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“Bank Runs and Asset Price Collapses”

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Abstract

To study the relationship between bank runs and asset prices, we consider a banking model that incorporates a secondary market for long-term assets. Adverse selection arises in this market because banks are better informed about the quality of their assets than other market participants. The model generates multiple equilibria. In one equilibrium, bank runs cannot occur. In another equilibrium, asset prices can be low and bank runs can occur. This can be interpreted as a financial crisis. In this framework, a liquidity requirement for banks might cause bank runs.

JEL Classification: D82, G01, G21

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1 Introduction

A collapse in asset prices can have a detrimental effect on the banking sector, leading to a banking crisis and a financial crisis. A typical example is the 2007–09 financial crisis in the US.[1] This raises several questions: Why do asset prices collapse? How is a decline in asset prices related to a banking crisis?

In this study, we examine how the interaction between banks and financial markets can lead to a collapse in asset prices and bank runs. Specifically, we introduce a secondary market for assets into the standard banking model that was developed by Diamond and Dybvig (1983). In the model, banks can invest in two types of assets: safe short-term assets and risky long-term assets. Further, there are two types of long-term assets: high-quality assets and low-quality assets. Banks can sell long-term assets in the secondary market.

The key feature of the model is that banks are better informed about the quality of their assets than other market participants. That is, banks can observe the quality of their long-term assets, but others cannot. Thus, in the secondary market, buyers cannot distinguish high-quality assets from low-quality assets. This may lead to an adverse selection problem (Akerlof, 1970).

The model might generate multiple equilibria. In one equilibrium, banks expect that the market price of long-term assets will be high and decide to invest all their resources in long-term assets. In this case, they cannot meet the demands of depositors without participating in the secondary market and selling some of their assets. This is the case regardless of whether banks hold high- or low-quality assets. This means that some high-quality assets will also be for sale. If the average quality of assets is high enough, the market price will be higher than the return on short-term assets. This justifies the initial decision. Then, bank runs cannot occur because even low-quality assets can be sold at a high price. Following Malherbe (2014), we call this equilibrium a "high-liquidity equilibrium."

In another equilibrium, banks hold sufficient short-term assets and will not need to participate in the asset market to obtain additional liquidity. This leads buyers to think that high-quality assets will not be for sale, and that the assets traded in the market are only of low quality. As a result, these assets are traded at low prices. Hence, the market is illiquid, and holding sufficient short-term assets is justified. This liquidity hoarding behavior leads to a decline in asset prices, which in turn causes bank runs. Banks with low-quality assets choose to default. This equilibrium is called a "low-liquidity equilibrium," as described in Malherbe (2014). Because asset price collapses and bank runs occur in a low-liquidity equilibrium, this can be interpreted as

[1] See Brunnermeier (2009) for a detailed description of the 2007–09 financial crisis.

a financial crisis.

Moreover, we study the impact of a liquidity requirement on asset prices and bank runs and show that imposing a liquidity requirement on banks might cause bank runs. Under a liquidity requirement, banks have to hold liquid assets, and banks with high-quality assets do not need to participate in the market. This depresses the market price, which might cause bank runs because banks with low-quality assets cannot obtain enough liquidity through the market. We compute the threshold level of a liquidity requirement above which a high-liquidity equilibrium cannot exist. This threshold depends on the return on assets; the lower the return on assets, the lower the threshold. This result implies that to reduce the possibility of a financial crisis, a countercyclical liquidity requirement might be needed.

Several studies, such as Bolton et al. (2011), Kirabaeva (2011), Malherbe (2014), and Heider et al. (2015), explore how asymmetric information leads to fire sales and market freezes. In particular, our model is largely based on that of Malherbe (2014). We extend the framework presented in Malherbe (2014) by incorporating a standard banking model. This extension allows us to study the relationship between bank runs and asset prices.

Several studies have constructed models of banks and asset markets, such as Allen and Gale (2004a), Allen and Gale (2004b), and Allen, Carletti, and Gale (2009). Allen and Gale (2004a) provide a general framework that includes both financial intermediaries and financial markets, and examine the efficiency of the economy. Allen and Gale (2004b) show that small shocks to the demand for liquidity can cause financial crises. Allen, Carletti, and Gale (2009) consider a central bank intervention. All of these papers focus on the cash-in-the-market pricing effect (Allen and Gale, 1994). In the cash-in-the-market pricing model, buyers do not have enough liquidity to clear the market at "fundamental" values. Conversely, we assume the existence of deep-pocket buyers who have enough resources and a perfectly elastic demand for assets. In this sense, we turn off the cash-in-the-market effect and instead study the effect of asymmetric information on asset prices and banks. In this sense, our study complements the above-mentioned literature.

