

As the study area, the two stations located in Brahmaputra-Jamuna River were selected shown in Fig. 1. The time series discharge data of the upstream station Bahadurabad was chosen for an upstream boundary and the data of downstream station, Aricha was selected for water level data.

2. Methodology

Firstly the raw data of discharge shown in Fig. 2 were considered and checked. Then from this dataset annual maximum dataset were calculated as shown in Fig. 3. Frequency analysis was performed to calculate the maximum magnitude of the flood of different return period. Prior to frequency analysis, the data series should be checked for outlier, homogeneity, and randomness. Several statistical tests exist to examine the outlier indicating any observation point that is distant from other observations of the data series (Hodge & Austin, 2004). Here, 'Grubb's' test was adopted which is commonly used for univariate data set (Grubbs, 1969). Then to check the independency of the data series of random variables 'Turning point' test was performed (Heyde & Seneta, 1963). To check the randomness of the data series, non-parametric statistical test names as 'Run test' that checks a randomness hypothesis for data sequence was done (Barton, 1957). The data dependency and stationary were checked using Pettit's and Dickey-Fuller test

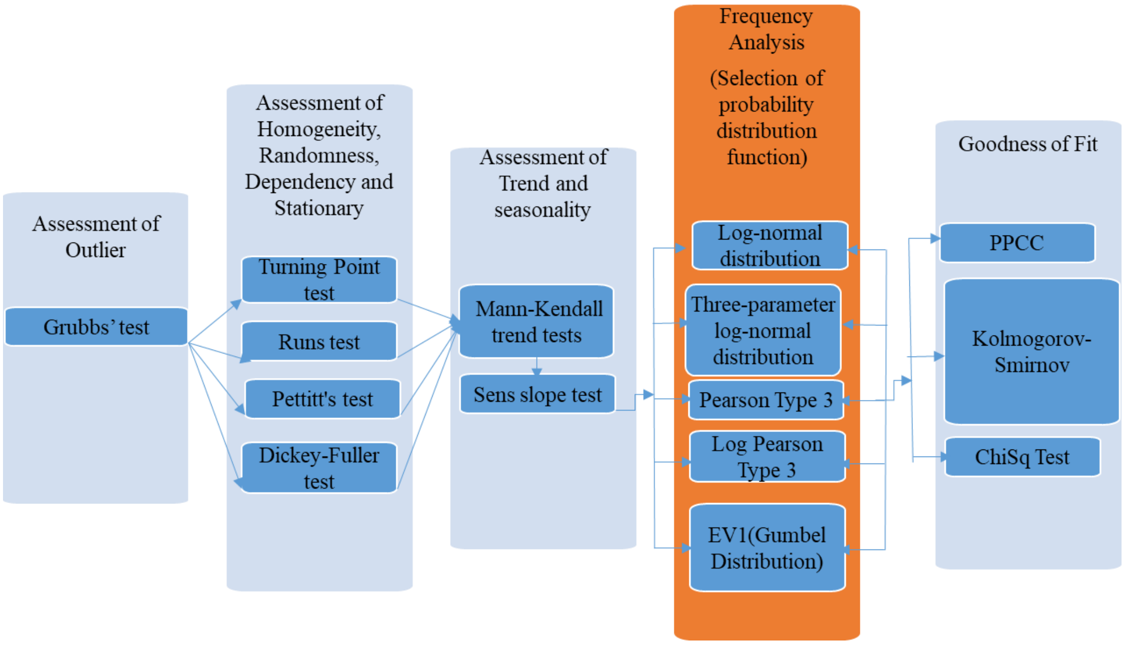

Fig. 4 Chart showing the methodology of frequency analysis

respectively (Dickey & Fuller, 1979; Pettitt, 1979). The existence of trend and seasonality within the data series was assessed by performing "Mann-Kendall" test (Kendall, 1975; Mann, 1945) and Sens's slope test (Sen, 1968).

In this study, widely used five distribution functions – Log-Normal (LN), Log-Normal type III (LN3), Pearson type III (P3), Log Pearson type III (LP3) and Extreme Value Distribution I/Gumbel Distribution (EV1) were tested. The probability distribution function was listed in Table 1. Here, a,b,c are the scale, shape and location parameters.

