KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.997

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Been-Lon Chen, Yunfang Hu and Kazuo Mino

August 2018

KYOTO UNIVERSITY

KYOTO, JAPAN

Does Nonlinear Taxation Stabilize Small Open Economies?*

Been-Lon Chen,† Yunfang Hu‡ and Kazuo Mino§

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Abstract

This paper examines the stabilization effect of income taxation rules in small open economies. We show that if endogenous growth is not allowed, belief-driven fluctuation will not emerge, but the economy displays total instability under certain conditions and nonlinear income tax may recover saddle point stability. If endogenous growth is possible and if the taxation rule specifies the rate of income tax held in the balanced growth equilibrium, then equilibrium indeterminacy will not arise either. However, if the long run tax rate is not predetermined, then, equilibrium path of the economy may be indeterminate, and an appropriate taxation rule can establish determinacy.

Keywords: Taxation Rule, Indeterminacy, Small Open Economy, Endogenous Growth

JEL Classification: E62, O41

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*We are grateful to Jang-Ting Guo for his insightful comments on an earlier version of this paper. Our research is financially supported by JSPS KAKEN Project (No. 17H02524).

†Institute of Economics, Academia Sinica, 128 Academia Road, Section 2, Taipei, Taiwan, bchen@econ.sinica.edu.tw

‡Graduate School of Economics, Kobe University. 2-1 Rokodai-cho, Nada-ku, Kobe 657-8501, Japan. E-mail: yhu@econ.kobe-u.ac.jp

§*Corresponding Author,* Faculty of Economics, Doshisha University, Karasuma Higashi-iru, Imadegawa, Kamigyo-ku, Kyoto, 602-8580, kmino@mail.doshisha.ac.jp

1 Introduction

Does the income taxation rule act as a built-in stabilizer? This long-standing question in public finance has attracted a renewed interest, ever since Guo and Lancing (1998) revealed that progressive income taxation contributes to stabilizing an economy in which production externalities generate belief-driven business fluctuations. Using a one-sector real business cycle model with external increasing returns, Guo and Lansing (1998) demonstrated that the progressive income tax schedule narrows the parameter space in which equilibrium indeterminacy emerges. The subsequent studies have reconsidered Guo and Lansing's finding in alternative settings such as two-sector real business cycle models, models with productive public investment, models with utility-enhancing public spending, as well as models of endogenous growth.[1] Those studies have shown that taxation rules may have a decisive role for eliminating belief-driven business cycles in various settings.

So far, the research on the stabilization effect of income tax schedules has focused on closed economy models, and the effect of income taxation rules on stability of open economies has not been well explored. The purpose of this paper is to investigate the relation between income tax schedules and stability of small open economies. We examine both exogenous and endogenous growth settings in order to inspect whether or not the stabilization effect of tax schedules found in closed economy models still holds in small-open economies.

The baseline analytical framework of our study is an open economy version of Benhabib and Farmer's (1994) one-sector real business cycle model in which the aggregate production technology exhibits external increasing returns. We open up the Benhabib-Farmer model by allowing free international financial transactions. According to the standard formulation in open-economy macroeconomics, we introduce adjustment costs of real investment so that households' selection between financial and real assets is uniquely determined in every moment. As to the specification of taxation scheme, we follow the formulation used by Guo and Lansing (1998), because it has become the standard formulation in the literature.

In this paper, we first examine a model economy in which endogenous growth is not allowed. Given this assumption, the steady state level of aggregate income is constant.

[1] A sample includes Ben-Gad (2003), Chen and Guo (2015, 2016), Gokan (2013), Greiner (2006), Guo and Harrison (2001, 2015), Lloyd-Braga, Modesto and Seegmuller (2008), and Parka and Philippopoulos (2003), and Zhang (2000).

As a result, the reference income given in the Guo and Lansing's tax schedule is fixed at the level of the steady state income. We show that in the case of exogenous growth, the small open economy will not exhibit equilibrium indeterminacy regardless of the degree of external increasing returns. We also confirm that under certain conditions, the model exhibits diverging behavior and that an appropriate choice of tax schedule may recover saddle stability. We inspect these instability conditions in detail and consider what kind of taxation scheme can avoid diverging behavior of the economy.

We then analyze an endogenous growth version of the base model. A key difference between exogenous and endogenous growth settings is that in the endogenous growth model, the reference income cannot stay constant but it should grow at the balanced growth rate of actual income. We show that if the reference income is assumed to be coincided with the actual income on the balanced growth path, which means that the long-run rate of income tax is predetermined, then equilibrium indeterminacy will not arise. Furthermore, as well as in the model with exogenous growth, the economy is completely unstable under certain conditions. If this is the case, again, an appropriate choice of nonlinear tax rule may contribute to stabilizing the economy. We inspect these instability conditions to find that the source of instability and implication of taxation rules are similar to those obtained in the exogenous growth model. On the other hand, if the ratio of reference and actual incomes on the balanced growth path is not specified (so that the long run rate of income tax is not predetermined), then the economy will not exhibit diverging behavior. However, if the tax rate on the balanced growth path is not specified, the equilibrium path of the economy may be indeterminate.

It is to be emphasized that, unlike the closed economy models, the stabilization effect of taxation rule has two different meanings in small open economy models. One is stabilization of an unstable dynamic system and the other is elimination of multiple equilibria. If the target rate of income tax held in the steady state is predetermined, then an appropriate choice of tax schedule may stabilize the economy in the first sense. As mentioned above, in this case, policy implication of taxation scheme in endogenous growth model is similar to that held in the exogenous growth counterpart. However, if the long run tax rate realized on the balanced growth path is not predetermined, the stabilization effect of a taxation rule has the second meaning, that is, establishing a unique equilibrium. If this is the case, the stabilization effect of tax schedule in the endogenous growth model is substantially different from that in the corresponding exogenous growth model.

Related Literature

The central concern of our study is closely related to the following literature.

(i) Indeterminacy in small-open economies

Early studies on equilibrium indeterminacy in small open economies such as Weder (2001), Lahiri (2001), Meng (2003) and Meng and Velasco (2003, 2004) utilized two sector models in which consumption goods are traded, while investment goods are not traded. These contributions show that small open economies tend to be volatile because indeterminacy holds under weaker conditions than in closed economy counterpart. We see that this conclusion does not hold in the standard one-sector model that is frequently employed in open economy macroeconomics literature.[2]

(ii) *The role of investment adjustment costs*

Kim (2003) introduced investment adjustment costs into the baseline model of Benhabib and Farmer (1994), and found that the possibility of equilibrium indeterminacy will decline in the presence of adjustment costs. On the other hand, Chin et al. (2012) showed that in a small open economy with endogenous growth, indeterminacy may emerge in the presence of investment adjustment costs, while equilibrium is determinate without adjustment costs. We confirm that the finding by Chin et al. (2012) fails to hold if endogenous growth is not possible, while their conclusion can hold in the endogenously growing, small open economy, if the rate of income tax realized in the balanced growth equilibrium is not predetermined.

(iii) *Exogenous versus endogenous growth*

Chen and Guo (2015) reveal that the stabilization effect of tax rules can be reserved if the model economy allows endogenous growth. Chen and Guo (2015) introduced a nonlinear taxation rule à la Guo and Lansing (1998) into an AK growth model of a closed economy to show that progressive taxation generates equilibrium indeterminacy, while regressive taxation ensures equilibrium determinacy. Chen and Guo (2017) confirmed that their finding also holds in an AK growth model with variable labor supply, as long as the economy has a

[2] See Chapter 6 in Mino (2017) for a detailed discussion on equilibrium indeterminacy in open economy models.

unique balanced growth path. As mentioned above, we show that in a small open economy version of Chen and Guo (2015, 2017), their main outcomes does not hold if the reference and actual incomes are the same on the balanced growth path. However, their findings hold, if the long run rate of income tax is not predetermined.

(iv) *Fiscal rules in small open economies*

A few authors have investigated the stabilization effect of fiscal policy rules in small open economies. Among others, Huang et al. (2017) introduced the balanced budget rule à la Schmitt-Grohè and Uribe (1997) into a two-sector small open economy model with variable labor supply. The authors demonstrated that destabilizing effect of balanced budget rule emphasized by Schimitt-Grohè and Uribe (1997) does not necessarily hold in their small open economy model. To our knowledge, Zhang (2017) is the most closely related study to our present paper. Using a two-sector small open economy model with exogenous growth in which capital goods are not traded, Zhang (2017) examined the stabilization effect of the balanced budget rule under Guo and Lansing's (1998) taxation scheme. Although the research concern of Zhang's study and our paper overlap each other, we also treat an endogenous growth model in which the level of the reference income is endogenously determined, whereas Zhang (2017) focuses on an exogenous growth model in which the reference income is fixed over time. Therefore, Zhang (2017) and our study are complements rather than substitutes.

The rest of the paper is organized as follows. The next section constructs the baseline model. Section 3 inspects the role of taxation rules in a model without endogenous growth. Sections 4 and 5, which display our main findings, investigate the model in which endogenous growth is sustained. In Section 6 summaries our discussion.

2 Baseline Setting

2.1 Production and Consumption

The baseline setting of our model is a small open economy version of Benhabib and Farmer (1994). The home country and the rest of the world produce homogeneous goods, and the aggregate production function of the home country is given by

$$Y_t = AK_t^a N_t^{1-a} \bar{K}_t^{\alpha-a} \bar{N}_t^{\beta-(1-a)} \quad A > 0, \quad 0 < a < 1, \quad a < \alpha \leq 1, \quad \beta > 1-a,$$

where Y_t is the total output and K_t and N_t, respectively, denote capital and labor, and $\bar{K}_t$ and $\bar{N}_t$ represent country-specific, external effects associated with the aggregate levels of capital and labor, respectively. In our representative-agent economy, the mass of agents is normalized to one, so in equilibrium, $\bar{K}_t = K_t$ and $\bar{N}_t = N_t$ hold for all $t \geq 0$. Therefore, the social production function is

$$Y_t = AK_t^{\alpha} N_t^{\beta}. \tag{1}$$

The final good and factor markets are assumed to be competitive, and thus the factor prices are given by

$$r_t = aAK_t^{\alpha-1} N_t^{\beta}, \quad w_t = (1-a)\, AK_t^{\alpha} N_t^{\beta-1}, \tag{2}$$

where r_t is the rate of return to capital and w_t is the real wage rate.