Uhlig (2010) develops a model of a systemic bank run. Unlike his study, liquidity hoarding behavior by the banks is the key to bank runs and the collapse of asset prices in our model.

There have also been studies on liquidity hoarding behavior. Notable examples are Acharya and Skeie (2011), Acharya, Shin, and Yorulmazer (2011), Diamond and Rajan (2011), and Gale and Yorulmazer (2012). Compared with these studies, our primary interest is in the role played by adverse selection in the banking sector and asset markets.

Our study also contributes to the literature on liquidity regulation.[2] For example, Farhi, Golosov, and Tsyvinski (2009) introduce hidden trades into a standard banking model and show that a constrained efficient allocation can be attained by imposing a liquidity requirement. Diamond and Kashyap (2016) examine the effects of regulations similar to the liquidity coverage ratio and the net stable funding ratio. The mechanism through which a liquidity requirement might lead to an unintended consequence is discussed in Malherbe (2014). Our contribution is to show that this mechanism could also cause bank runs.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we apply the model, show that there might be multiple equilibria, and discuss the relationship between bank runs and asset prices. Section 4 analyzes the effects of liquidity requirements on asset prices and bank runs. Section 5 concludes.

2 The model

Time is divided into three periods ($t = 0, 1, 2$). Moreover, period 1 is divided into two subperiods. There is a measure one of banks. Each bank raises funds from consumers in exchange for a deposit contract.

Consumers are endowed with one unit of the good in period 0 and nothing thereafter, and are uncertain about their time preferences. With probability λ, a consumer is an early consumer who only values consumption in period 1. With probability $1-\lambda$, the consumer is a late consumer who only values consumption in period 2. The consumer's utility function is given by

$$U(\tilde{c}_1, \tilde{c}_2) = \begin{cases} \ln(\tilde{c}_1) & \text{w.pr. } \lambda, \\ \ln(\tilde{c}_2) & \text{w.pr. } 1-\lambda, \end{cases}$$

where $\tilde{c}_t$ denotes consumption in period $t = 1, 2$. At the beginning of period 1, each consumer learns whether he is an early or late consumer. We assume that banks can do anything that consumers can do. Therefore, there is no loss of generality in assuming that consumers deposit their entire endowments in a bank in period 0.

In this economy, there are two types of assets: short-term assets and long-term assets. A short-term asset can be thought of as a storage technology or reserve, which transforms one unit of the good in period t into one unit of the good in period $t+1$, where $t = 0, 1$. A long-term asset represents a project that takes one unit of the good in period 0 and yields an uncertain payoff in period 2. This yields R_H units of the good with probability π and R_L units of the good with probability $1-\pi$ in period 2.

[2] Allen and Gale (2017) survey the literature on liquidity regulations.

We assume that $0 \leq R_L < R_H$ and $R_L < 1 < \pi R_H + (1-\pi)R_L$. This means that on average, long-term assets yield higher returns than short-term assets, but they yield less than short-term assets in the case of failure.

At the beginning of period 1, owners of long-term assets privately observe their long-term assets' quality, that is, whether the payoffs of their assets will be R_H or R_L. We assume that quality is common across all the projects of a given owner. However, quality is independent across banks, and thus by the law of large numbers, average quality is deterministic.

We assume that the long-term assets cannot be physically liquidated in period 1. However, banks can issue claims in relation to the payoffs of their projects in a competitive asset market, which we describe later.

2.1 Secondary Market and Buyers

The asset market opens in the first subperiod of period 1. Banks can issue perfectly divisible shares of their long-term assets in this market. We assume that short sales are prohibited.

There is a measure one of risk-neutral "deep-pocket" buyers who do not have access to long-term assets but can hold short-term assets and participate in the market. They have enough resources available at period 1 to clear the market at the expected value of the underlying payoffs.

