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Intertemporal efficiency does not imply a common price forecast: a leading example

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Abstract

We define an efficient temporary equilibrium (ETE) within the framework of a two period economy. We show by example that ETE in this setting can lead to intertemporally efficient allocations without the agents forecasts being coordinated on a perfect foresight price. There is a one dimensional set of such efficient allocations for generic endowments.

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1 Introduction

Imagine a sequence of commodity markets with no uncertainty, where there is a riskless bond market in each period. The classical temporary equilibrium analysis (Grandmont, 1977) asks if there are market clearing prices in a particular period in question for arbitrarily given anticipated prices for the markets in the following periods. Since market clearing for subsequent periods is not required, the analysis hardly explains how a sequence of market prices are determined over time, not to mention that it is completely mute on welfare analysis for intertemporal allocations of goods. On the other hand, a

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perfect foresight (more generally, a rational expectations) equilibrium (Radner, 1982) predicts a sequence of market prices in a determinate way, and that the resulting allocation is Pareto efficient. But this approach incurs a serious cost in that perfect foresight is assumed, rather than derived. That the assumption of perfect foresight is extraordinarily strong is a view expressed by various scholars; a case in point is Radner's own critique of perfect foresight.[1]

So the following question seems very natural: First, require that all the spot markets clear in the temporary equilibrium analysis. That is, even when the households engaging in trade anticipated wrong prices in the past, describe how they consume and save in every period so that one can address the welfare issue. Secondly, suppose that the markets are so elaborated that the resulting sequence of consumption constitutes an efficient allocation, not only within each period but also intertemporally. The question is, does it imply a perfect foresight equilibrium?

At first sight, the answer might appear positive, because of the second fundamental theorem of welfare economics. As is known, since the markets are complete with the bond market, a perfect foresight equilibrium can be reduced to an Arrow-Debreu equilibrium, and vice versa. If the intertemporal consumption allocation is required to be efficient, there should be the supporting prices in the sense of Arrow-Debreu economy, which should determine the candidates for prices. Since trade is voluntary, the initial endowments should be available under these prices, and hence we have a Arrow-Debreu equilibrium, and hence a perfect foresight equilibrium.

In this note, we present an extremely simple two-period model with two households, which can be reduced to an Edgeworth box, which shows that the conjecture above is incorrect. We consider a standard competitive two-period exchange economy with inside money. There is one perishable consumption good in each period to be traded. In the first

[1] On page 942, Radner (1982) writes "Although it is capable of describing a richer set of institutions and behaviour than is the Arrow-Debreu model, the perfect foresight approach is contrary to the spirit of much of competitive market theory in that it postulates that individual traders must be able to forecast, in some sense, the equilibrium prices that will prevail in the future under all alternative states of the environment. Even if one grants the extenuating circumstances mentioned in previous paragraphs, this approach still seems to require of the traders a capacity for imagination and computation far beyond what is realistic."

period, a bond which pays off one unit in units of account (dollar) in the second period is traded. We restrict attention to agents forming point forecasts as this allows for a more transparent comparison of our approach to the perfect foresight approach and makes our argument simpler.[2] We define an efficient temporary equilibrium (ETE), which in addition to requiring market clearing in each of the two periods, guarantees efficiency. An ETE is a particular variant of a perfectly contracted equilibrium (Chatterji and Ghosal, 2013).[3] Even in this simple environment, we demonstrate the existence of a continuum of ETE for generic endowments.

2 The Model

We consider a standard competitive two-period exchange economy with inside money. There is one perishable consumption good in each period to be traded. There are 2 households, labelled by $h = 1, 2$. Household h is endowed with e_h^0 units of good in the first period (period 0) and e_h^1 units in the second period (period 1). We write $e_h = \left(e_h^0, e_h^1\right)$. To keep the specification symmetric for ease of computation, assume that $\left(e_1^0, e_1^1\right) = (1 - \varepsilon, \varepsilon)$ and $\left(e_2^0, e_2^1\right) = (\varepsilon, 1 - \varepsilon)$ for some $\varepsilon \in [0, 1]$. Households have identical preferences for consumption bundles represented by $u_h\left(x^0, x^1\right) = \ln x^0 + \ln x^1$.

In the first period, period 0, a bond which pays off one unit in units of account (dollar) in the second period is traded. The net supply of the asset is zero, so it is inside money whose real return is determined in the markets.