Our formulation of a small open economy is the conventional one: domestic households freely lend to or borrow from foreign households and international lending and borrowing are carried out by trading foreign bonds under a given world interest rate. The objective function of the representative household is given by the following lifetime utility:

$$U = \int_0^{\infty} e^{-\rho t} \left(\log C_t - \frac{N_t^{1+\gamma}}{1+\gamma} \right) dt, \quad \rho > 0, \quad \gamma > 0,$$

where ρ denotes a given time discount rate. Household's flow budget constraint is

$$\dot{B}_t = (1-\tau_{y,t})\,(r_t K_t + w_t N_t) + (1-\tau_b)\, RB_t - \left[\frac{I_t}{K_t} + \frac{\theta}{2} \left(\frac{I_t}{K_t} \right)^2 \right] K_t - C_t, \quad \theta > 0, \tag{3}$$

where B_t denotes the stock of foreign bond (net asset position) held by the domestic households, R is a given world interest rate, τ_b is a fixed rate of taxes on interest income, $\tau_{y,t}$ is the rate of factor income taxes, and I_t denotes gross investment on capital. Here, the term $(\theta/2)\,(I_t/K_t)^2\, K_t$ represents the adjustment costs of investment. The capital stock changes according to

$$\dot{K}_t = I_t - \delta K_t, \quad 0 < \delta < 1, \tag{4}$$

where δ denotes the rate of the depreciation of capital.

The household maximizes U by controlling C_t, N_t and I_t subject to (3) and (4) together with the initial condition on K_t and B_t as well as with the no-Ponzi-game condition:

$$\lim_{t\to\infty} e^{-(1-\tau_b)Rt} B_t \geq 0. \tag{5}$$

Following Guo and Lansing (1998), we assume that the fiscal authority adjusts the rate of factor income tax according to the following rule:

$$\tau_{y,t} = 1 - \eta \left(\frac{Y_t^*}{Y_t}\right)^{\phi}, \quad 0 < \eta < 1, \quad \phi_0 < \phi < 1, \tag{6}$$

where $Y_t = r_t K_t + w_t N_t$ denotes factor income of the household and Y_t^* is a reference level of aggregate income[3]. Guo and Lansing (1998) assume that Y_t^* is the aggregate level of income realized in the steady state equilibrium. In the above, the restriction on η means that when $Y_t = Y_t^*$ holds, the rate of average tax is in between 0 and 1. In addition, parameter ϕ_0 is given by

$$\phi_0 = \max\left\{\frac{\eta - 1}{\eta}, \frac{\alpha - 1}{\alpha}\right\}.$$

In this taxation scheme, the rate of income tax is endogenously determined out of the steady state, but it becomes an exogenously given flat rate, $1-\eta$, .at the steady state. Since this formulation is helpful to elucidate the dynamic impact of nonlinear taxation during the transition process of the economy, many subsequent studies on the stabilization effect of taxation rule utilize Guo and Lansing's (1998) formulation.

The restriction on ϕ ensures that if $Y_t = Y_t^*$, the after-tax income of the representative household increases with Y_t and that the after tax rate of return on the private capital

[3]For analytical simplicity, we assume that the interest income obtained from holding foreign bond is taxed in the rest of the world, and the nonlinear taxation rule applies to the domestic income alone. Alternatively, we may assume a residential taxation rule in which nonlinear tax scheme applies to the national income, $Y_t + RB_t$, as a whole. In this case, the after-tax income of the household is $\eta\,(Q_t^*/Q_t)^{\phi}$, where $Q_t = Y_t + RB_t$ and Q_t^* is the reference level of national income. Although the steady state conditions are essentially the same as that of our formulation, the dynamic behavior of optimal consumption out of the steady state becomes more complex than that held under our formulation.

decreases with K_t[4]. In addition, (6) means that the marginal tax rate is given by

$$\frac{d}{dY_t}\left(\tau_{y,t}Y_t\right) = 1 - (1-\phi)\,\eta\left(\frac{Y_t^*}{Y_t}\right)^{\phi},$$

which is higher (or lower) than the average tax rate, τ_{yt}, if $0 < \phi < 1$ (or $-\phi_0 < \phi < 0$). Thus, the taxation is progressive (regressive) if $0 < \phi < 1$ (or $\phi_0 < \phi < 0$).

Denoting the government consumption as G_t, the flow budget constraint for the government is

$$G_t = \tau_{y,t}Y_t = \left[1 - \eta\left(\frac{Y_t^*}{Y_t}\right)^{\phi}\right]Y_t. \tag{7}$$

We assume that the government simply consumes its tax revenue, so that the government spending affects neither household's welfare nor production activities.

2.2 The Optimal Conditions

To derive the optimization conditions for the household, we set up the following Hamiltonian function:

$$\begin{aligned} H_t &= \log C_t - \frac{N_t^{1+\gamma}}{1+\gamma} + q_t\left(I_t - \delta K_t\right) \\ &\quad + \lambda_t\left[(1-\tau_{y,t})Y_t + (1-\tau_b)\,RB_t - \left[\frac{I_t}{K_t} + \frac{\theta}{2}\left(\frac{I_t}{K_t}\right)^2\right]K_t - C_t\right], \end{aligned}$$

where q_t and λ_t respectively denote the implicit prices of K_t and B_t. In the above, the after tax income is expressed as

$$(1-\tau_{y,t})Y_t = \eta\left(\frac{Y_t^*}{r_tK_t + w_tN_t}\right)^{\phi}\left(r_tK_t + w_tN_t\right).$$

Remember that when selecting the optimal levels of C_t, N_t and I_t, the representative household takes sequences of $\{r_t, w_t, Y_t^*\}_{t=0}^{\infty}$ as given. Therefore, the first-order conditions for an

[4] Note that the after-tax income is $(1-\tau_{y,t})\,Y_t = Y_t - \eta Y_t^{1-\phi}Y_t^{*\phi}$ and the after tax rate of return on private capital, $(1-\tau_{y,t})\,r_t$ is given by

$$(1-\tau_{y,t})\,aY_t/K_t = a\eta Y_t^{*\phi}A^{1-\phi}K_t^{\alpha(1-\phi)-1}\bar{K}_t^{(1-\alpha)(1-\phi)}N_t^{(1-\phi)(1-a)}\bar{N}_t^{(1-\phi)[\beta-(1-a)]},$$

which decreases with private capital, K_t.

optimum include the following:

$$\max_{C_t} H_t \Longrightarrow \frac{1}{C_t} = \lambda_t, \tag{8a}$$

$$\max_{N_t} H_t \Longrightarrow N_t^{\gamma} = \lambda_t \eta \left(1-\phi\right) \left(\frac{Y_t^*}{Y_t}\right)^{\phi} w_t, \tag{8b}$$

$$\max_{I_t} H_t \Longrightarrow q_t = \lambda_t \left[1 + \theta \frac{I_t}{K_t}\right], \tag{8c}$$

$$\dot{q}_t = (\rho+\delta) q_t - \lambda_t \left[\eta(1-\phi)\left(\frac{Y_t^*}{Y_t}\right)^{\phi} r_t + \frac{\theta}{2}\left(\frac{I_t}{K_t}\right)^2\right], \tag{8d}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - \left(1-\tau_b\right) R\right], \tag{8e}$$

together with the transversality condition: $\lim_{t\to\infty} e^{-\rho t} q_t K_t = 0$ and $\lim_{t\to\infty} e^{-\rho t} \lambda_t B_t = 0$.

We find that conditions ($8a$) and ($8b$) yield

$$C_t N_t^{\gamma} = \eta \left(1-\phi\right) \left(\frac{Y_t^*}{Y_t}\right)^{\phi} w_t, \tag{9}$$

which states that the marginal rate of substitution of consumption for labor equals the after-tax real wage. In addition, condition ($8c$) yields

$$\frac{I_t}{K_t} = \frac{1}{\theta}\left(\frac{q_t}{\lambda_t} - 1\right), \tag{10}$$

showing that gross investment is positive as long as the utility price of capital, q_t, exceeds the utility price of foreign bond, λ_t.

3 Exogenous Growth

We first assume that $0 < \alpha < 1$, so that the marginal productivity of capital is diminishing and continuing growth cannot be sustained. The steady state of the small open economy is realized when K_t, B_t C_t, and Y_t stay constant over time. In this exogenous growth environment, following Guo and Lansing (1998), we assume that the fiscal authority uses the steady state level of aggregate income as the reference income. Hence, we set $Y_t^* = \bar{Y}$ for all $t \geq 0$, where $\bar{Y}$ denotes the aggregate income realized in the steady state equilibrium.

As usual, we should set $\rho = (1-\tau_b) R$ to make λ_t constant even out of the steady sate. Given this condition, $(8a)$ shows that C_t does not change during the transition:

$$C_t = \bar{C} \quad \text{for all } t \geq 0,$$

where $\bar{C}$ is determined endogenously to satisfy the intertemporal budget constraint for the representative household.

3.1 Dynamic System

From (9), we obtain $C_t N_t^{1+\gamma} = \eta (1-\phi)(1-a) Y^{*\phi} \left(A K_t^{\alpha} N_t^{\beta} \right)^{1-\phi} \frac{1}{N_t}$. Hence, the equilibrium level of hours worked is expressed as

$$N_t = \left[\frac{1}{\bar{C}} \eta (1-\phi)(1-a) A^{1-\phi} Y^{*\phi} K_t^{\alpha(1-\phi)} \right]^{\frac{1}{1+\gamma-(1-\phi)\beta}} \equiv N\left(K_t; \bar{C}\right). \tag{11}$$

Notice that under a given level of $\bar{C}$, the relation between N_t and K_t satisfies

$$\operatorname{sign} \frac{\partial N_t}{\partial K_t} = \operatorname{sign} \ (1+\gamma-(1-\phi)\beta).$$

Using (4), $(8d)$ and (11), we find that the dynamic behavior of the aggregate capital, K_t, and its utility value, q_t, are respectively given by the following:

$$\dot{K}_t = K_t \left[\frac{1}{\theta} \left(\frac{q_t}{\bar{\lambda}} - 1 \right) - \delta \right], \tag{12a}$$

$$\dot{q}_t = (\rho+\delta) q_t - \bar{\lambda} \left[\eta (1-\phi) \left(\frac{Y^*}{Y_t} \right)^{\phi} r_t + \frac{1}{2\theta} \left(\frac{q_t}{\bar{\lambda}} - 1 \right)^2 \right], \tag{12b}$$

where $r_t = \alpha A K_t^{\alpha-1} N\left(K_t, \bar{C}\right)^{\beta}$. Once the levels of $\bar{C}$ $\left(= 1/\bar{\lambda}\right)$ and Y^* are specified, $(12a)$ and $(12b)$ constitute a complete dynamic system with respect to the aggregate capital, K_t, and its utility price, q_t.