Because asset quality is private information, the expected value of an asset depends on the average quality of traded assets. As the deep-pocket buyers have access to short-term assets, the buyers' no-arbitrage condition implies that the market price p is equal to the expected value of an asset. Thus, the market price p is given by

$$p(q) = R_L + q(R_H - R_L), \tag{1}$$

where q denotes the proportion of high-quality assets in the market.

Equation (1) implies that p increases with q. That is, the more long-term assets that are sold, the higher the price. Because the market is a source of liquidity, adverse selection undermines liquidity provision.

2.2 Timing

The timing of events is as follows. In period 0, agents deposit all their endowments in banks. Banks choose the deposit contract $((c_1, \{c_{2i}\}_{i\in\{L,H\}}))$ and the amount of long-term assets (y).[3] In the first subperiod of period 1, consumers privately observe their

[3] Let the index $i \in \{L, H\}$ denote the realization of R_i for the bank.

types. Banks privately observe their long-term assets' quality and choose how much of their long-term assets to sell ($\{x_i\}_{i\in\{L,H\}}$). Then, the market closes. In the second subperiod of period 2, early consumers withdraw. Late consumers observe the market price and how much of the long-term assets a bank sold, that is, they observe a pair (p, x_i). Given this, they choose whether or not to withdraw in period 1. Consumers who did not withdraw in period 1 withdraw in period 2.

3 Equilibria

In what follows, we assume that bank runs as a result of a coordination failure among depositors cannot occur. That is, bank runs cannot occur as long as the incentive compatibility condition is satisfied.[4]

3.1 The Problem of the Bank

Banks choose how much of their resources to invest in long-term assets and the deposit contract to maximize the expected utility of a depositor in period 0.

In period 1, banks choose how much of their long-term assets to sell. Note that y and c_1 are predetermined in period 1 and p is taken as given.

If a bank run occurs, the banks have to sell all their assets. Consider the case where late consumers do not participate in a run. First, we consider banks with low-quality assets. We call these banks L. It is obvious that they sell all their assets if $p > R_L$. As the long-term assets cannot be liquidated in period 1 other than by sale in the market, there is no loss of generality in assuming that the banks also sell all their assets when $p = R_L$. This means that it is always necessary for L banks to sell all their assets because under (1), $p \in [R_L, R_H]$. Formally, this can be written as

$$x_L = y.$$

Next, consider the banks with high-quality assets. We call these banks H. Note that equation (1) implies that $p < R_H$. Therefore, they sell their long-term assets only when they cannot meet the demands of early consumers with short-term assets, that is,

$$x_H = \begin{cases} 0 & \text{if } \lambda c_1 < 1 - y, \\ \frac{\lambda c_1 - (1-y)}{p} & \text{if } \lambda c_1 \geq 1 - y. \end{cases}$$

In period 0, banks choose how much of their resources to invest in long-term assets.

[4]The incentive compatibility condition is given by equation (5).

Banks maximize the expected utility of a depositor. The problem is written as

$$\max_{y,c_1,\{c_{2i}\}_{i\in\{L,H\}}} (1-\pi)\left[\lambda \ln(c_1) + (1-\lambda)\ln(c_{2L})\right] + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln(c_{2H})\right], \quad (2)$$

subject to

$$\begin{aligned} \lambda c_1 &\leq 1 - y + p x_i, & (3)\\ (1-\lambda)c_{2i} &= 1 - y + p x_i - \lambda c_1 + (y - x_i)R_i, & (4)\\ 0 \leq & y \leq 1. \end{aligned}$$

Equations (3) and (4) are resource constraints. In period 1, a fraction λ of consumers withdraw. Banks have to hold enough liquidity to meet this demand. In period 2, banks pay all they have to depositors.

If banks want to avoid runs, additional constraints are needed.

$$c_1 \leq c_{2i}, \quad (5)$$

for $i \in \{L, H\}$. If $c_1 > c_{2i}$, late consumers who deposited their endowments in period 0 in bank i receive more by withdrawing in period 1 than in period 2, and a bank run occurs. In this case, both early and late consumers receive $1 - y + py$.

It is easy to show that $x_H < y = x_L$. Therefore, late consumers correctly infer whether or not their bank has high-quality assets by observing the market price and how much of their long-term assets the banks sell.

In the following proposition, we summarize the investment decision and the optimal deposit contract given the market price p.