Normalize the price of the good in period 0 to one. Let r be the market nominal interest rate prevailing in period 0: that is, if a household saves z units in period 0, $(1 + r)\, z$ dollars is delivered at the beginning of period 1. Thus, a negative saving corresponds to borrowing. Writing z_h for the amount of saving of household h, the consumption of household h in period 0 is therefore $x_h^0 = e_h^0 - z_h$.

There is no futures market which might help predict the price of the good in the second period. Thus we assume that each household h first anticipates the price $\hat{p}_h$ of the good in period 1 in order to decide consumption and saving/borrowing in period

[2]Since there is no private information in this set up, probabilistic forecasts appear to be unnecessary.

[3]A perfectly contracted equilibrium is defined using reduced form intertemporal contracts. The present paper considers an explicit decentralized scheme, the bond market, for organizing these transfers.

0. Specifically, at the prevailing market interest rate r, household h expects the budget $\hat{p}_h \left(x_h^1 - e_h^1\right) \leq (1+r)\, z_h$ if his saving is z_h. There is no limit for saving/borrowing, so by eliminating z_h household h faces in effect the following budget constraint for consumption goods:

$$\left(x_h^0 - e_h^0\right) + \frac{\hat{p}_h}{(1+r)} \left(x_h^1 - e_h^1\right) \leq 0. \tag{1}$$

It is readily seen that if $\left(x^0, x^1\right)$ satisfies (1), then there is z with which the budget is met in both periods. Note that the monotonicity of u_h implies that the equality will hold at the optimum.

We denote the market price of the good in period 1 by p. That is, household h is subject to the constraint $p\left(x_h^1 - e_h^1\right) \leq (1+r)\, z_h$, i.e., the market value of the net consumption must be equal to the nominal return from the bond holding. Notice that z_h is already determined before satisfying the period 0 budget. So the realized consumption path $\left(x_h^0, x_h^1\right)$ must satisfy the following equation:

$$\left(x_h^0 - e_h^0\right) + \frac{p}{(1+r)} \left(x_h^1 - e_h^1\right) \leq 0. \tag{2}$$

Note that although inequality (2) is not taken into account in period 0, household h will spend all the income in period 1 at the market price, i.e., $p\left(x_h^1 - e_h^1\right) = (1+r)\, z_h$ will hold, by the monotonicity of u_h. Thus the equality holds for (2) at the optimum.

The following is the notion of equilibrium that we characterize:

Definition 1 *An efficient temporary equilibrium (ETE) is a tuple* $\left(x^*, r^*, (\hat{p}_h)_{h=1}^2, p^*\right) \in \left(\mathbb{R}_{++}^2\right)^2 \times \mathbb{R} \times (\mathbb{R}_+)^2 \times \mathbb{R}_+$ *such that:*
(1) x^* *is Pareto efficient;*
(2) for each h*, there exists* $\hat{x}_h^1$ *such that* $\left(x_h^{0*}, \hat{x}_h^1\right)$ *maximizes utility under budget (1) given* $(r^*, \hat{p}_h)$*;*
(3) $\left(x_h^{0*}, x_h^{1*}\right)$ *satisfies constraint (2) at* $p = p^*$*.*

Note that the efficiency requirement (1) implies in particular that the total demand meets the total supply in both periods. Condition (2) says that period 0 market is in *temporal equilibrium* given forecasts $(\hat{p}_h)_{h=1}^H$, and condition (3) says that the period 1 market is also in temporal equilibrium. Thus at an ETE, all spot markets clear, and the consumption allocation is efficient.

Clearly, a perfect foresight equilibrium, where all forecasts are necessarily coordinated, constitutes an ETE. We shall demonstrate that, unless $\varepsilon = 0, \frac{1}{2}, 1$, there is a continuum of ETE, whereas there is a unique perfect foresight equilibrium that corresponds to the Arrow-Debreu allocation.

3 Existence of an ETE

Since the total supply is one for both goods, a Pareto efficient allocation entails that each household consumes the same amount in both periods; that is, allocation (x_1, x_2) is efficient if and only if it can be written as $(x_1, x_2) = (\alpha\,(1,1)\,, (1-\alpha)\,(1,1))$ with $\alpha \in [0,1]$. There is a unique perfect foresight equilibrium that corresponds to the Arrow-Debreu allocation of this economy, where both consume $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\frac{\bar{p}}{1+\bar{r}} = 1$.