3.2 Tax Schedule and Stability

Given $\bar{C}$ and $\bar{\lambda} = 1/\bar{C}$, in the steady state where $Y_t = Y^*$ holds, the steady state values of q_t

and r_t respectively satisfy the following:

$$q^* = (\theta\delta + 1)\bar{\lambda}, \tag{13}$$

$$(\rho + \delta)(\theta\delta + 1) = \alpha\eta(1-\phi) r^* + \frac{\theta\delta^2}{2}. \tag{14}$$

The steady state rate of return to capital is determined by (14). In addition, in view of (11), $r^* = aAK^{*\alpha-1}N^{*\beta}$ gives

$$r^* = aAK^{*\alpha-1}\left[\frac{1}{\bar{C}}\eta(1-\phi)(1-a)A^{1-\phi}Y^{*\phi}K_t^{*\alpha(1-\phi)}\right]^{\frac{\beta}{1+\gamma-(1-\phi)\beta}}. \tag{15}$$

It is easy to see that under a given $\bar{C}$, the right hand side of (15) is a monotonic function of K^*, Therefore, there is a unique level of K^* that fulfills (15). Notice that the steady state level of capital depends on $\bar{C}$.

To determine $\bar{C}\left(=1/\bar{\lambda}\right)$, we use the intertemporal budget constraint for the household:

$$\begin{aligned} & B_0 + \int_0^\infty e^{-(1-\tau_b)Rt}(1-\tau_t)Y_t dt \\ = \bar{C}\int_0^\infty e^{-(1-\tau_b)Rt}dt \quad & + \int_0^\infty e^{-(1-\tau_b)Rt}\left[\frac{1}{\theta}\left(\frac{q_t}{\bar{\lambda}}-1\right)+\frac{1}{2\theta}\left(\frac{q_t}{\bar{\lambda}}-1\right)^2\right]K_t dt. \end{aligned} \tag{16}$$

Once the paths of $\{Y_t, K_t, \tau_t\}_{t=0}^{\infty}$ are determined by $(12a)$ and $(12b)$ under a given initial capital, K_0, the value of $\bar{C}\left(=1/\bar{\lambda}\right)$ is determined by (16). Finally, the steady state level of asset holding is given by $\dot{B}_t = 0$ condition in (3), which yields

$$B^* = \frac{\left[\delta + \frac{\theta\delta^2}{2}\right]K^* + \bar{C} - \eta Y^*}{(1-\tau_b)R}, \tag{17}$$

where $Y^* = AK^{*\alpha}N\left(K^*;\bar{C}\right)^{\beta}$. It is to be noted that as well as K^*, the steady state level of B_t depends on the level of $\bar{C}$.

As for equilibrium (in)determinacy, inspection of the dynamic system consisting of $(12a)$ and $(12b)$ leads to the following:

Proposition 1 *The one-sector small open economy with exogenous growth and nonlinear* *income taxation is completely unstable, if and only if*

$$1+\gamma > (1-\phi)\,\beta > [1-(1-\phi)\alpha]\,(1+\gamma)\,.$$

Otherwise, the economy holds saddle-point stability and equilibrium determinacy.

Proof. See Appendix 1. ■

As is well known, in the closed economy model of Benhabib and Farmer (1994), a necessary condition for equilibrium indeterminacy is $\beta > 1+\gamma$. Given this condition, the labor demand curve is steeper than the Frisch labor supply curve. Thus, a rise in consumption demand caused by a positive sunspot shock shifts the labor supply curve upward, so that equilibrium level of hours worked increases. This raises the current output, meaning that the sunspot-driven consumption increase is materialized. Namely, the economy displays belief-driven fluctuation. Proposition 1 reveals that such a mechanism will not work in the small-open economy. As shown in Appendix 1, the steady state of the dynamic system is a source if and only if

$$1+\gamma > (1-\phi)\,\beta > [1-(1-\phi)\,\alpha]\,(1+\gamma). \tag{18}$$

On the other hand, if the above condition is not fulfilled, the steady state is a saddle point , so that there is a unique conversing path under a given level of initial capital stock, K_0. In other words, if the degree of progressiveness of taxation, fulfills

$$\phi < 1-\frac{1+\gamma}{\beta} \quad \text{or } \phi > \frac{\beta-\alpha(1+\gamma)}{\beta+\alpha\,(1+\gamma)}, \tag{19}$$

then the economy has a unique, stable equilibrium path. Figure 1 classifies the (β,ϕ) space according to the dynamic behavior of the economy under given level of α and γ. (For simplicity of exposition, in the figure we assume the case of indivisible labor , i.e. $\gamma = 0$). As the figure shows, if the external effect associated with aggregate labor is relatively small (so that β is relatively small) , then a higher degree of progressiveness tax, (a higher ϕ) is useful to avoid diverging behavior of the economy. If β is relatively large, then instability can be eliminated under a relatively high or low level of ϕ.

Figure 1

To obtain an intuition behind the stabilization effect of tax schedule, note that (9) is written as

$$\frac{C_t}{\eta\left(1-\phi\right)Y^{*\phi}}A^{\phi}K_t^{\alpha\phi}N_t^{\gamma+\phi\beta}=\left(1-a\right)AK_t^{\alpha}N^{\beta-1}. \tag{20}$$

Under given levels of C_t and K_t, the left-hand side of (20) represents the Frisch labor supply curve and the right-hand side expresses the labor demand curve. We see that in the presence of nonlinear income tax, the elasticity of labor supply is affected by the level of ϕ. We also see that that if $1+\gamma>1-\phi\beta$, the labor demand curve is steeper than the Frisch labor supply curve. Now suppose that the small open economy initially stays at the steady state and that a positive sunspot shock hits the economy. This shock makes the households increase their consumption level, $\bar{C}$. Since the labor demand curve is steeper than the labor supply curve, such an increase in consumption demand raises the equilibrium level of hours worked. Hence, output will increase to meet the rise in consumption demand. However, this does not mean that the change in expectation caused by the sunspot shock is self fulfilled. As Figure 2 (a) depicts, the equilibrium path of ($12a$) and ($12b$) under (18) exhibits cyclical, unstable motion, so that there is no feasible equilibrium path out of the steady state.

In contrast, if the conditions in (19) hold, the labor demand curve is less steeper than the labor supply curve, and thus a rise in $\bar{C}$, which yields an upward shift of the labor supply curve, lowers the hours worked so that output declines. This contradicts the initial anticipation of the households. In fact, under (19), the phase diagram of ($12a$) and ($12b$) shows that there exist stable saddle paths that ensures the presence of unique equilibrium path under given level of K_0 : see Figure 2 (b).

Figures 2(a), 2(b)

4 Endogenous Growth

Following Benhabib and Farmer (1994), we now set $\alpha=1$ in order for continuing growth to be possible. The resulting social production function is

$$Y_t=AK_tN_t^{\beta}. \tag{21}$$

Hence, the economy has an Ak technology with variable labor input. The competitive factor prices are thus given by

$$r_t = aAN_t^{\beta}, \tag{22}$$

$$w_t = (1-a)\, AK_t N_t^{\beta-1}. \tag{23}$$

4.1 Dynamic System

A key difference of tax schedule between exogenous and endogenous growth settings is that the reference income, Y_t^*, in an endogenously growing economy is not fixed at the steady state level of aggregate income, but it grows at a constant rate realized in the balanced growth equilibrium. Note that from $(8a)$ and $(8b)$, consumption grows at a constant rate even out of the balanced growth path:

$$\frac{\dot{C}_t}{C_t} = -\frac{\dot{\lambda}_t}{\lambda_t} = (1-\tau_b)\, R - \rho. \tag{24}$$

Since the reference income, Y_t^*, is assumed to change at the balanced-growth rate given above, Y_t^* is related to C_t in such a way that

$$Y_t^* = \bar{\mu} C_t, \tag{25}$$

where $\bar{\mu}$ is a positive constant. In view of (25), the optimal condition (9) is written as

$$C_t N_t^{\gamma} = \eta\,(1-\phi)\left(\frac{\bar{\mu} C_t}{AK_t N_t^{\beta}}\right)^{\phi}(1-a)\, AK_t N_t^{\beta-1},$$

which gives the equilibrium level of hours worked as follows:

$$N_t = \left[\eta\,(1-\phi)\,(1-a)\, A^{1-\phi}\bar{\mu}^{\phi} z_t^{\phi-1}\right]^{\frac{1}{1+\gamma-(1-\phi)\beta}} \equiv N\left((z_t;\bar{\mu}\right), \tag{26}$$

where $z_t = C_t/K_t$. As a consequence, it holds that

$$\begin{aligned}
\frac{Y_t^*}{Y_t} &= \frac{\bar{\mu}\bar{C}}{AK_t N_t{}^{\beta}} = \frac{1}{A}\left(\frac{\bar{\mu}C_t}{K_t}\right)\left[\eta\left(1-\phi\right)\left(1-a\right)A^{1-\phi}\bar{\mu}^{\phi}\left(\frac{C_t}{K_t}\right)^{\phi-1}\right]^{-\frac{\beta}{1+\gamma-(1-\phi)\beta}} \\
&= A^{-\left[1+\frac{\beta}{1+\gamma-(1-\phi)}\right]}\left[\eta\left(1-\phi\right)\left(1-a\right)\right]^{-\frac{\beta}{1+\gamma-(1-\phi)\beta}}\bar{\mu}^{1-\frac{\phi\beta}{1+\gamma-(1-\phi)\beta}}z_t^{1+\frac{(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}} \\
&\equiv \Lambda\left(\bar{\mu}\right)z_t^{\varepsilon}, \qquad (27)
\end{aligned}$$

where

$$\begin{aligned}
\Lambda\left(\bar{\mu}\right) &= A^{-\varepsilon}\left[\eta\left(1-\phi\right)\left(1-a\right)\right]^{-\frac{\phi\beta}{1+\gamma-(1-\phi)\beta}}\bar{\mu}^{1-\frac{\beta}{1+\gamma-(1-\phi)\beta}}, \\
\varepsilon &= 1+\frac{(1-\phi)\beta}{1+\gamma-\left(1-\phi\right)\beta}.
\end{aligned}$$

As shown above, the ratio of the reference and the realized realized income increases (or decreases) with the consumption-capital ratio, z_t, if $1+\gamma > (1-\phi)\beta$ (or $1+\gamma < (1-\phi)\beta$).