The goodness of fit of the above-mentioned function was done using probability plot correlations coefficient (PPCC), Kolmogorov-Smirnov and Chi-square test. The ranking was made based on each test and the lowest ranked distribution was assumed to be the best fit distribution of the data. The methodology of Frequency analysis as shown in Fig. 4.

Table 1 Probability Distribution Functions (PDF) of the distributions used in this study.

Distributions	PDF	Range
Log normal (Johnson, Kotz, & Balakrishnan, 1994)	$f(x) = \frac{1}{x}.\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$	x > 0
Log normal type III (Ahrens, 1957)	$f(x) = \frac{1}{(x-a)c\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{-(\ln(x-a)-b)^2}{c^2}\right)\right)$	Where, $0 \leq \gamma < x$, -∞<μ<∞, $\sigma > 0$
Pearson type III (Pearson, 1895)	$f(x) = \frac{1}{a\Gamma(b)}(\frac{x-c}{a})^{b-1} exp\left(-(\frac{x-c}{a})\right)$	If$a > 0, x \geq c$. If $a < 0,\ x \leq c$
Log Pearson type III (Singh, 1998)	$f(x) = \frac{1}{ax\Gamma(b)}(\frac{y-c}{a})^{b-1} exp\left(-(\frac{y-c}{a})\right)$	$a > 0, b > 0$ and $0 < c < y$
Extreme Value Distribution I (Gumbel, 1941)	$f(x) = a \exp(-a(x-b) - e^{-a(x-b)})$	$a > 0$, -∞<b<x