Proposition 1. *The optimal investment decision is written as*

$$y(p) \in \begin{cases} \left\{\frac{\pi R_H + (1-\pi)R_L - 1}{(R_H-1)(1-p)}\right\} & \text{if } p < 1 \text{ and } \frac{\pi R_H + (1-\pi)R_L - 1}{(R_H-1)(1-p)} \leq \frac{1-\lambda}{1-\lambda+\lambda R_H}, \\ (0, 1-\lambda) & \text{if } p < 1 \text{ and } \frac{\pi R_H + (1-\pi)R_L - 1}{(R_H-1)(1-p)} > \frac{1-\lambda}{1-\lambda+\lambda R_H}, \\ [1-\lambda, 1] & \text{if } p = 1, \\ \{1\} & \text{if } p > 1. \end{cases}$$

The optimal deposit contract is given by

$$c_1(p) = \begin{cases} 1 - y(p) + y(p)R_H & \text{if } p < 1 \text{ and } \frac{\pi R_H + (1-\pi)R_L - 1}{(R_H-1)(1-p)} \leq \frac{1-\lambda}{1-\lambda+\lambda R_H}, \\ \frac{1-y(p)}{\lambda} & \text{if } p < 1 \text{ and } \frac{\pi R_H + (1-\pi)R_L - 1}{(R_H-1)(1-p)} > \frac{1-\lambda}{1-\lambda+\lambda R_H}, \\ 1 & \text{if } p = 1, \\ p & \text{if } p > 1. \end{cases}$$

Moreover, bank runs can only occur if $p < 1$.

Proof: See Appendix A.

If banks expect the market to be liquid, that is, $p > 1$, then they invest only in long-term assets, $y = 1$.

Conversely, if banks anticipate that $p < 1$, they will want to hold more liquid assets, that is, short-term assets. In this case, the incentive constraint for L banks is not satisfied and both early and late consumers withdraw in period 1 and receive payments $1 - y + py$. We consider the situation whereby if bank runs do not occur, the consumption in period 1 is the same regardless of whether the banks hold high- or low-quality assets. This raises the possibility that default might be optimal, because default allows for a greater degree of contingency in consumption (Allen and Gale, 1998).

3.2 Asset Sales

Given the investment decision and the optimal allocation given by Proposition 1, we can obtain the asset sale functions $x_L(p)$ and $x_H(p)$.

Lemma 1. *The optimal asset sale is given by*

$$x_L(p) = y(p),$$

$$x_H(p) \in \begin{cases} \{0\} & \text{if } p < 1, \\ [0, \lambda] & \text{if } p = 1, \\ \{\lambda\} & \text{if } p > 1. \end{cases}$$

3.3 Average Quality

Next, we consider the average quality of assets traded in the market. Let $q(p)$ represent the proportion of good assets for a given p. This can be written as

$$q(p) \equiv \frac{\pi x_H(p)}{(1-\pi)x_L(p) + \pi x_H(p)}.$$

This can be computed as follows.

Lemma 2. *The proportion of high-quality assets is given by*

$$q(p) \in \begin{cases} \{0\} & \text{if } p < 1, \\ \left[0, \frac{\pi\lambda}{1-\pi+\pi\lambda}\right] & \text{if } p = 1, \\ \left\{\frac{\pi\lambda}{1-\pi+\pi\lambda}\right\} & \text{if } p > 1. \end{cases} \tag{6}$$

3.4 Equilibria

From equations (1) and (6), we define the implied price correspondence:

$$p'(p) \equiv R_L + q(p)[R_H - R_L],$$

where $p'(p)$ is the market price corresponding to a proportion of high-quality assets $q(p)$. Then, $p'(p) = p$ determines an equilibrium for this economy. The corresponding values of y and q are derived from Proposition 1 and Lemma 2, respectively.

In the following, we characterize the equilibria. In the first equilibrium, which we call a high-liquidity equilibrium, banks expect the market to be liquid, that is, $p > 1$. From Proposition 1, we have $y = 1$, $c_1 = p$, $c_{2L} = p$, and $c_{2H} = R_H$. In this case, bank runs cannot occur because the incentive constraint (5) is satisfied. This equilibrium arises when $1 < p = R_L + q(p)(R_H - R_L)$. This implies that

$$R_H > \frac{1 - \pi(1-\lambda)}{\pi\lambda} - \frac{1-\pi}{\pi\lambda} R_L. \tag{7}$$

Thus, if R_H is sufficiently high, banks may invest all their resources in long-term assets. Even if H banks have to sell some of their assets to meet the demands of early withdrawers, the price can be high, which justifies the initial decision, $y = 1$. In addition, L banks can sell their assets at the high price, and so bank runs cannot occur.