Equation $(8c)$ gives

$$\frac{I_t}{K_t} = \frac{1}{\theta}(v_t - 1), \qquad (28)$$

where $v_t = q_t/\lambda_t$. Using this expression, we respectively rewrite (3), $(8d)$ and (28) in the following manner:

$$\frac{\dot{B}_t}{B_t} = \eta\left[\Lambda\left(\bar{\mu}\right)z_t^{\varepsilon}\right]^{\phi}\frac{A}{b_t}N(z_t;\bar{\mu})^{\beta} + \left(1-\tau_b\right)R - \left[\frac{1}{\theta}(v_t-1)+\frac{1}{2\theta}\left(v_t-1\right)^2\right]\frac{1}{b_t} - \frac{z_t}{b_t}, \qquad (29)$$

$$\frac{\dot{q}_t}{q_t} = \rho+\delta-\frac{1}{v_t}\left[\eta(1-\phi)\left[\Lambda\left(\bar{\mu}\right)z_t^{\varepsilon}\right]^{\phi}aAN(z_t;\bar{\mu})^{\beta}+\frac{1}{2\theta}\left(v_t-1\right)^2\right], \qquad (30)$$

$$\frac{\dot{K}_t}{K_t} = \frac{1}{\theta}(v_t-1)-\delta, \qquad (31)$$

where $b_t = B_t/K_t$.

Using $(8e)$, (29), $(30)$and (31), we derive the following complete dynamic system with respect to $b_t\left(=B_t/K_t\right)$, $v_t\left(=q_t/\lambda_t\right)$ and $z_t\left(=C_t/K_t\right)$:

$$\begin{aligned}
\dot{b}_t &= \left[\left(1-\tau_b\right)R+\delta-\frac{1}{\theta}\left(v_t-1\right)\right]b_t \\
&\quad +\eta A\left[\Lambda\left(\bar{\mu}\right)z_t^{\varepsilon}\right]^{\phi}N\left(z_t;\bar{\mu}\right)^{\beta}-\left[\frac{1}{\theta}(v_t-1)+\frac{1}{2\theta}\left(v_t-1\right)^2\right]-z_t, \qquad (32a)
\end{aligned}$$

$$\dot{v}_t = [\delta + (1-\tau_b)\, R]v_t - \left[\eta(1-\phi)aA[\Lambda\,(\bar{\mu})\, z_t^{\varepsilon}]^{\phi} N\,(z_t;\bar{\mu})^{\beta} + \frac{1}{2\theta}\,(v_t-1)^2\right], \tag{32b}$$

$$\dot{z}_t = z_t\left[(1-\tau_b)\, R - \rho - \frac{1}{\theta}(v_t-1) + \delta\right]. \tag{32c}$$

4.2 Balanced Growth Equilibrium

On the balanced growth equilibrium, consumption, capital and foreign bond grow at a common, constant rate and the hours worked stay constant over time. This means that b_t, v_t and z_t stay constant on the balanced growth path, so that the steady state levels of (b_t, v_t, z_t), denoted by (b^*, v^*, z^*), satisfy the following conditions:

$$\rho b^* + \eta A[\Lambda\,(\bar{\mu})\, z_t^{\varepsilon}]^{\phi} N\,(z^*;\bar{\mu})^{\beta} = \frac{1}{\theta}(v^*-1) + \frac{1}{2\theta}\,(v^*-1)^2 + z^*, \tag{33a}$$

$$[\delta + (1-\tau_b)\, R]v^* = \eta(1-\phi)aA[\Lambda\,(\bar{\mu})\, z^{*\varepsilon}]^{\phi} N\,(z^*;\bar{\mu})^{\beta} + \frac{1}{2\theta}\,(v^*-1)^2\,, \tag{33b}$$

$$(1-\tau_b)\, R - \rho = \frac{1}{\theta}(v^*-1) + \delta. \tag{33c}$$

Moreover, following Chen and Guo (2015, 2016), we impose that $Y_t^* = Y_t$, holds on the balanced growth path. Thus from (27), it holds that

$$\frac{Y_t^*}{Y_t} = \frac{\bar{\mu} z_t}{AN_t{}^{\beta}} = \Lambda\,(\bar{\mu})\, z^{*\varepsilon} = 1. \tag{34}$$

First, condition $(33c)$ gives a unique level of v^* :

$$v^* = \theta\,[(1-\tau_b)\, R - \rho - \delta] + 1. \tag{35}$$

Then, using $(33b)$ and (34), we derive

$$[\delta + (1-\tau_b)\, R]v^* = \eta(1-\phi)aAN^{*\beta} + \frac{1}{2\theta}\,(v^*-1)^2\,,$$

which yields a unique level of N^* in the following manner:

$$N^* = \left[\frac{\delta + (1-\tau_b)\, R]v^* - \frac{1}{2\theta}\,(v^*-1)^2}{\eta(1-\phi)aA}\right]^{\frac{1}{\beta}}. \tag{36}$$

In addition, given condition (34) , the steady state expression of (9) is

$$z^* = \eta (1 - \phi) (1 - a) AN^{*\beta - (1+\gamma)}.$$

Hence, z^* is uniquely determined as well. As a result, (34) gives the level of $\bar{\mu}$ in such a way that

$$\bar{\mu} = \frac{AN^{*\beta}}{z^*}. \tag{37}$$

Finally, from (33a) the steady state level of b_t is given by

$$b^* = \frac{1}{\rho} \left[\frac{1}{\theta} (v^* - 1) + \frac{1}{2\theta} (v^* - 1)^2 + z^* - \eta AN^{*\beta} \right]. \tag{38}$$

In sum, we have found:

Proposition 2 *In an endogenously growing, small open economy under the nonlinear taxation rule given by* (6) , *there is a unique balanced growth equilibrium.*

This result has been pointed out by Chin et al. (2012) in the corresponding model without nonlinear taxation. This is a major divergence from the closed economy model. Benhabib and Farmer (1994) demonstrated that in an endogenous growth version of their baseline real business cycle model with external increasing returns, the model economy may have dual balanced growth paths if labor externality is sufficiently large to satisfy $\beta > 1 + \gamma$. In this case, BGP with a higher growth exhibits local indeterminacy, whereas BGP with a lower growth rate holds local determinacy. As mentioned in Section 1, Chen and Guo (2016) revealed that the same conclusion holds in a model under the nonlinear taxation. Since the balanced growth rate of our small open economy is exogenously specified as $(1 - \tau_b) R - \rho$, the economy has a unique balanced growth equilibrium, regardless of the magnitude of external effects associated with aggregate labor.

4.3 Equilibrium Dynamics

The analysis of equilibrium dynamics of the system derived above leads to the following outcome:

Proposition 3 *In a small-open, growing economy in which the reference income, Y_t^*, coincides with the actual income on the balanced growth path, there is no equilibrium path converging to the steady state if $\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma$. Otherwise, the economy satisfies saddle-point stability and equilibrium determinacy.*

Proof. See Appendix 2. ■

As Appendix 2 shows, the steady state of the dynamic system consisting of $(32b)$, $(32c)$ is a source if and only if

$$\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma. \tag{39}$$

Thus, under (39), the solution of $(32b)$, $(32c)$ and $(32a)$ diverges from the balanced growth equilibrium. In contrast, if ϕ is set to satisfy

$$\phi < \frac{\beta}{\beta+1+\gamma} \quad \text{or} \quad \phi > 1-\frac{1+\gamma}{\beta}, \tag{40}$$

then the balanced growth equilibrium establishes local saddle stability. Therefore, unlike the closed economy model examined by Chen and Guo (2016), the small-open economy with endogenous growth and variable labor supply does not exhibit equilibrium indeterminacy, regardless of the degree of external effect associated with aggregate labor. However, as well as in the small open economy with exogenous growth, the degree of progressiveness of taxation, ϕ, plays an important role to establish tability of the balanced growth path.

Figure 3 classifies the (β, ϕ) space into the saddle stability and completely unstable regions. Compared to Figure 1, we find that the parameter space in which the equilibrium path is diverging or it satisfies saddle stability are similar to those of the exogenous growth model.

Figure 3

We note that if the tax rate is flat (i.e. $\phi = 0$), then conditions (39) and (40) respectively reduce to $\beta < 1+\gamma$ and $\beta > 1+\gamma$. This means that under a flat rate of income tax, the small open economy is completely unstable if $\beta < 1+\gamma$, whereas it exhibits saddle stability and equilibrium determinacy if $\beta > 1+\gamma$. This is a stark contrast to the corresponding closed economy model with endogenous growth in which determinacy holds under $\beta < 1+\gamma$, and indeterminacy may arise under $\beta > 1+\gamma$. Such a difference stems from the fact that we should consider the behavior of $b_t\,(= B_t/K_t)$ in our small open economies. If we require that b_t does not exhibit diverging behavior, we should treat a dynamic system that include b_t, which yield a substantial difference in stability conditions between the closed and open economy models[5].

5 An Alternative Taxation Rule

So far, we have assumed that on the balanced growth path, it holds that $Y_t^* = Y_t$. This assumption means that the rate of income tax on the balanced growth path is fixed at the level of $1 - \eta$. Of course, the long-run target rate of income tax may take another value, such as $Y_t^*/Y_t = \bar{x} \neq 1$. If this is the case, the long-run target rate of income tax become $1 - \eta \bar{x}^{\phi}$ rather than $1 - \eta$. It is easy to confirm that when the long-run level of Y_t^*/Y_t is predetermined, the dynamic property of the model economy is essentially the same as that in the case of $\bar{x} = 1$.