We might as well normalize $r = 0$ in search for a continuum of ETE. Also, if $\varepsilon = \frac{1}{2}$, (e_1, e_2) is efficient and hence no trade can occur; that is, the unique ETE is the perfect foresight equilibrium. So assume $\varepsilon \neq \frac{1}{2}$ from now on.

The demand for good in period 0, i.e., the result of utility maximization given (1) is:

$$x_1^0 = \frac{1}{2}\left[(1-\varepsilon) + \hat{p}_1\varepsilon\right]$$
$$x_2^0 = \frac{1}{2}\left[\varepsilon + \hat{p}_2\,(1-\varepsilon)\right]$$

Then a period 0 temporary equilibrium occurs when $x_1^0 + x_2^0 = 1$, that is,

$$\varepsilon\hat{p}_1 + (1-\varepsilon)\,\hat{p}_2 = 1. \tag{3}$$

We shall construct an ETE as follows. Set $x_h^{0*} = x_h^{1*}$ for both h, i.e., we assign the same consumption level for both periods. Then x^* is an efficient allocation. Market clearing occurs in period 0 because of (3), and the saving market clearing occurs automatically because of the budget constraint. Thus it remains to find a period 1 price p^* such that household h demands x_h^{1*} in period 1.

Let

$$p^* = -\frac{x_1^{0*} - e_1^0}{x_1^{0*} - e_1^1}, \tag{4}$$

which is strictly positive if and only if

$$\min(e_1^0, e_1^1) < x_1^{0\,*} < \max(e_1^0, e_1^1) \tag{5}$$

In particular, when $\varepsilon \neq 0$, through $x_1^{0*} = \frac{1}{2}\left[(1-\varepsilon) + \hat{p}_1\varepsilon\right]$, inequalities (5) can be equivalently written as

$$\max\left(0, \min(3 - \frac{1}{\varepsilon}, \frac{1}{\varepsilon} - 1)\right) < \hat{p}_1 < \max(3 - \frac{1}{\varepsilon}, \frac{1}{\varepsilon} - 1). \tag{6}$$

By direct computation, it can be readily seen that (2) holds for $h = 2$ as well. Therefore, we have constructed an ETE $\left(x^*, r^*, (\hat{p}_h)_{h=1}^2, p^*\right)$.

This ETE is different from the unique rational expectation equilibrium unless $\varepsilon = 0$ or 1, since $x_h^{0*} \neq \frac{1}{2}$ unless $\varepsilon = 0$ or 1.

Thus, our findings are summarized as follows:

Result 1 Assume $\varepsilon \neq \frac{1}{2}$. For each Pareto efficient allocation (x_1^*, x_2^*) satisfying (5), there exist anticipated prices $(\hat{p}_1, \hat{p}_2) \in \mathbb{R}_{++}^2$ satisfying (3) such that the corresponding ETE has allocation (x_1^*, x_2^*) and price p^* of (4).

Result 2 Assume $\varepsilon \neq \frac{1}{2}$. For any $(\hat{p}_1, \hat{p}_2) \in \mathbb{R}_{++}^2$ satisfying (3) and (6) (when $\varepsilon = 0$, only (3) is needed), there is an ETE corresponding to $(\hat{p}_1, \hat{p}_2)$.

The construction of the ETE above makes it clear that the key for the existence of a non trivial ETE is that the real return on saving is determined only after period 1 market price is fixed, and thus there is a degree of freedom on households' anticipation about the real value of period 1 good. If there were some saving device whose real return is fixed a priori, then the households must agree on the present value of period 1 price as early as in period 0. Consequently, an ETE would require a common price forecast, i.e., an Arrow-Debreu equilibrium will result.

4 Welfare Distribution

Note that by normalization of $r = 0$, anticipated price $\hat{p}_h$ effectively represents the discounted present value of an anticipated nominal price. So for any pair of anticipated nominal prices $\hat{p}_1$ and $\hat{p}_2$, by choosing r^* such that $\varepsilon\hat{p}_1 + (1-\varepsilon)\,\hat{p}_2 = 1 + r^*$, we can construct an ETE $\left(x^*, r^*, (\hat{p}_h)_{h=1}^2, p^*\right)$ as long as (6) is satisfied for the adjusted price, $\frac{\hat{p}_1}{1+r^*}$. Since the interest rate and the second period commodity price are determined in the respective competitive markets, our finding may be interpreted that the market interest rate is adjusted in such a way that the discounted present value of anticipated prices satisfy (3).