In the second equilibrium, which we call a low-liquidity equilibrium, banks anticipate an illiquid market, that is, $p < 1$. Then, they initially choose to hold enough short-term assets. This means that H banks need not participate in the market and sell their assets. Hence, only low-quality assets are traded in the market, that is, $x_H = 0$, and thus $p = R_L < 1$. In this case, L banks choose to default, as seen in Proposition 1. This can be interpreted as a financial crisis. It is easy to see that a low-liquidity equilibrium always exists. If equation (7) is satisfied, there are multiple equilibria.[5]

There are also equilibria corresponding to $p = 1$, where $y \in [1-\lambda, 1]$, $c_1 = 1$, $c_{2L} = 1$, and $c_{2H} = R_H$. Moreover, neither type of bank defaults.[6]

We summarize these results in Proposition 2.

Proposition 2. *A low-liquidity equilibrium always exists. In this equilibrium, banks with low-quality assets choose to default. If $R_H > \frac{1-\pi(1-\lambda)}{\pi\lambda} - \frac{1-\pi}{\pi\lambda} R_L$, there are multiple equilibria. In a high-liquidity equilibrium, bank runs cannot occur.*

Proposition 2 states that even if the returns on assets are sufficiently high, a self-

[5] If equation (7) is satisfied, then $\frac{\pi R_H + (1-\pi)R_L - 1}{(R_H - 1)(1-p)} > \frac{1-\lambda}{1-\lambda+\lambda R_H}$. Therefore, if a high-liquidity equilibrium exists, in a low-liquidity equilibrium, $c_1 = \frac{1-y}{\lambda}$ from Proposition 1.

[6] These equilibria corresponding to $p = 1$ are unstable.

fulfilling financial crisis can occur as a result of a coordination failure among banks, not depositors. Another implication is that under the condition of low returns on assets, which can be thought as an economic downturn, only a low-liquidity equilibrium exists, so bank runs occur. This is consistent with the "business cycle" view of bank runs advocated by Gorton (1988).

4 Liquidity Requirements

To investigate the effects of liquidity requirements on asset prices and bank runs, we conduct a simple exercise. Consider the following liquidity requirement in relation to the initial investment decision of banks:

$$1 - y \geq \kappa,$$

where $0 \leq \kappa < 1$. This implies that banks are forced to invest a fraction κ of their initial resources in short-term assets.

Proposition 3. *Assume that* $R_H > \frac{1-\pi(1-\lambda)}{\pi\lambda} - \frac{1-\pi}{\pi\lambda} R_L$. *A liquidity requirement strictly reduces the expected utility of a consumer in period* 0 *in a high-liquidity equilibrium. Moreover, there exists a* $\bar{\kappa}$*, above which a high-liquidity equilibrium cannot exist.* $\bar{\kappa}$ *is given by*

$$\bar{\kappa} \equiv \frac{\pi\lambda(R_H - 1) - (1-\pi)(1-R_L)}{\pi(R_H - 1) - (1-\pi)(1-R_L)}.$$

Proof: See Appendix A.

First, note that in a high-liquidity equilibrium, the liquidity requirement is binding. Obviously, a liquidity requirement reduces the expected utility of a consumer in period 0, not only because banks are forced to invest in dominated assets (i.e., short-term assets), but also because the average quality of assets traded in the market is lower, and thus the market price is also lower.

Second, because an increase in κ decreases the asset sales of H banks and depresses the market price, there exists a $\bar{\kappa}$ in which a market price greater than one is unsustainable. In other words, if κ is sufficiently high, a high-liquidity equilibrium might not exist and a low-liquidity equilibrium will be the unique equilibrium, wherein L banks default.[7]

Finally, $\bar{\kappa}$ is increasing in R_H and R_L. This implies that when economic fundamentals are bad and returns on assets are low, a looser requirement should be imposed to guarantee the existence of a high-liquidity equilibrium. That is, a countercyclical liquidity requirement might be needed.

[7] If $\kappa = 1$, bank runs cannot occur.

5 Conclusion

In this study, we examine the relationship between bank runs and asset prices. To this end, we consider a banking model that incorporates a secondary market for long-term assets. This secondary market might be illiquid as a result of adverse selection.