In the model with exogenous growth, Y_t stays constant in the steady state, so that it is natural to assume that Y_t^* is a positive constant. If Y^* does not equal the steady state level of Y_t, then the long-run tax rate becomes $1 - \eta \left(\frac{Y^*}{\bar{Y}}\right)^{\phi} \neq 1 - \eta$, where $\bar{Y}$ denotes the steady state level of Y_t. However, as long as Y^* is a given constant, the property of the reduced dynamic system is the same as the case of $Y^*/\bar{Y} \;= 1$. By contrast, in an endogenous growth model, there is no restriction on the level of Y_t^* except that it grows at the rate of $(1 - \tau_b)\,R - \rho$. As demonstrated in the previous section, if we assume that $Y_t^* = Y_t$ is one of the balanced growth conditions, the small open economy does not yield equilibrium indeterminacy. However, if the rate of income tax on the balanced growth path is not predetermined, we cannot rule out the possibility of equilibrium indeterminacy. To see this, remember that (32b) and (32c) constitute a complete dynamic system with respect to v_t and z_t *under a given level of* $\bar{\mu}$. Additionally, once $\bar{\mu}$ is given, the dynamic system, (32b) and (32c), involves a unique set of steady state values of (v_t, z_t) that are determined by (33b) and (33c). In what follows, we re-examine the dynamic property of our small open economy under an alternative assumption in which the taxation rule does not not specify the

[5] Chin et al. (2012) examine a small open economy model with endogenous growth in which there is no income tax. The authors focus on a dynamic system that excludes the dynamic equation of b_t. Their determinacy/indeterminacy conditions, therefore, differ form those shown in our paper.

rate of income tax realized on the balanced growth path.

Case (i) $\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma$

As discussed in the previous section, in this case, the steady state of the dynamic system, $(32b)$ and $(32c)$, is a source. and thus $v_t = v^*$, $z_t = z^*$ and $N_t = N^*$ for all $t \geq 0$. Denoting the steady state value of $Y_t^*/Y_t = x^*$, from (27) we obtain

$$x^* = Y_t^*/Y_t = \Lambda(\bar{\mu}) z_t^{*\varepsilon}. \tag{41}$$

Now notice that he intertemporal budget constraint for the household is expressed as.

$$B_0 + \int_0^\infty e^{-(1-\tau_b)Rt} \eta x^{*\phi} K_t N^{*\beta} dt$$
$$= \int_0^\infty e^{-(1-\tau_b)Rt} C_t dt \quad + \int_0^\infty e^{-(1-\tau_b)Rt} \left[\frac{1}{\theta}(v^* - 1) + \frac{1}{2\theta}(v^* - 1)^2\right] K_t dt.$$

Since $K_t = e^{gt} K_0$ and $g = (1-\tau_b) R - \rho$, in view of (26) and (41), the above equation is rewritten as

$$b_0 + \frac{\eta}{\rho}\left(\Lambda(\bar{\mu}) z_t^{*\varepsilon}\right)^\phi [N(z^*; \bar{\mu})]^\beta = \frac{1}{(1-\tau_b) R}\left[z^* + \frac{1}{\theta}(v^* - 1) + \frac{1}{2\theta}(v^* - 1)^2\right]. \tag{42}$$

Equation (35) shows that v^* does not depend on $\bar{\mu}$. Hence, we see that $(33b)$ and (42) determine z^* and $\bar{\mu}$. As a result, the steady sate level of $N^* = N(z^*, \mu)$ is determined as well.

Ftom (42), $(32a)$ is expressed as

$$\dot{b}_t = \rho b_t + \eta A^{1-\phi} (\bar{\mu} z^*)^\phi - \left[\frac{1}{\theta}(v^* - 1) + \frac{1}{2\theta}(v^* - 1)^2\right] - z^*.$$

Using (42), we find that the above becomes

$$\dot{b}_t = \rho (b_t - b_0).$$

This means that b_t stays constant over time, and thus $b_0 = b^*$. Therefore, the economy always stays on the balanced growth path, which establishes equilibrium determinacy.

Case (ii) $(1-\phi)\beta > 1+\gamma$ or $(1-\phi)\beta < \phi(1+\gamma)$

In this case, Appendix 2 reveals that there is a stable saddle path in $(v_t,\ z_t)$ space, which has a positive slope:

$$v_t = \xi\left(z_t;\bar{\mu}\right). \quad \xi_z\left(z_t;\bar{\mu}\right) > 0.$$

Hence, a complete dynamic system is summarized as the following equations:

$$\begin{aligned} \dot{b}_t &= [(1-\tau_b)\, R + \delta - \frac{1}{\theta}\left(\xi\left(z_t;\bar{\mu}\right) - 1\right)]b_t \\ &\quad + \eta A\left[\Lambda\left(\bar{\mu}\right) z_t^{\varepsilon}\right]^{\phi} N\left(z_t;\bar{\mu}\right)^{\beta} - \left[\frac{1}{\theta}(\xi\left(z_t;\bar{\mu}\right) - 1) + \frac{1}{2\theta}\left(\xi\left(z_t;\bar{\mu}\right) - 1\right)^2\right] - z_t, \quad \text{(43a)} \end{aligned}$$

$$\dot{z}_t = z_t\left[(1-\tau_b)\, R - \rho - \frac{1}{\theta}(\xi\left(z_t;\bar{\mu}\right) - 1) + \delta\right]. \quad \text{(43b)}$$

As shown in Appendix 2, the above system has a saddle point property, so that there is one dimensional stable saddle path in $(b_t,\ z_t)$ space. Since equations $(43a)$ and $(43b)$ involve $\bar{\mu}$, the stable saddle path also depends on $\bar{\mu}$. We express the stable saddle path as

$$z_t = \zeta\left(b_t;\bar{\mu}\right). \quad (44)$$

As Figure 4 depicts, we can confirm that the stable saddle path described by (44) has a negative slope. Now suppose that the economy initially stays on the steady state (Point E_0 in Figure 5). Then suppose further that a positive sunspot shock hits the economy, and the households anticipate that their future income will rise, so that they increases their initial consumption level, C_0. In view of (44), the aggregate consumptions at $t = 0$ satisfies the following relation.

$$C_0 = \zeta\left(\frac{B_o}{K_0};\bar{\mu}\right) K_0.$$

This relation shows that a sunspot shock yields a change in $\bar{\mu}$ under given B_0 and K_0. Thus a positive sunspot shock yields an upward shift of the stable saddle path, which gives rise to a change in $\bar{\mu}$. Therefore, in response to the sunspot shock, the economy jumps up from E_0 to Point B in the figure. If there is no further shock, the economy moves along the new saddle path towards to the new steady state (Point E_1) that corresponds to the new level of $\bar{\mu}$. In this sense, if the steady state level of Y_t^*/Y_t is not predetermined (so that the long run rate of income tax is not specified), then equilibrium indeterminacy emerges, which yields sunspot-driven business fluctuations.

Figure 4, Figure 5

In sum, we have shown:

Proposition 4 *Suppose that the steady state rate of income tax is not predetermined. Then the balanced growth path of the small open economy is locally determinate if* $\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma$. *Otherwise, the balanced growth path is locally indeterminate.*

Figure 6 classifies $(\phi,\ \beta)$ space according to equilibrium determinacy/indeterminacy conditions in Proposition 3. We see that the parameter space in Figure 3 where the economy is completely unstable now becomes the regions that holds determinacy, while the parameter space in which determinacy holds in Figure 3 turns out to be regions that holds intermediacy in Figure 6. Those results mean that specification of taxation rule is crucial when evaluating the stabilization effect of nonlinear tax schedules.

6 Summary of Main Findings

Table 1 summarizes our findings. This table reveals that $Y_t^* = Y_t$ holds in the long run equilibrium. f imposed, dynamic behavior of the small open economy does not show significant differences between the cases of exogenous and endogenous growth. In both cases, the economy exhibits total instability under similar conditions. As mentioned earlier, Chen and Guo (2015, 2017) reveal that the stabilization effect of progressive (regressive) taxation is completely opposite in exogenous and endogenous growth models. Our study shows that if the consistency condition, $Y_t^* = Y_t$, is imposed, the small open economy does not show significant differences in the stabilization effect of income taxation rules. However, if the condition $Y_t^* = Y_t$ is not added to a set of balanced growth conditions, then equilibrium indeterminacy may arise and nonlinear taxation can act as an automatic stabilizer in the sense that it eliminates the possibility of sunspot-driven fluctuations.

	Exogenous Growth	Endogenous Growth $Y_t^* = Y_t$ on BGP	Endogenous Growth $Y_t^* \neq Y_t$ on BGP
Instability	$1+\gamma > (1-\phi)\,\beta$ $> [1-(1-\phi)\,\alpha]\,(1+\gamma)$	$1+\gamma > (1-\phi)\,\beta$ $> \phi(1+\gamma)$	NA
Determinacy	$1+\gamma < (1-\phi)\,\beta$ or $[1-(1-\phi)\,\alpha]\,(1+\gamma)$ $> (1-\phi)\,\beta$	$1+\gamma < (1-\phi)\,\beta$ or $\phi\,(1+\gamma) > (1-\phi)\,\beta$	$1+\gamma > (1-\phi)\,\beta$ $> \phi(1+\gamma)$
Indeterminacy	NA	NA	$1+\gamma < (1-\phi)\,\beta$ or $\phi\,(1+\gamma) > (1-\phi)\,\beta$

Table 1: Stability of the Baseline Small Open Economy under Nonlinear Income Tax (NA=not applicable, BGP = balanced growth path)

7 Conclusion

In this paper, we have explored the stabilization effect of income taxation rules in small open economies. Using a conventional modeling of one sector, small open economy with free capital mobility, we first examine an economy in which endogenous growth is not allowed. In this case, we find that in contrast to the closed economy counterpart, equilibrium indeterminacy will not emerge, regardless of the degree of external increasing returns. We see that the economy is completely unstable under certain conditions and that an appropriate choice of tax parameters may recover saddle stability. When endogenous growth is possible, the level of the reference income is be endogenously determined. If the fiscal rule specifies the rate of income tax realized on the balanced growth path, then equilibrium intermediacy will not arise but the economy could be totally unstable. If this is the case, an appropriate choice of tax parameter may recover saddle stability. On the other hand, if the fiscal rule does not specify the target rate of income tax realized on the balanced growth path, then equilibrium indeterminacy may hold and nonlinear taxation can eliminate sunspot driven fluctuations.

Finally, it is worth emphasizing again that unlike the closed economy counterpart, the stabilization effect of tax rule has two different meanings in small open economies. One is stabilizing an unstable economy, and the other is eliminating equilibrium indeterminacy. We have demonstrated that the stabilization effect of taxation schedule in both senses critically depends on the environment to which theses rules are applied as well as on the form of taxation rules employed by the fiscal authority.