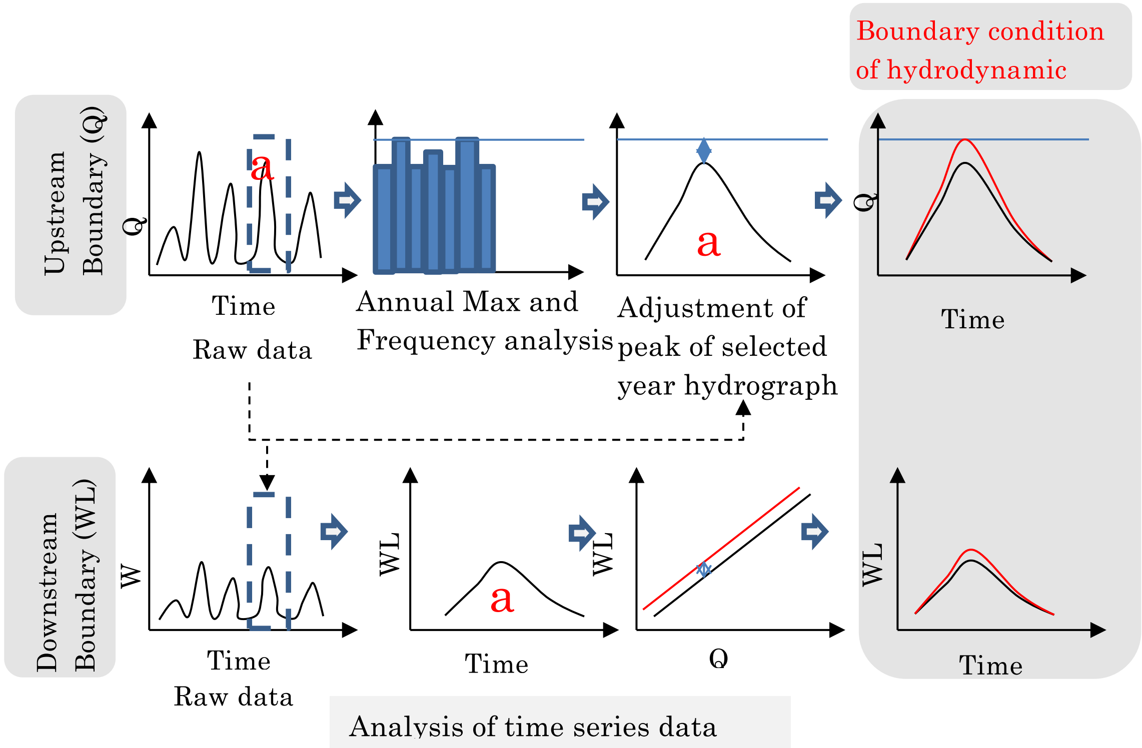

Fig. 5 Methodology of the study

Then the floods of different return period were calculated and the magnitude was matched with the annual maximum data series and the shape of the recent hydrograph which was the best match with the distribution function was chosen as the shape of the hydrograph of that defined return period. The total hydrograph was recalculated with the adjustment of the peak discharge. For downstream boundary, the stage-discharge relation was calculated for that particular year and the deviation of water level due to the adjustment of the peak of the discharge was computed. Then the downstream boundary conditions were defined by the adjustment of the peak discharge. A graphical representation of the methodology was shown in Fig. 5.

3. Results

3.1 Outlier, Randomness, and dependency

In this study, daily river discharge and water level data for the sixty years from 1956 to 2016 were considered as the time series data. The analysis of the time series annual maximum discharge data shows that the maximum discharge varies within this period from 103128.8 m^3/s (Q_{max}) to 40900.0 m^3/s (Q_{min}) having the mean discharge (Q_{mean}) of 68129.25 m^3/s. The standard deviation, σ was found 13249.37 m^3/s. The non-parametric test result for outlier, randomness and dependency are listed in Table 2.

Table 2 Test for outlier, Randomness and dependency

Test	Z_{crit}	Z_{obs}	Comments
Grub's	3.14	2.90	Non-outliered
Run	30.47	26.00	Random
Turning Point	1.96	0.22	Independent
Pettitt	157	55	Homogeneous
Dickey-Fuller	-3.48	-7.281	Stationary

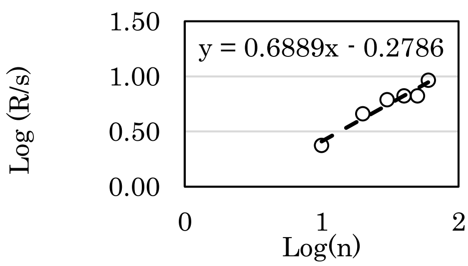

Fig. 6 Hurst exponent test summary

Grubb's test result indicates that the G value was

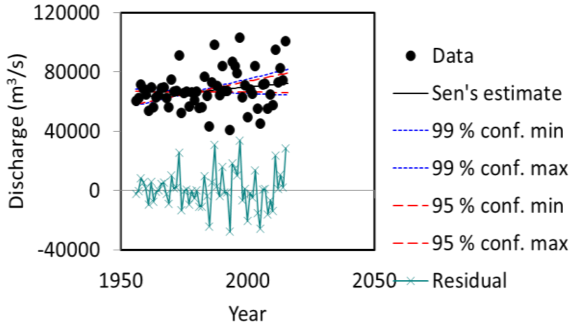

Fig. 7 Trend analysis of the data series

found 2.90 where $G_{critical}$ was 3.14. As the $Z_{observed} < Z_{critical}$ no outlier exists in the data set. The test result for randomness and short-term dependency also showed the similar result. However, to check the

Long-term dependency Hurst test was performed by splitting the data series on a decadal basis. the $Z_{observed}$ in Hurst test was 0.69 and $Z_{critical}$ was 0.50 $Z_{observed} > Z_{critical}$ it indicates slight persistency in the data series As shown in Figure 6. The homogeneity was tested using Pettitt's test where p-value was found 0.407 indicating the rejecting null hypothesis of inhomogeneous.

3.2 Trend and seasonality

The result of the Mann-Kendal test of the time series data shows that Z_{obs} was 1.89 which refers to a significant trend existed in the data set. Then the Sen's slope test was performed as shown in Fig. 7. This figure indicates the presence of a positive trend in the data series. Hence the data series was de-trended using linear regression method using the correlation equation of the trend analysis (187.73x+62403) shown in Fig. 8. De-seasonalised was done by splitting the data into decadal basis.