We find that the model may generate multiple equilibria. When banks invest all their resources in long-term assets and do not hold liquid assets, even banks with high-quality assets have to sell some of their long-term assets in the secondary market to meet the demands of early withdrawers. Then, the market price can be high. In this case, bank runs cannot occur. Conversely, if banks hold enough liquid assets, banks with high-quality assets do not participate in the market and assets can only trade at low prices, which causes bank runs.

In addition, we examine the impact of a liquidity requirement on asset prices and bank runs. Imposing a liquidity requirement on banks reduces the need for the participation of banks with high-quality assets in the market, which might result in a collapse in asset prices and bank runs.

Our model cannot explain why a liquidity requirement should be imposed. Introducing the mechanism highlighted by this study into the model where a liquidity requirement has a positive effect on the economy would be a useful extension. Moreover, by using this model, analyzing the effects of other policy tools, such as deposit insurance, on banks and asset markets would be an interesting research topic.

Appendix A Proofs

Proof of Proposition 1

In the following, we solve the banks' problem without the incentive constraints, and then check whether or not the incentive constraints are satisfied.

i) Consider the case where $p > 1$. In this case, the long-term asset dominates the short-term asset. Therefore, banks invest all their resources in long-term assets. That is, $y = 1$. Then, the banks' problem (2) can be rewritten as:

$$\max_{c_1} (1-\pi)\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\frac{p-\lambda c_1}{1-\lambda}\right)\right] + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\frac{1-\frac{\lambda c_1}{p}}{1-\lambda}R_H\right)\right].$$

Note that $x_L = y = 1$ and $x_H = \frac{\lambda c_1}{p}$ because $\lambda c_1 \geq 0 = 1 - y$. It is easy to show that $c_1 = p$. Therefore, $c_{2L} = p$ and $c_{2H} = R_H \geq p$. This implies that the incentive constraint (5) is satisfied for both L and H banks, and thus bank runs cannot occur.

ii) Consider the case where $p = 1$. In this case, $c_{2L} = \frac{1-\lambda c_1}{1-\lambda}$. Suppose that $\lambda c_1 < 1 - y$. Then, $x_H = 0$ and $c_{2H} = 1 - y - \lambda c_1 + yR_H$. This implies that the objective function is increasing in y and banks choose to invest all their resources in long-term assets, that is, $y = 1$. However, this means that $\lambda c_1 < 1 - y = 0$, and thus we obtain a contradiction. Hence, $\lambda c_1 \geq 1 - y$ and $\lambda c_1 = 1 - y + px_H$. Then, $c_{2L} = \frac{1-\lambda c_1}{1-\lambda}$ and $c_{2H} = \frac{(y-x_H)R_H}{1-\lambda}$. The problem is as follows:

$$\max_{c_1,y}\ (1-\pi)\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\frac{1-\lambda c_1}{1-\lambda}\right)\right] + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\frac{y-x_H}{1-\lambda}R_H\right)\right],$$

subject to

$$\begin{aligned} \lambda c_1 &= 1 - y + x_H, \\ 0 \leq & y \leq 1. \end{aligned}$$

Therefore, we have $c_1 = 1$ and $y - x_H = 1 - \lambda$. Then, $c_{2L} = 1$ and $c_{2H} = R_H$. This means that the incentive constraints are satisfied for both types of banks, and thus the banks do not default.

iii) Consider the case where $p < 1$. In this case, the incentive constraint for L banks is either binding or violated. If the condition is binding, the banks' maximization problem is

$$\max_{y,c_1,c_{2H}} (1-\pi)\ln(1-y+py) + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln(c_{2H})\right],$$

subject to

$$\begin{aligned} c_1 &= 1 - y + py, \\ \lambda c_1 &\leq 1 - y + px_H, \\ (1-\lambda)c_{2H} &= 1 - y + px_H - \lambda c_1 + (y - x_H)R_H, \\ 0 \leq & y \leq 1. \end{aligned}$$

If the incentive constraint is violated and banks choose to default, their problem is as follows:

$$\max_{y,c_1,c_{2H}} (1-\pi)\ln(1-y+py) + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln(c_{2H})\right], \tag{8}$$

subject to

$$\begin{aligned} \lambda c_1 &\leq 1 - y + px_H, \\ (1-\lambda)c_{2H} &= 1 - y + px_H - \lambda c_1 + (y - x_H)R_H, \\ 0 \leq & y \leq 1. \end{aligned}$$

Since the additional constraint $c_1 = 1 - y + py$ is imposed when banks want to avoid bank runs and the incentive constraint is binding, it is clear that the expected utility is higher when banks choose to default. That is, when $p < 1$, L banks go bankrupt.