Appendices

Appendix 1: Pfoof of Proposition 1

We first focus on $(12a)$ and $(12b)$ which constitute a complete dynamic system with respect to K_t and q_t. Evaluating the Jacobian matrix of the dynamic system at the steady state, we obtain

$$J_1 = \begin{bmatrix} 0 & K^* \dfrac{\bar{C}}{\theta} \\ \dfrac{\partial \dot{q}}{\partial K} |_{K_t = K^*, \quad q_t = q^*} & \rho \end{bmatrix}.$$

In the above, it holds that

$$\frac{\partial \dot{q}}{\partial K} |_{K_t = K^*, \quad q_t = q^*} = -\bar{\lambda}\eta (1-\phi) \frac{\partial}{\partial K_t} \left(\frac{Y^*}{Y_t} \right)^{\phi} r_t = -\bar{\lambda}\eta (1-\phi) \frac{\partial}{\partial K_t} \left(\frac{Y_t^{1-\phi}}{K_t} \right).$$

From (11), we see that

$$\frac{Y_t^{1-\phi}}{K_t} = AK_t^{(1-\phi)\alpha - 1} N_t^{(1-\phi)\beta} = AK_t^{(1-\phi)\alpha-1} \left[\frac{1}{\bar{C}} \eta (1-\phi)(1-a) A^{1-\phi} Y^{*\phi} K_t^{\alpha(1-\phi)} \right]^{\frac{(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}},$$

which leads to the following relation:

$$\text{sign} \frac{\partial \dot{q}}{\partial K} |_{K_t = K^*, \quad q_t = q^*} = \text{sign} \frac{[1-(1-\phi)\alpha](1+\gamma) - (1-\phi)\beta}{1+\gamma-(1-\phi)\beta}.$$

Thus $\dfrac{\partial \dot{q}}{\partial K} |_{K_t = K^*, \quad q_t = q^*} < 0$, if and only if

$$1+\gamma > (1-\phi)\beta > [1-(1-\phi)\alpha](1+\gamma). \tag{A1}$$

Given $(A1)$, both $\det J_1$ and trace of J_1 have positive values, so that the steady state is a source. Since the initial level of K_0 is historically given, there is no equilibrium path converging to the steady state, unless $K_0 = K^*$. Remember that K^*depends on $\bar{C}$. Therefore, in view of (16) , it is possible to realize $K^* = K_0$ by selecting an appropriate level of $\bar{C}$ that satisfies

$$r^* = aAK^{*\alpha-1}\left[\frac{1}{\bar{C}}\eta\left(1-\phi\right)\left(1-a\right)A^{1-\phi}Y^{*\phi}K_0^{\alpha(1-\phi)}\right]^{\frac{\beta}{1+\gamma-(1-\phi)\beta}}. \tag{A2}$$

If this is the case, $q_t = q^*$ and $K_t = K^*$ from the outset. However, if $(K_t, q_t) = (K^*, q^*)$ for all $t \geq 0$, then the intertemporal budget constraint (16) reduces to

$$B_0 + \frac{\eta AK^{*\alpha}N\left(K^*;\bar{C}\right)^\beta}{\left(1-\tau_b\right)R} = \frac{\bar{C}}{\left(1-\tau_b\right)R} + \frac{(\delta+\delta^2\theta/2)K^*}{\left(1-\tau_b\right)R}. \tag{A3}$$

Since $\bar{C}$ cannot fulfill (A2) and (A3) simultaneously under given levels of K_0 and B_0, there is no stable equilibrium path.

On the other hand if (A4) is not fulfilled, then (K^*, q^*) is a saddle point, meaning that (K_t, q_t) converges to the steady state under a given level of K_0. Hence, $\bar{C}$ can be set to satisfy (A3) . Note that (A3) is rewritten as

$$B_0 = \frac{\left[\delta + \frac{\theta\delta^2}{2}\right]K^* + \bar{C} - \eta Y^*}{\left(1-\tau_b\right)R}.$$

Hence, from (17) it holds that $B^* = B_0$, and, therefore, the stock of foreign bond stays at its given initial level during the transition towards the steady state. This means that the small open economy establishes a determinate and stable equilibrium.

Appendix 2: Pfoof of Proposition 3

Since the dynamic behaviors of v_t and z_t are independent of b_t, we first analyze (32b) and (32c) . Note that the term, $[\Lambda\left(\bar{\mu}\right)z_t^\psi]^\phi N\left(z_t;\bar{\mu}\right)^\beta$, involved in the right hand side of (32b) is expressed as

$$\begin{aligned} & [\Lambda\left(\bar{\mu}\right)z_t^\psi]^\phi N\left(z_t;\bar{\mu}\right)^\beta \\ = \; & \Lambda\left(\bar{\mu}\right)^\phi z_t^{\frac{\phi(1+\gamma)}{1+\gamma-(1-\phi)\beta}}\left[\eta\left(1-\phi\right)\left(1-a\right)A^{1-\phi}\bar{\mu}^\phi z_t^{\phi-1}\right]^{\frac{\beta}{1+\gamma-(1-\phi)\beta}} \\ & \Psi z_t^{\frac{\phi(1+\gamma)-(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}}. \end{aligned}$$

where $\Psi\,(>0)$ ummarizes a set of parameters. The coefficient matrix of the dynamic system of v_t and z_t evaluated at the steady state is

$$J_2 = \begin{bmatrix} \rho, & \frac{\partial \dot{v}_t}{\partial z_t}. \\ -z^* \frac{1}{\theta}, & 0 \end{bmatrix},$$

where

$$\begin{aligned} \left.\frac{\partial \dot{v}_t}{\partial z_t}\right|_{v_t=v^*, z_t=z^*} &= -\eta(1-\phi)\alpha A \frac{\partial}{\partial z}\left\{[\Lambda(\bar{\mu}) z_t^{\psi}]^{\phi} N(z_t;\bar{\mu})^{\beta}\right\} \\ &= -\frac{\phi(1+\gamma)-(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}\eta(1-\phi)aA\Psi z^{*\frac{\phi(1+\gamma)-(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}-1}. \end{aligned}$$

Since $\det J_2 = -z^* \frac{1}{\theta}\zeta \left.\frac{\partial \dot{v}_t}{\partial z_t}\right|_{v_t=v^*, z_t=z^*}$, we see that

$$\text{sign } \det J_1 = \text{ sign } \frac{\phi(1+\gamma)-(1-\phi)\beta}{1+\gamma-(1-\phi)\beta}.$$

As a result, we see that if

$$\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma,$$

then both $\det J_2$ and the trace of J_2 have positive signs, which means that the subsystem is completely unstable. On the other hand, if

$$\phi(1+\gamma) > 1+(1-\phi)\beta \text{ or } (1-\phi)\beta < 1+\gamma,$$

then the steady state of the subsystem is saddle point.

Case (i) $\phi(1+\gamma) < (1-\phi)\beta < 1+\gamma$

In this case, $v_t = v^*$ and $z_t = z^*$ from the outset, because both v_t and z_t are jump variables. As shown in the main text, the steady state levels of v^* and N^* are independent of $\bar{\mu}$. Thus, under the consistency condition,, $Y_t^* = Y_t$, the steady state expression of (9) is

$$z^* N^{*\gamma} = (1-a) A N^{*\beta-1}.$$

Therefore, the steady state value of z^* is not related to $\bar{\mu}$ either. This means that the consistency condition, $Y_t^*/Y_t = 1$, determines $\bar{\mu}$ in such a way that $\bar{\mu} = \frac{AN^{*\beta}}{z^*}$. At the same time, b_t changes according to

$$\dot{b}_t = \rho b_t + \eta A N^{*\beta} - \left[\frac{1}{\theta}(v^* - 1) + \frac{1}{2\theta}(v^* - 1)^2 \right] - z^*. \tag{A4}$$

Since this system is completely unstable and b_t $(= B_t/K_t)$ is not a jump variable, b_t continues diverging unless $b_0 = b^*$. This means that unless $b_0 = b^*$, the transversailty condition will be violated if $b_0 > 0$, while the no-Ponzi game condition fails to hold if $b_0 < 0$. As a consequence, the small open economy cannot have a feasible perfect foresight equilibrium.[6]

Case (ii) $(1 - \phi)\beta > 1 + \gamma$

In this case, there is a positively sloped, stable saddle path in (v_t, z_t) space. We denote the stable saddle path as

$$v_t = \xi(z_t;\ \bar{\mu}), \quad \xi_z(z_t;\ \bar{\mu}) > 0, \qquad v^* = \xi(z^*; \bar{\mu}).$$

Notice that the relation between v_t and z_t *on* the stable saddle path depends on $\bar{\mu}$. Then a complete dynamic system reduces to

$$\begin{aligned} \dot{b}_t &= [(1-\tau_b) R + \delta - \frac{1}{\theta}(\xi(z_t;) - 1)] b_t \\ &\quad + \eta A[\Lambda(\bar{\mu}) z_t^{\psi}]^{\phi} N(z_t; \bar{\mu})^{\beta} - \left[\frac{1}{\theta}(\xi(z_t; \bar{\mu}) - 1) + \frac{1}{2\theta}(\xi(z_t; \bar{\mu}) - 1)^2 \right] - z_t, \end{aligned} \tag{A5}$$

$$\dot{z}_t = z_t \left[(1-\tau_b) R - \rho - \frac{1}{\theta}(\xi(z_t; \bar{\mu}) - 1) + \delta \right]. \tag{A6}$$

The coefficient matrix of the system of (A5) and (A6) is

$$J_2 = \begin{bmatrix} \rho, & \frac{\partial \dot{b}_t}{\partial z_t} \\ 0, & -z^* \frac{1}{\theta} \xi'(z^*) \end{bmatrix},$$

Since $\det J_2 = -\rho z^* \xi'(z^*)/\theta < 0$, the steady state satisfies saddle stability so that determinacy holds.

[6] It is to be noted that in the exogenous growth model dicussed in Section 3, consumption stays constant, so that the intertemporal budget constraint for the househld may determine $\bar{C}$ that establishes $B^* = B_0$.unless $t(K_t, q_t)$ satisfy saddle-point stability. Such kind of adjustment is not possible, because $\bar{\mu}$ is not involved the dynamic equation of b_t :see (A4).