Although the existence of an ETE is warranted for a large variety of anticipated prices, the equilibrium consumption allocation depends on them. It can be readily seen that the set of equilibrium allocations is one dimensional. Since both an ETE and the Arrow-Debreu equilibrium allocations are efficient, there must be welfare gains and losses in an ETE allocation when it is compared with the bench mark Arrow-Debreu allocation: i.e., one household is better off and the other household is worse off.

Since the households consume the same amount in both periods, the household who is led to consumes more, i.e., save less, than at the Arrow-Debreu equilibrium is better off. Therefore, it is the household who anticipates a higher price, expecting a higher inflation so to speak, who is better off. Although the process of anticipation is non-strategic by assumption in this model, the welfare consideration above suggests that there is no strong incentive for a household to anticipate the *correct* price, and in particular to coordinate with the other for a common price expectation. That is, even in the simple market environment we considered, efficiency does not necessitate perfectly coordinated forecasts across the traders.

We note that an ETE allocation is not necessarily "individually rational" in the sense that a household might be worse off in an ETE than at the initial endowments. Of course, the intended consumption is utility maximizing under (1) and thus it is no worse than the initial endowments. But the realized consumption might meet (1) with inequality, so the revealed preference argument does not work. Since the initial endowments satisfy budget (2), a household might want to choose no trade *ex post*, but it is infeasible in period 1. Intuitively, imagine a household who anticipates a very low price, thus it saves eagerly in period 0, but the actual price turns out to be much higher. Then the purchasing power of the saving is much lower than anticipated, and the household would regret that it saved at all.

To see this in the example numerically, we set $\varepsilon = \frac{9}{25}$, so the endowments are $e_1 = (\frac{16}{25}, \frac{9}{25})$ and $e_2 = (\frac{9}{25}, \frac{16}{25})$. Assume that household 1 has a low anticipated price $\hat{p}_1 = \frac{4}{9}$ while household 2 holds the anticipated price $\hat{p}_2 = \frac{21}{16}$ which is approximately twice higher than the anticipated price of household 1. It can be readily seen that (5) and (6) are satisfied. By calculation, we pin down the ETE allocation $(x_1^*, x_2^*) = \big((\frac{2}{5}, \frac{2}{5}), (\frac{3}{5}, \frac{3}{5})\big)$ and the ETE price at period 1 $p^* = 6$. Thus, the anticipated price of household 1 is significantly lower than the actual ETE price, and household 1 is better off at the initial endowment, i.e., $u_1(x_1^{0\,*}, x_1^{1\,*}) = \ln \frac{2}{5} + \ln \frac{2}{5} < \ln \frac{16}{25} + \ln \frac{9}{25} = u_1(e_1^0, e_1^1)$ (see the box below).

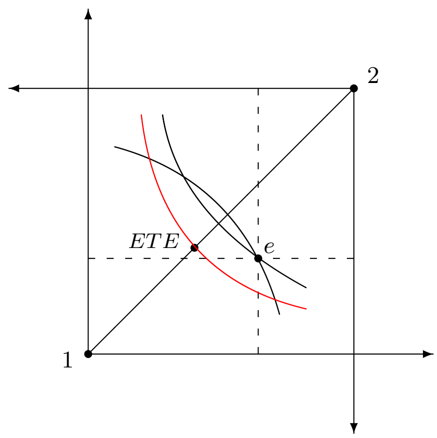

Figure 1: The ETE allocation is not individually rational for agent 1

Perhaps the term "individual rational" is not appropriate in this context, since the households do the best they can given their anticipated prices. If there is anything "irrational" about a household, it must be attributed to its anticipation. But notice that "individual rationality" fails when a household anticipates an excessively low price *relative to* the other household's anticipation, which is beyond control of the individual.

5 Conclusion

We have presented a simple example of a two period economy where the agents forecasts need not be fully coordinated in order for efficient allocations to result from decentralized trade. There is a one dimensional set of such efficient allocations for generic endowments. This finding extends to more general economies with more than two households (Chatterji et al., 2018).

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