Suppose that $x_H > 0$. Then, $x_H = \frac{\lambda c_1 - (1-y)}{p}$ and $c_{2H} = \left[y - \frac{\lambda c_1 - (1-y)}{p}\right] \frac{R_H}{1-\lambda}$. Substituting this into equation (8), the objective function is as follows:

$$\max_{y,c_1} \ (1-\pi)\ln(1-y+py) + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\left[y - \frac{\lambda c_1 - (1-y)}{p}\right]\frac{R_H}{1-\lambda}\right)\right].$$

Since this is decreasing in y, banks do not invest in long-term assets at all, that is, $y = 0$. However, this means that $\lambda c_1 \geq 1$ and $c_{2H} \leq 0$, and we obtain a contradiction. Therefore, $x_H = 0$ and $c_{2H} = \frac{1-y-\lambda c_1 + yR_H}{1-\lambda}$.

The problem is as follows:

$$\max_{y,c_1} \ (1-\pi)\ln(1-y+py) + \pi\left[\lambda \ln(c_1) + (1-\lambda)\ln\left(\frac{1-y-\lambda c_1 + yR_H}{1-\lambda}\right)\right],$$

subject to

$$\lambda c_1 \leq 1-y, \tag{9}$$
$$0 \leq y \leq 1. \tag{10}$$

Ignoring constraint (9), the first-order condition with respect to c_1 is as follows:

$$\frac{\lambda}{c_1} - \lambda(1-\lambda)\frac{1}{1-y-\lambda c_1 + yR_H} = 0.$$

Rearranging this, we have

$$c_1 = 1 - y + yR_H. \tag{11}$$

Ignoring constraints (9) and (10), the first-order condition with respect to y is as follows:

$$-\frac{(1-\pi)(1-p)}{1-y+py} + \frac{\pi(1-\lambda)(R_H-1)}{1-y-\lambda c_1 + yR_H} = 0.$$

Rearranging this and substituting (11) into it, we obtain

$$y = \frac{\pi R_H + (1-\pi)p - 1}{(R_H-1)(1-p)}.$$

Since $\pi R_H + (1-\pi)R_L - 1 > 0$ and $p \geq R_L$, $y > 0$. If

$$y = \frac{\pi R_H + (1-\pi)p - 1}{(R_H-1)(1-p)} \leq \frac{1-\lambda}{1-\lambda+\lambda R_H},$$

constraints (9) and (10) are satisfied. Moreover, we have $c_{2H} = 1 - y + yR_H = c_1$, and thus the incentive constraint for H banks is satisfied.

If

$$\frac{\pi R_H + (1-\pi)p - 1}{(R_H - 1)(1-p)} > \frac{1-\lambda}{1-\lambda+\lambda R_H}, \tag{12}$$

constraint (9) is binding. In this case, the problem is as follows:

$$\max_y \ (1-\pi)\ln(1-y+py) + \pi\left[\lambda \ln\left(\frac{1-y}{\lambda}\right) + (1-\lambda)\ln\left(\frac{yR_H}{1-\lambda}\right)\right],$$

subject to

$$0 \leq y \leq 1.$$

Ignoring the constraint, the first-order condition is as follows:

$$-\frac{(1-\pi)(1-p)}{1-y+py} - \frac{\pi\lambda}{1-y} + \frac{\pi(1-\lambda)}{y} = 0.$$

Define the function $F : [0,1] \to \mathbb{R}$

$$F(y) \equiv -\frac{(1-\pi)(1-p)}{1-y+py} - \frac{\pi\lambda}{1-y} + \frac{\pi(1-\lambda)}{y}.$$