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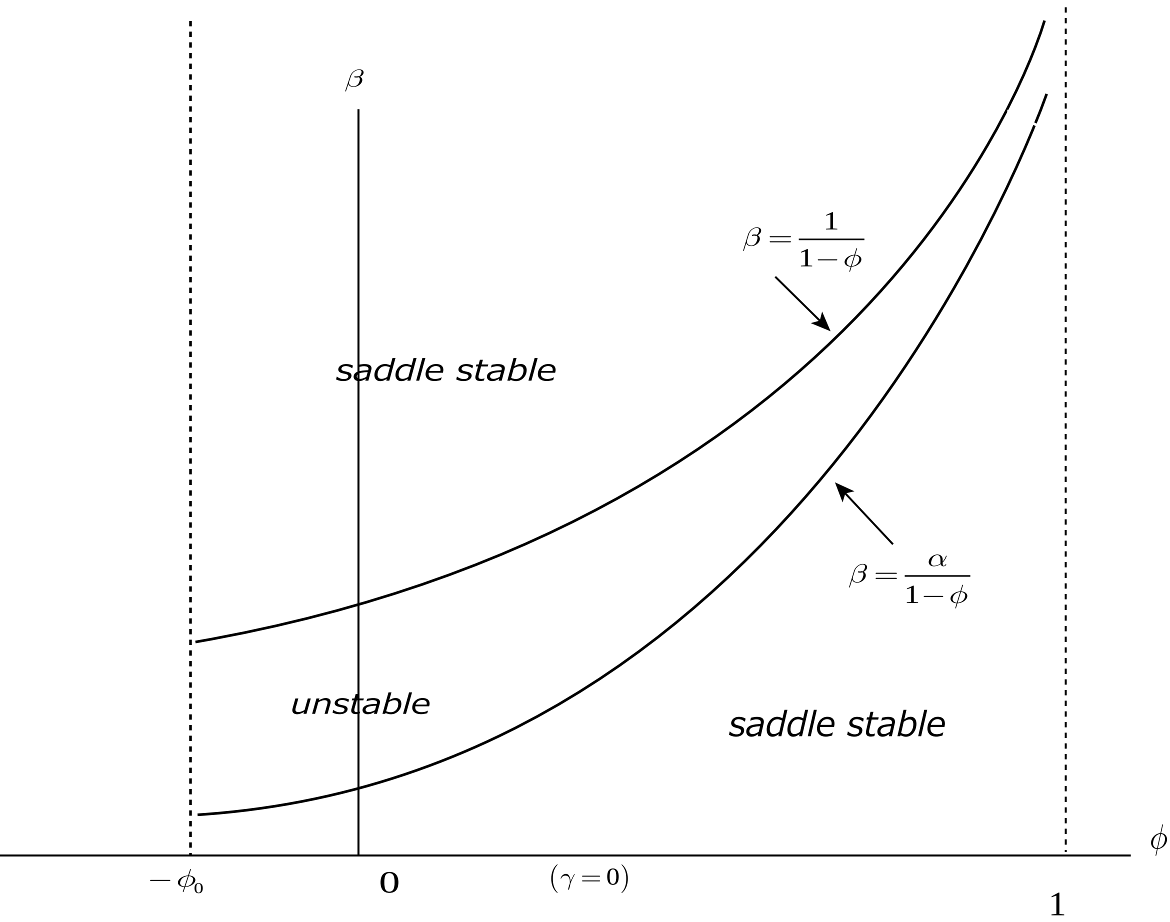

Figure 1: Classification of (β, ϕ) space under exogenous growth $(\gamma = 0)$

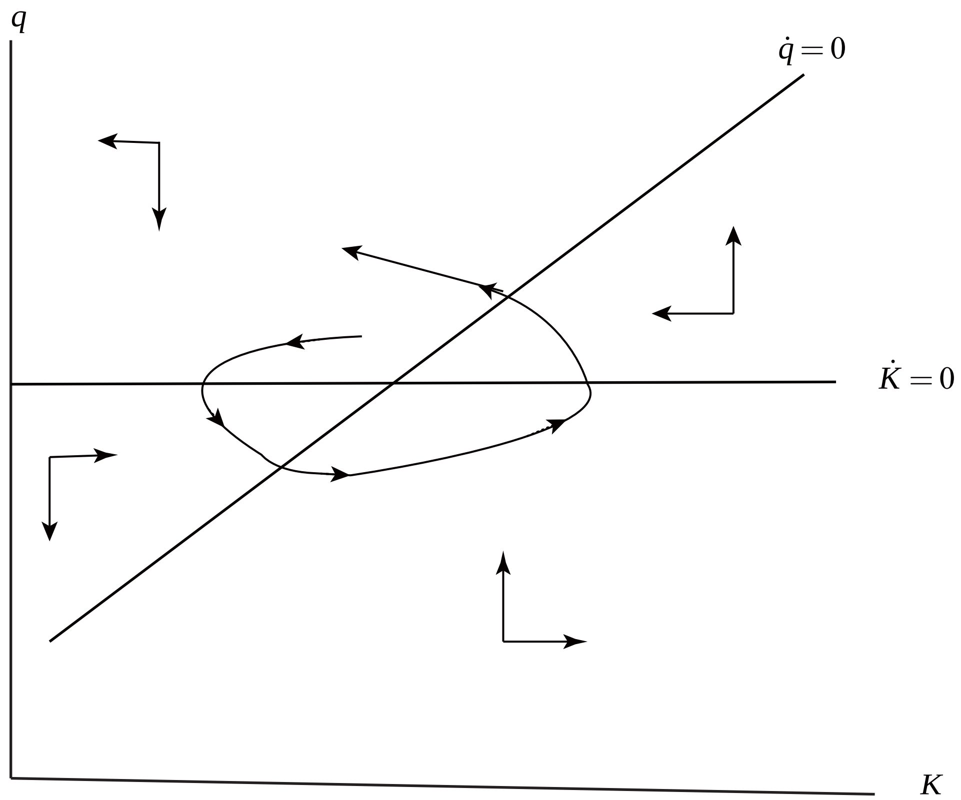

Figure 2 (a) : $1+\gamma>(1-\phi)\beta>[1-(1-\phi)\alpha](1+\gamma)$

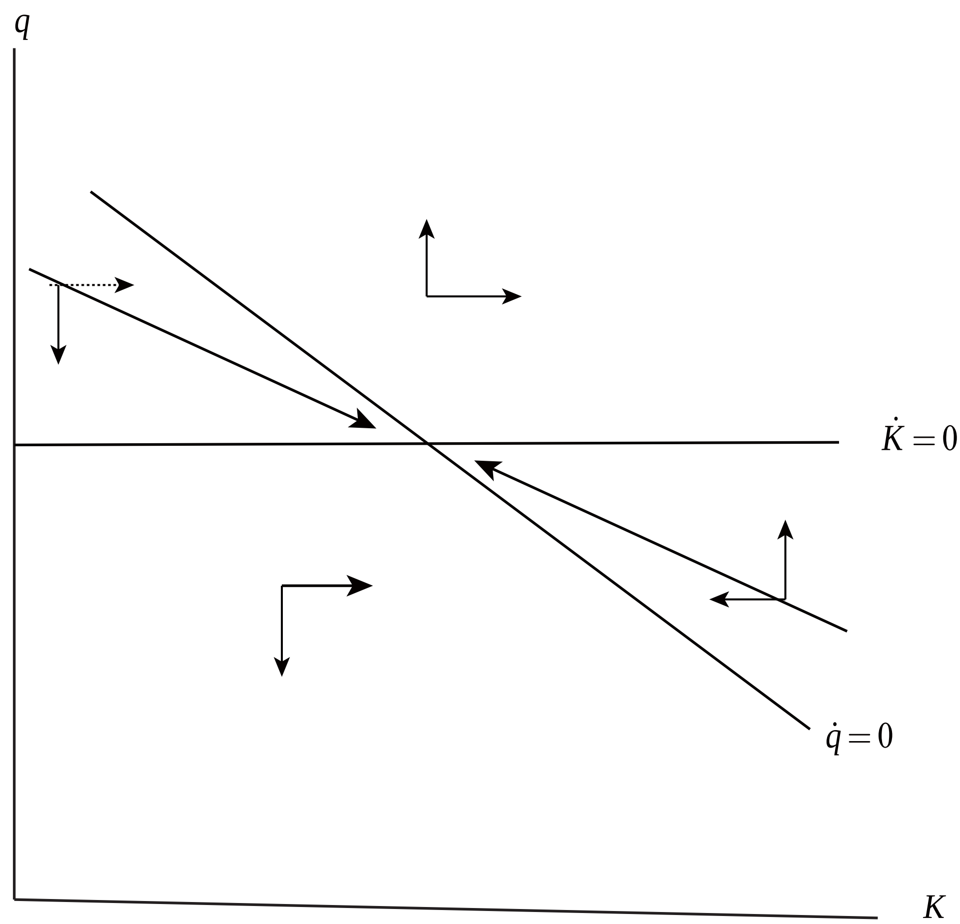

Figure 2 (b) : $(1-\phi)\beta>1+\gamma$ or $(1-\phi)\beta<[1-(1-\phi)\alpha](1+\gamma)$