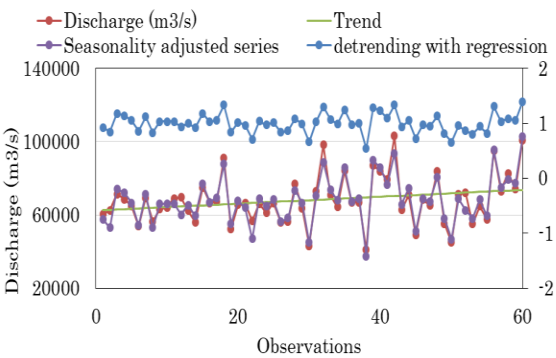

Fig. 8 De-trended of the time series data

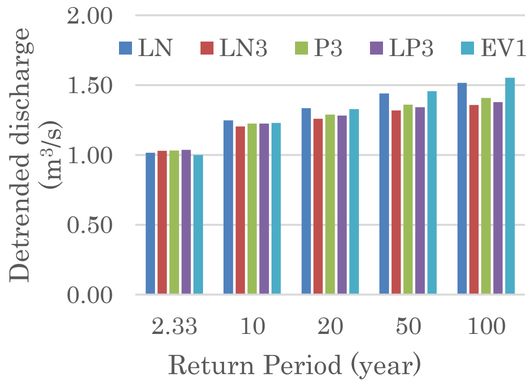

Fig. 9 Frequency distribution of de-trended data

3.3 Frequency distribution

Based on the de-trended, de-deseasonalized time series data frequency distribution analysis was done showing in Fig. 9. This figure indicates that Log-normal type three (LN3) predicts the lowest discharge except for the return period 2.33 where Extreme value distribution (EV 1) predicts the highest one. However, the best-fitting of distribution was assumed from the goodness of fit test the results of which is shown in Table 3. This table indicates that among the five distribution Log-Pearson type III are the most suitable distribution for the flood prediction of different return periods.

3.4 Estimating boundary conditions for the hydrodynamic model

The de-trended discharge data obtained from Log-Pearson type III distribution again re-trended and plotted with the annual maximum discharge shown in Fig. 10. From this figure, five recent year flood was chosen the magnitude of which is nearest

Table 3 Goodness of fit for different distribution

Fitted distribution	Kolmogorov-Smirnov	PPCC	Chai-sq	Total Rank
LN	0.9284	0.9875	7.8301	10.00
LN3	0.9867	0.9856	7.8948	9.000
P3	0.9867	0.9877	7.9075	8.000
LP3	0.9284	0.9845	7.9675	7.000
EV1	0.8133	0.9888	6.8760	11.00

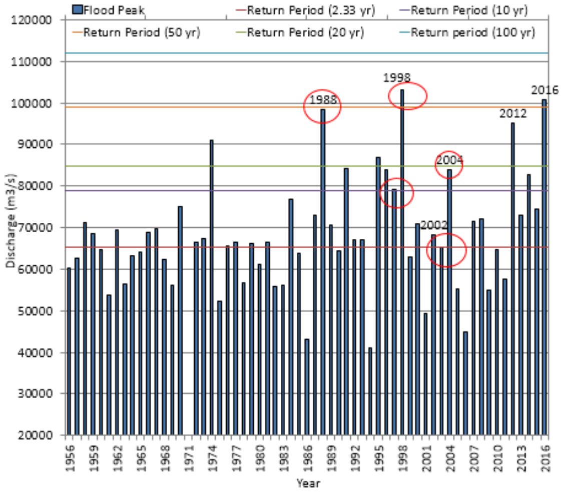

Fig. 10 Selection of hydrograph from annual maximum discharge

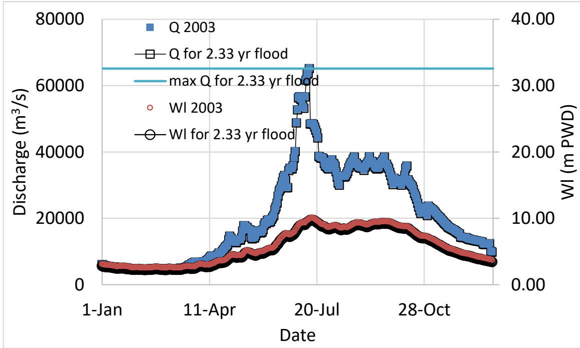

Fig. 12 Adjusted upstream and downstream boundary condition for 2.33yr return period flood.

