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<th>Pathological quotient singularities which are not log canonical in positive characteristic</th>
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<tr>
<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>代数幾何学シンポジウム記録 2018: 152-152</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2018</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/236420">http://hdl.handle.net/2433/236420</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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Kyoto University
Pathological quotient singularities which are not log canonical in positive
class characteristic

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Abstract
In characteristic zero, quotient singularities are log terminal. Moreover, we can check whether a quotient variety is canonical
or not by using only the cyclic subgroups of the relevant finite group if the group does not have pseudo-reflections. In positive
class characteristic, a quotient variety is not log terminal, in general. In this paper, we give an example whose singularity cannot
be determined by looking at proper subgroups.

1 Introduction
Quotient singularities form one of the most basic classes of sin-
gularities. They behave well in characteristic zero. In charac-
teristic zero, any quotient variety has log terminal singularities.
Moreover, for a finite group $G \subset \text{GL}(d,\mathbb{C})$ without pseudo-
reflection, we say that $G$ is small, we can use Reid-Shepherd-
Barron-Tai criterion [2, Theorem 3.21] to know the singularity
of $\mathbb{C}^d/G$. Namely, the following two conditions are equivalent.

$\bullet$ $\mathbb{C}^d/G$ is canonical (resp. terminal).

$\bullet$ $\mathbb{C}^d/C$ is canonical (resp. terminal) for all cyclic subgroup $C$ of $G$.

In positive characteristic, if the given finite group is tame, then the quotient variety is again log terminal and we can use
the Reid-Shepherd-Barron-Tai criterion.

2 Main Theorem
If the group is wild, there exists a quotient variety which is not
log terminal. We give an even more pathological example.

Theorem (Main Theorem). Let $C_3$ be the cyclic group
of order three and $C_2^3$ be the product of two copies of it.
Suppose that the group $C_3^2$ is embedded in $\text{SL}(3,K)$ and this
embedding makes $C_3^2$ small, where $K$ is algebraically
closed field of characteristic three. Then the quotient vari-
ety $\mathbb{A}^3_K/C_3^2$ is not log canonical.

The pathological point of this example is that:

$\mathbb{A}^3_K/C$ is canonical for all cyclic subgroup $C \subset C_3^2$
because $\# C = 3$ for any cyclic subgroup $C$, see [3]. But
$\mathbb{A}^3_K/C_3^2$ is not log canonical.

This is in contrast to the fact that, in characteristic zero, we
can use the Reid-Shepherd-Barron-Tai criterion.

We give the proof of the main theorem by explicit compu-
tation. The all small action of $C_3^2$ are given in [1]. The actions
are parametrized by $a \in K \setminus \mathbb{F}_3$, $b \in K$. Then, we can compute
the explicit form of the quotient varieties $X$ for each action of
$C_3^2$. We will find that the quotient varieties are classified in two
types separating by whether $b = 0$ or not about the parame-
ter $b$ of the action. Finally, we construct the proper birational
morphism $Y \to X$ with exceptional divisors which discrepancy
is smaller than $-1$, which shows the quotient varieties are not
log canonical. This construction given by a few times blow up
along the singular loci in each case.

3 Application
As an application of the main result, we give a criterion when
a quotient variety associated to a small wild finite group is log
terminal in dimension three and characteristic three. Accord-
ing to the criterion, we can judge the singularity of a quotient
variety by seeing the order of the acting group.

Corollary. Let $G$ be a wild small finite group of $\text{GL}(3,K)$
where $K$ is an algebraically closed field. We write $\#G = 3^n$ where $r, n$
are positive integer and $n$ is not divided by three.

(i) If $r = 1$ then $\mathbb{A}^3_K/G$ is log terminal.

(ii) If $r \geq 2$ then $\mathbb{A}^3_K/G$ is not log canonical.

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