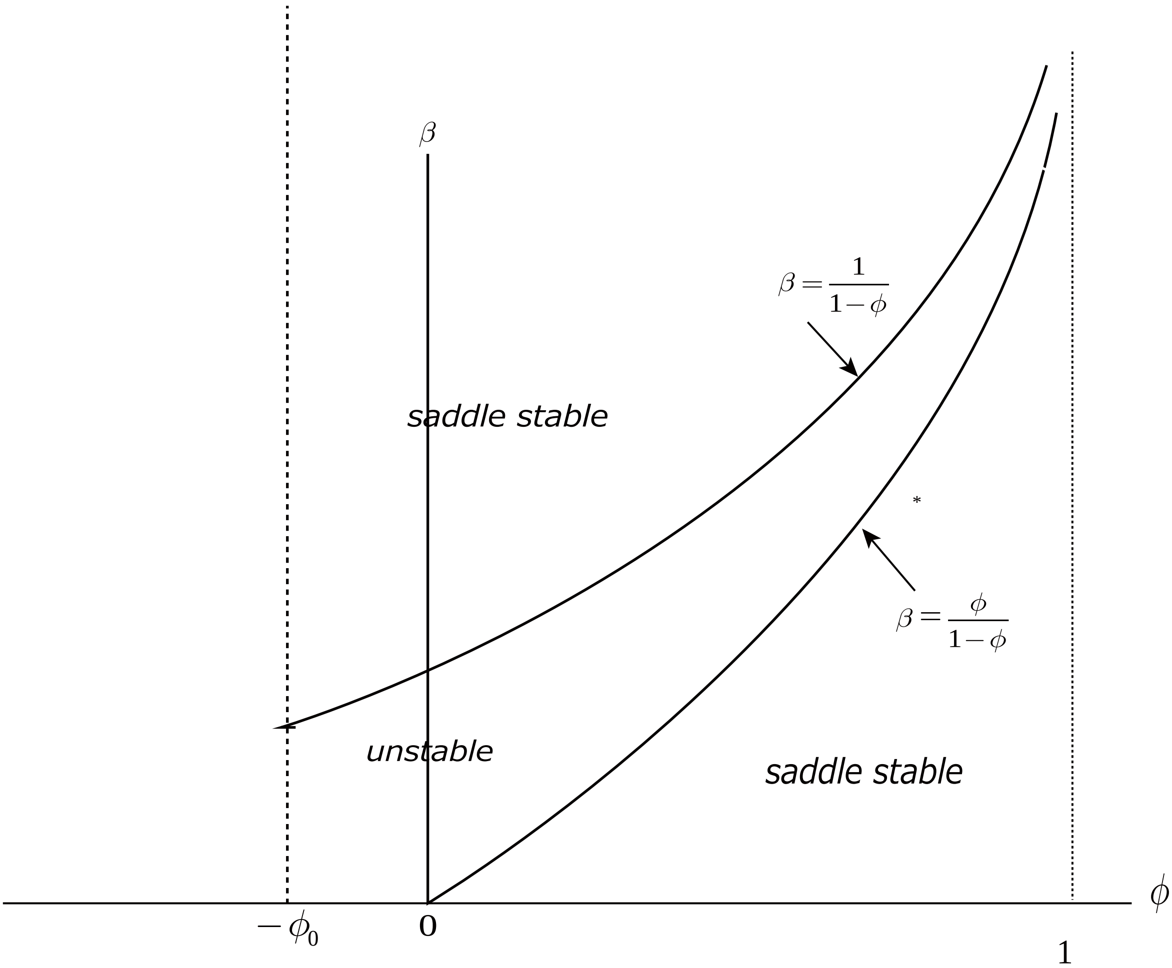

Figure 3: Classification of (β, ϕ) space under $Y^*_t = Y_t \quad (\gamma = 0)$

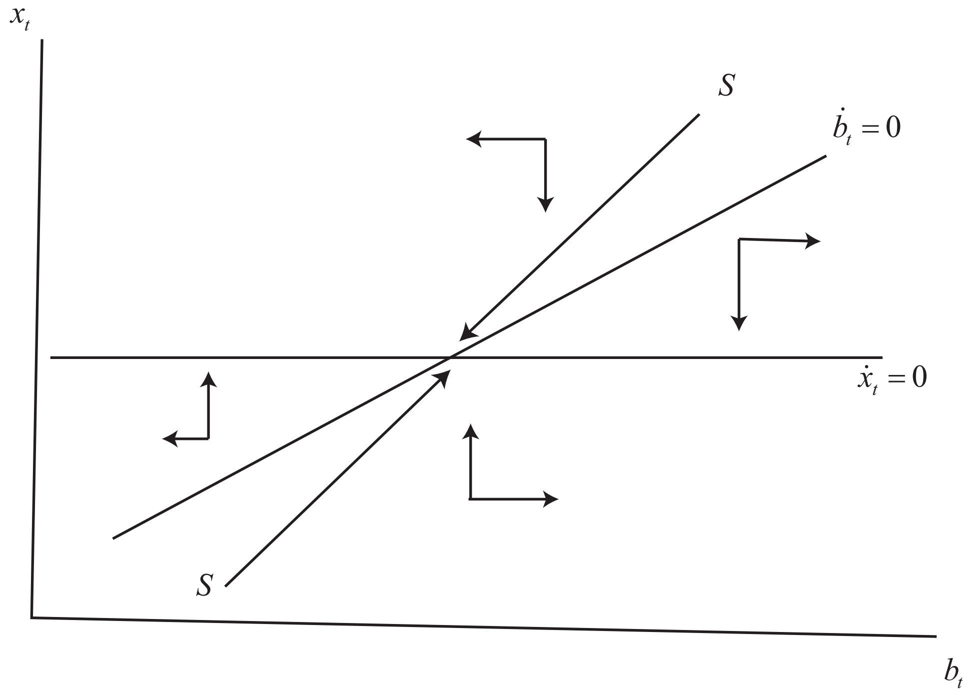

Figure 4: Stable saddle paths towards the steady state

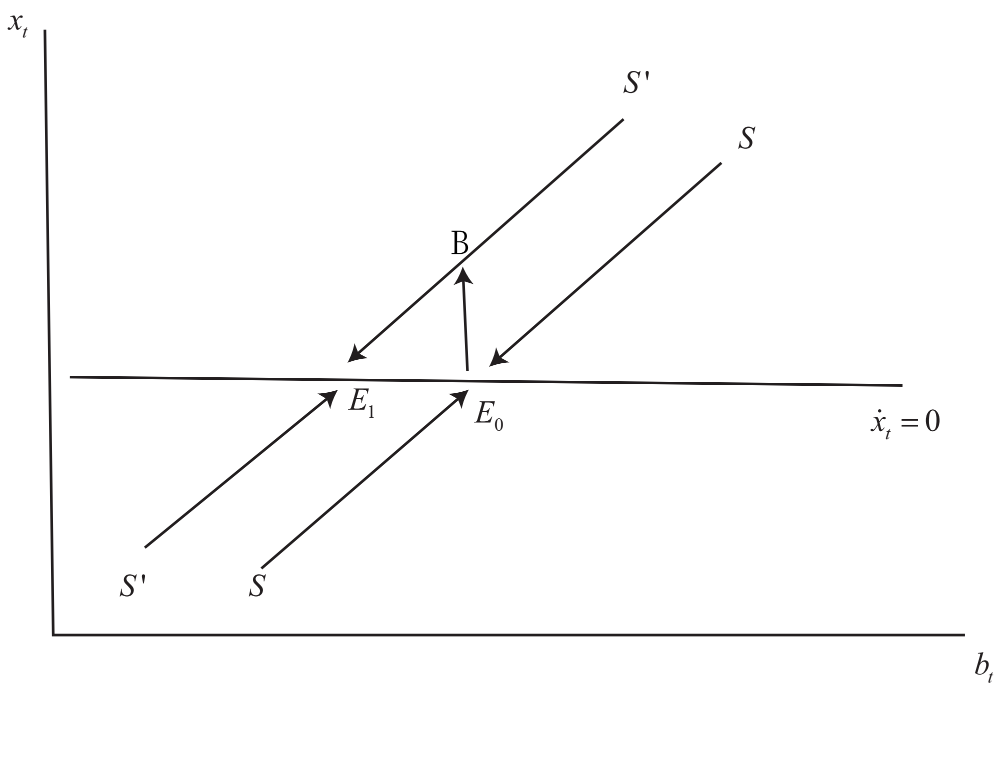

Figure 5: The effect of a sunspot shock

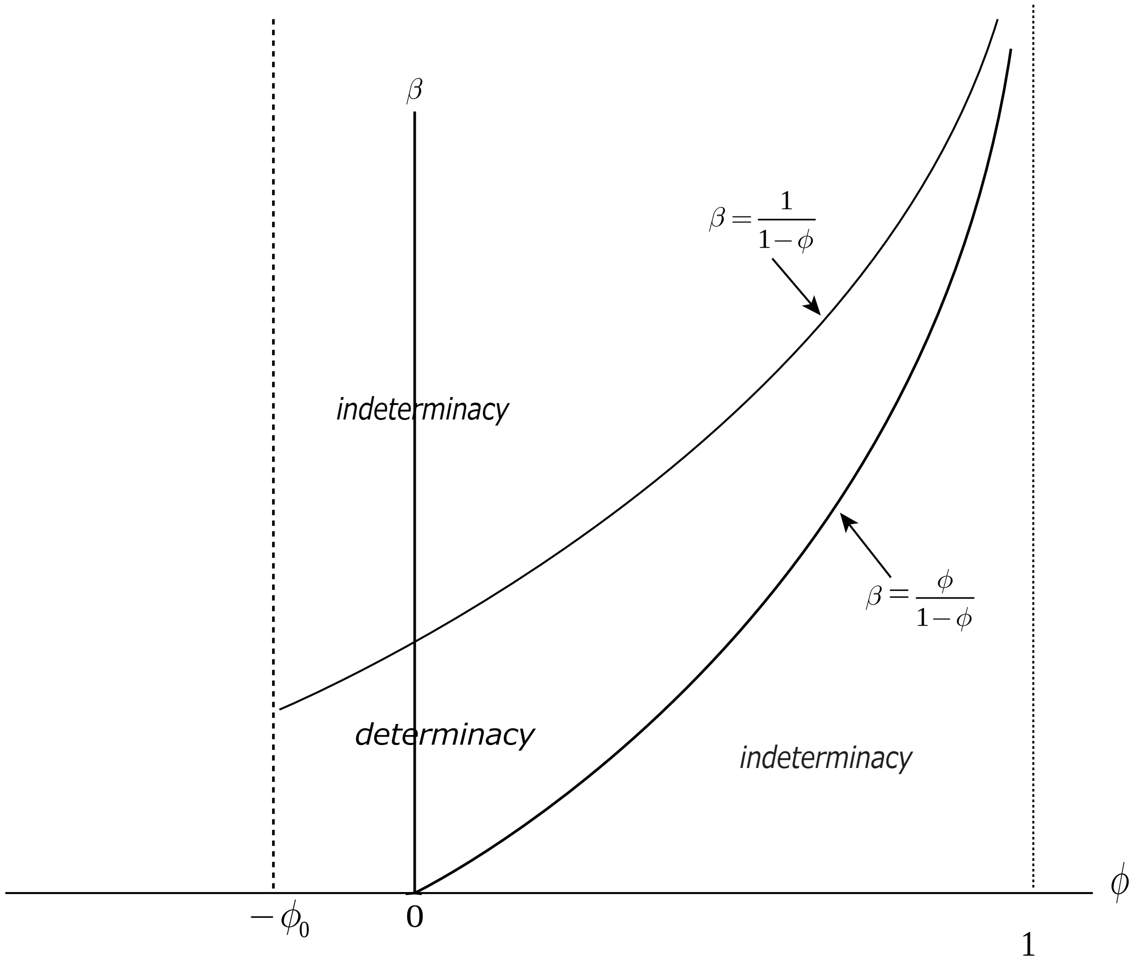

Figure 6: Classification of (β, ϕ) space under $Y^*_t \neq Y_t$ $(\gamma = 0)$