to the magnitude of different return period flood. For example for 2.33,10,20,50 and 100 year return period 2003, 1997, 2004, 1988 and 1998 year hydrograph were selected. The peak of the hydrographs was then adjusted according to the magnitude of different return period floods shown in

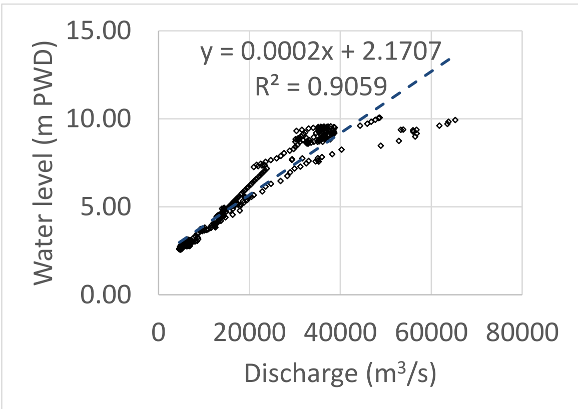

Fig. 11 Stage-Discharge relationship for the year 2003.

Table 3 Adjustment of hydrograph due to different return period floods

Return Period	2.33	10	20	50	100
Adjustment u/s (m^3/s)	-147	-485	872	728	8847
Adjustment d/s (m)	-0.3	-1.3	1.17	1.32	4.25

Table 4.

The stage-discharge relationship was assessed at the same time and from that relationship, the adjustment of water level due to the adjustment of discharge was calculated and also shown in Table 4. An example of such relationship is shown in Fig. 11. The adjustment of upstream and downstream boundary condition for 2.33 year return period is shown in Fig. 12.

4. Conclusions

In this study, a procedure has been illustrated to define the boundary conditions of a hydrodynamic model for a particular event from time series data. As a test case 2.33,10, 20, 50 and 100 year return period flood of Brahmaputra-Jamuna river was chosen. We conclude the followings:

Here the magnitude of different return period flood was chosen by doing frequency analysis. Therefore, prior to do any frequency analysis the dataset should be free from outlier, dependency, and trend. Hence the firstly the data were checked for such conditions using nonparametric Grubb's, Runs and Turning point test and the results indicated the there was no outlier, no dependency and random but the Mann-Kendal test for trend showed significant increasing trend persists within the data series. This trend was removed by linear regression method.

Through The frequency analysis five distribution functions - Lognormal (LN), Log-Normal type III (LN3), Pearson type III (P3), Log-Pearson type III (LP3) and Extreme Value Distribution I/Gumbel Distribution (EV1) were tested to find out the best fit function to predict the flood distribution of the Brahmaputra-Jamuna River. The analysis of goodness of fit showed the Log-Pearson type III distribution was the best-matched function for predicting different magnitude flood. For 2.33, 10, 20, 50 and 100 year return period flood the maximum discharge were found 65140, 78735, 84818, 99028 and 111976 m^3/s.

Visual Comparison of annual maximum flood and the maximum flood obtained from the frequency analysis was made and the shape of the hydrograph for the year 2003, 1997, 2004, 1988 and 1998 year was chosen for representation the flood of 2.33, 10, 20, 50 and 100 year return periods. Nevertheless, 2003, 1997, 2004, 1988 and 1998 hydrographs should be adjusted by -147, -485, 872, 728 and 8847 m^3/s to represent the appropriate conditions of the of 2.33, 10, 20, 50 and 100 year return periods floods. At the same time, the stage-discharge relationship curves of the respected year suggests the downstream boundary, the water level should be adjusted -0.3,-1.3, 1.17, 1.32 and 4.52 m to get appropriate boundary condition for the downstream.

Acknowledgments

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