It is easy to check that $F(y)$ is strictly decreasing in y, $F(0) = +\infty$, and $F(1) = -\infty$. Moreover, $F(1-\lambda) = -\frac{(1-\pi)+(1-p)}{\lambda+p(1-\lambda)} < 0$. By continuity, there exists a $y \in (0, 1-\lambda)$ such that $F(y) = 0$. That is, constraint (10) is satisfied. This implies that $c_1 = \frac{1-y}{\lambda} > 1$. In addition, since $F\left(\frac{1-\lambda}{1-\lambda+\lambda R_H}\right) > 0$ under condition (12), $y > \frac{1-\lambda}{1-\lambda+\lambda R_H}$ and $c_1 = \frac{1-y}{\lambda} < \frac{yR_H}{1-\lambda} = c_{2H}$, which means that the incentive constraint for H banks is satisfied. ■

Proof of Proposition 3

First, it is obvious that the liquidity requirement reduces the expected utility in a high-liquidity equilibrium because the liquidity requirement is binding for $p > 1$.

Next, we want to show that there exists a $\bar{\kappa}$ such that under $\kappa > \bar{\kappa}$, a high-liquidity equilibrium cannot exist. Let $x_H(p,\kappa)$ and $q(p,\kappa)$ denote optimal asset sales of H banks and the average quality of a given pair (p,κ), respectively. Define the following function for $p \geq 1$ and $0 \leq \kappa < 1$:

$$G(p,\kappa) \equiv R_L + q(p,\kappa)(R_H - R_L) - p.$$

For the existence of a high-liquidity equilibrium, $G(p,\kappa) = 0$ needs to have a solution.

In a high-liquidity equilibrium, the liquidity requirement is binding, that is, $y = 1-\kappa$. Moreover, under $p > 1$, $x_H > 0$ and $x_H = \frac{\lambda c_1 - (1-y)}{p} = \frac{\lambda c_1 - \kappa}{p}$. Then, the banks' problem is

$$\max_{c_1}\ (1-\pi)\left[\lambda\ln(c_1)+(1-\lambda)\ln\left(\frac{\kappa+p(1-\kappa)-\lambda c_1}{1-\lambda}\right)\right]$$
$$+\pi\left[\lambda\ln(c_1)+(1-\lambda)\ln\left(\frac{1-\kappa-\frac{\lambda c_1-\kappa}{p}}{1-\lambda}R_H\right)\right].$$

Solving this problem, we obtain

$$c_1=p(1-\kappa)+\kappa.$$

Therefore, for $p>1$,

$$x_H(p,\kappa)=\frac{\lambda c_1-\kappa}{p}=\frac{\lambda p(1-\kappa)-(1-\lambda)\kappa}{p}, \tag{13}$$

which is increasing in p and decreasing in κ. Using this, we can compute the average quality

$$q(p,\kappa)=\frac{\pi x_H}{(1-\pi)x_L+\pi x_H}=\frac{\pi\frac{\lambda p(1-\kappa)-(1-\lambda)\kappa}{p}}{(1-\pi)(1-\kappa)+\pi\frac{\lambda p(1-\kappa)-(1-\lambda)\kappa}{p}}.$$

It is easy to check that q increases strictly with p and decreases strictly with κ.

Then, we have

$$G(p,\kappa)=R_L+\frac{\pi\frac{\lambda p(1-\kappa)-(1-\lambda)\kappa}{p}}{(1-\pi)(1-\kappa)+\pi\frac{\lambda p(1-\kappa)-(1-\lambda)\kappa}{p}}(R_H-R_L)-p,$$

which is increasing strictly with p and decreasing strictly with κ.

Define $\bar{p}\equiv R_L+\frac{\pi\lambda}{(1-\pi)(1-\lambda)+\pi\lambda}(R_H-R_L)$. Then, $G(\bar{p},0)=0$. This means that for $\kappa>0$, $G(\bar{p},\kappa)<0$. Since $G(p,\kappa)$ is continuous and strictly decreasing in p, for the nonexistence of a high-liquidity equilibrium it suffices to show that $G(1,\kappa)<0$. That is,

$$R_L+\frac{\pi(\lambda-\kappa)}{(1-\pi)(1-\kappa)+\pi(\lambda-\kappa)}-1<0.$$

Rearranging this, we have

$$\kappa>\frac{\pi\lambda(R_H-1)-(1-\pi)(1-R_L)}{\pi(R_H-1)-(1-\pi)(1-R_L)}\equiv\bar{\kappa}.$$

Therefore, under $\bar{\kappa}<\kappa<1$, a high-liquidity equilibrium cannot exist. ■

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